task_7el8zlzjglaazxtw_with_calculation

Student Group

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resonant circuit, exam ee2 SS2022

Exercise E13 Series Resonant Circuit
(written test, approx. 10 % of a 120-minute written test, SS2022)

A real capacitor can be modeled as an ideal capacitance C with an equivalent series resistance of $R=38.6 ~\rm m\Omega$ and an equivalent series inductance L.
At a certain frequency f_0 the impedance of the capacitor is purely resistive.
This case shall be given with the following values:

- $C=10 ~\rm nF$
- $L=60 ~\rm pH$

The resonance frequency is given as $f_{\rm r} = {{1}\over{2\pi\sqrt{LC}}}$

1. What is the impedance $\underline{Z}_{RLC}$ of this real capacitor for $f_0=100 ~\rm MHz$? (Phase and magnitude)

Path

The impedance $\underline{Z}_{RLC}$ is given by: $$\begin{aligned} \underline{Z}_{RLC} &= R + \underline{X}_{L} + \underline{X}_{C} \\ &= R + {\rm j}\omega L - {{\rm j}\over{\omega C}} \\ &= R + {\rm j}\cdot \left(\omega L - {{1}\over{\omega C}} \right)\\ &= R + {\rm j}\cdot {X}_{LC} \\ \end{aligned}$$

Putting in the numbers, only for the reactive part ${X}_{LC}$: $$\begin{aligned} {X}_{LC} &= 2\pi\cdot \quad \quad f_0 \quad \quad \cdot \quad L \quad \quad \quad \; - {{1}\over{2\pi \cdot \quad \quad f_0 \quad \quad \cdot \quad \quad C \quad\quad}} \\ &= 2\pi \cdot 100 \cdot 10^{6}{~\rm Hz} \cdot 60 \cdot 10^{-12}{~\rm H} - {{1}\over{2\pi \cdot 100 \cdot 10^{6}{~\rm Hz} \cdot 10 \cdot 10^{-9}{~\rm F}}} \\ &= -121.45... ~\rm m\Omega \\ \end{aligned}$$

With the real and imaginary parts, we can derive the magnitude and phase:
$$\begin{align*} Z_{RLC} &=\sqrt{R^2 + {X}_{LC}^2} \\ &=\sqrt{(88 ~\rm m\Omega)^2 + (-121.45 ~\rm m\Omega)^2} \\ &= 150.0... ~\rm m\Omega \\ \end{align*}$$

$$\begin{align*} \varphi &=\arctan \left({ {{X}_{LC}}\over{R}} \right)\\ &=\arctan \left({ {{-121.45 ~\rm m\Omega}\over{88 ~\rm m\Omega}}} \right)\\ &= -0.9437... = -54.07...° \\ \end{align*}$$