task_k4wrrhf8v46gct49_with_calculation

Student Group

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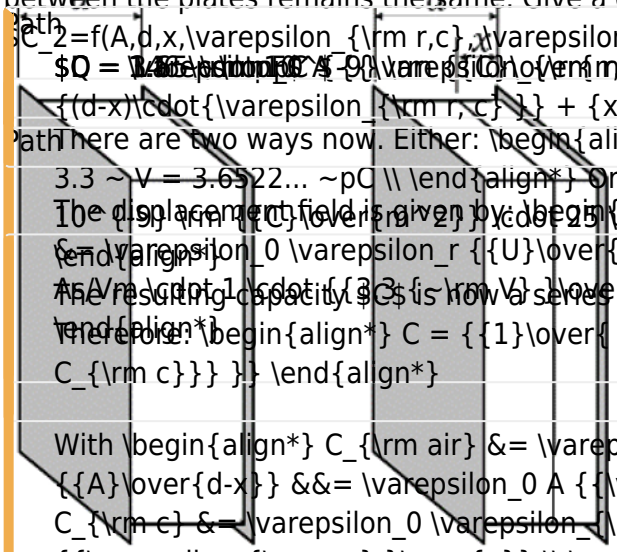
electrostatic, capacitor, plate capacitor, capacity, exam ee2 SS2024

Exercise E1 Capacitor
(written test, approx. 12 % of a 120-minute written test, SS2024)



The contaminant has $\varepsilon_{\rm r,c}>\varepsilon_{\rm r,air}$, while the distance between the plates remains the same. Give a generalized formula $C_2=f(A,d,x,\varepsilon_{\rm r,c},\varepsilon_{\rm r,air})$.

There are two ways now. Either: $$\begin{aligned} Q &= C \cdot U = 1.1 ~\rm pF \cdot 3.3 ~V = 3.6522... ~pC \\ \end{aligned}$$ Or: $$\begin{aligned} Q &= D \cdot A = 146 \cdot 10^{-9} ~\rm {C}\over{m^2}} \cdot 25 \cdot 10^{-6} ~m^2 = 3.6522... ~pC \\ \end{aligned}$$

The displacement field is given by: $$\begin{aligned} D &= \varepsilon_0 \varepsilon_r E \\ &= \varepsilon_0 \varepsilon_r {{U}\over{d}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 1 \cdot {{3.3 ~\rm V} }\over{200 \cdot 10^{-6} {~\rm m} }} \\ \end{aligned}$$

The resulting capacity C is now a series circuit of $C_{\rm air}$ and $C_{\rm c}$. Therefore: $$\begin{aligned} C = {{1}\over{ {{1}\over{C_{\rm air} }} + {{1}\over{ C_{\rm c}}} }} \end{aligned}$$

With $$\begin{aligned} C_{\rm air} &= \varepsilon_0 \varepsilon_{\rm r, air} {{A}\over{d-x}} &&= \varepsilon_0 A {{\varepsilon_{\rm r, air}}\over{d-x}} \\ C_{\rm c} &= \varepsilon_0 \varepsilon_{\rm r, c} {{A}\over{x}} &&= \varepsilon_0 A {{\varepsilon_{\rm r, c} }\over{x}} \\ \end{aligned}$$

This leads to: $$\begin{aligned} C = \varepsilon_0 A {{1}\over{ {{d-x}\over{\varepsilon_{\rm r, air} }} + {{x}\over{ \varepsilon_{\rm r, c}}} }} \end{aligned}$$

In the following such a sensor is given with:

- Plate area : $A=25 ~\rm mm^2$
- Distance between both plates: $d=200 ~\rm \mu m$
- Air between the plates: $\varepsilon_{\rm r,air}=1$
- Supply voltage: $3.3 ~\rm V$
- Boundary effects on the end of the layers shall be ignored in the following calculations.

$\varepsilon_{0} =8.854 \cdot 10^{-12} ~\rm F/m $

1. Calculate the capacity C.

Path

$$\begin{aligned} C &= \varepsilon_0 \varepsilon_r {{A}\over{d}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 1 \cdot {{25 \cdot 10^{-6} {~\rm m} }\over{200 \cdot 10^{-6} {~\rm m} }} \end{aligned}$$