task_tb6pi8dgh0m2e2pw_with_calculation

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Charging Capacitors, dc network analysis, pure resistor network simplification, delta wye transformation, exam ee1 WS2022

Exercise E1 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

4. Now, consider the light bulb as a resistor R_B when the switch S_2 is closed. The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and $R_\rm B$.

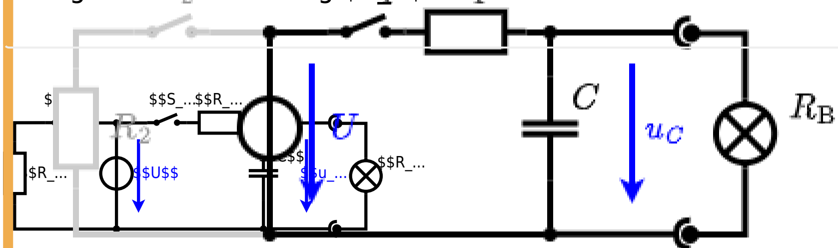
Result

\begin{align*} \Delta W_R = 3.792 ~\rm{mWs} \end{align*}

Solution

\begin{align*} \Delta W_R &= \Delta W_C \\ &= {{1}\over{2}} \cdot C \cdot U^2 \\ &= {{1}\over{2}} \cdot 100 ~\rm{µF} \cdot (12 ~\rm{V})^2 \end{align*}

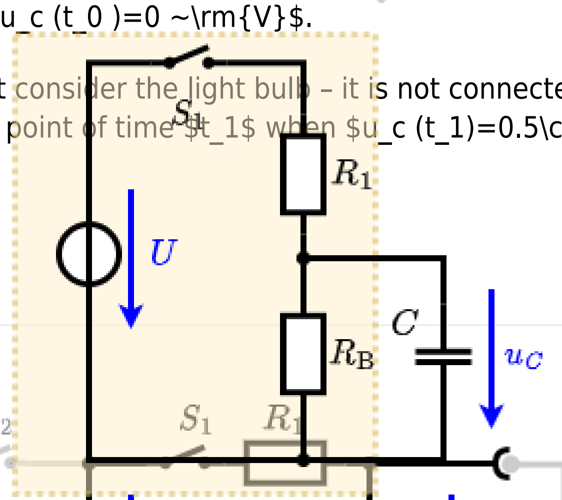
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2.

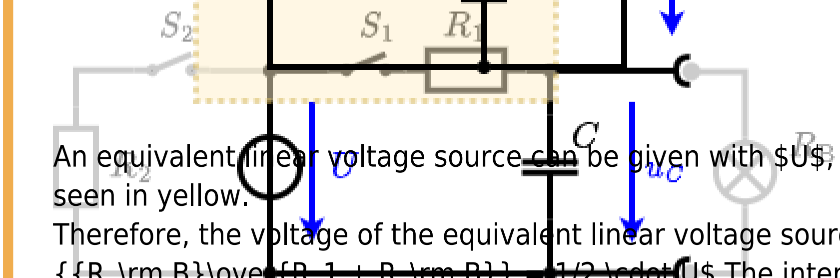


The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution





An equivalent linear voltage source can be given with U, R_1, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit).

So, here only R_1 and C gives the time constant $\tau = R_1 \cdot C$ \begin{align*} R_i &= R_1 || R_m || R \\ &= 10 ~\Omega \end{align*}

The following formula describes the time course of $U_C(t)$, which has to be set $U_C(t)=0.5 \cdot U_1$: \begin{align*} U_2 &= U_1 \cdot (1 - e^{t/(R_1 \cdot C)}) \\ &= {{1}\over{2}} \cdot U_1 \\ U_C(t) = U_1 \cdot (1 - e^{t/\tau}) = 0.5 \cdot U \\ 10 ~\Omega \cdot 100 ~\mu F \end{align*}
\end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}