

Exam Summer Semester 2023

Student Group

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Exam Summer Semester 2023

Additional permitted Aids

- non-programmable calculator,
- formulary (2 DIN A4 pages)

Hits

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Resistivity and temperature dependent Resistance (written test, approx. 7 % of a 60-minute written test, SS2023)

The resistivity of a dielectric material is given by the formula $\rho(T) = \rho_0 \exp\left(\frac{A}{T} + B T^2\right)$ where ρ_0 is a constant, A and B are material constants. The resistivity of the dielectric material is $\rho(20^\circ\text{C}) = 10^{17} \Omega\text{m}$ and $\rho(55^\circ\text{C}) = 80 \Omega\text{m}$. The temperature coefficients in the range 20°C to 55°C are given as $\alpha = -0.048 \text{ K}^{-1}$ and $\beta = +0.00057 \text{ K}^{-2}$. Calculate the resistivity of the dielectric material at $T = 35^\circ\text{C}$.

Solution
The resistivity of the dielectric material is $\rho(T) = \rho_0 \exp\left(\frac{A}{T} + B T^2\right)$.

For the given material the temperature coefficients in the range 20°C to 55°C are given as $\alpha = -0.048 \text{ K}^{-1}$ and $\beta = +0.00057 \text{ K}^{-2}$.

$$R(55^\circ\text{C}) = R(20^\circ\text{C}) \cdot (1 + \alpha \Delta T + \beta \Delta T^2 + \dots)$$

$$80 \Omega = 10^{17} \Omega \cdot (1 - 0.048 \text{ K}^{-1} \cdot (35 - 20) \text{ K} + 0.00057 \text{ K}^{-2} \cdot (35 - 20)^2 \text{ K}^2)$$

Calculate the resistance for the dielectric material for $20 \text{ }^\circ\text{C}$.

Solution

$$\begin{aligned} R(20 \text{ }^\circ\text{C}) &= \rho \cdot \frac{d}{A} \\ &= 10^{17} \text{ } \Omega \cdot \frac{0.8 \cdot 10^{-6} \text{ m}}{1 \text{ m}^2} \end{aligned}$$

Exercise E1 Resistivity and temperature dependent Resistance (written test, approx. 7 % of a 60-minute written test, SS2023)

The resistivity of a dielectric material is given by the Arrhenius equation: $\rho = \rho_0 \cdot \exp\left(\frac{E_a}{k_B T}\right)$. The activation energy E_a is 0.5 eV . Calculate the resistance R of a dielectric material with a thickness $d = 0.8 \text{ } \mu\text{m}$ and a cross-sectional area $A = 1 \text{ cm}^2$ at $T = 300 \text{ K}$ and $T = 400 \text{ K}$. The resistance at $T = 300 \text{ K}$ is $R_0 = 10^{17} \text{ } \Omega$.

Solution: The plates of \$d=0\$ and \$100\text{ }\mu\text{m}\$ (rm °C\$)

The resistivity of the dielectric material is \$10^{-6}\text{ }\Omega\text{m}\$ (rm °C\$) and \$10^{17}\text{ }\Omega\text{m}\$ (rm °C\$).

For the given material the temperature coefficients in the range of \$20\text{ }\text{ }^\circ\text{C}\$ and \$55\text{ }\text{ }^\circ\text{C}\$ are given as

\$\alpha = 48(20\text{ }\text{ }^\circ\text{C})\$ and \$\beta = 10^{-5}\text{ }^\circ\text{C}^{-2}\$.

\$\rho(T) = \rho_0(1 + \alpha(T - T_0) + \frac{\beta}{2}(T - T_0)^2)\$
 \$\rho_0 = 10^{-6}\text{ }\Omega\text{m}\$, \$T_0 = 20\text{ }\text{ }^\circ\text{C}\$, \$\alpha = 48\text{ }^\circ\text{C}^{-1}\$, \$\beta = 10^{-5}\text{ }^\circ\text{C}^{-2}\$
 \$\rho(55\text{ }\text{ }^\circ\text{C}) = 10^{-6}(1 + 48(55 - 20) + \frac{10^{-5}}{2}(55 - 20)^2)\$

Calculate the resistance for the dielectric material for \$20\text{ }\text{ }^\circ\text{C}\$.

Solution

\$R(20\text{ }\text{ }^\circ\text{C}) = \rho \cdot \frac{d}{A} = 10^{-6} \cdot \frac{100 \cdot 10^{-6}}{1 \cdot 10^{-2}} = 10^{17}\text{ }\Omega\$

Calculate the two impedances Z_L and Z_C and the resulting phase shift between the voltage $\underline{U}$ and the current $\underline{I}$. Choose an appropriate scaling factor and write it down.

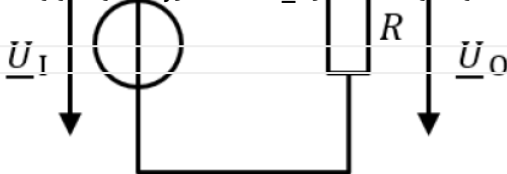
- $R = 1.1 \text{ k}\Omega$

Solution $L = 3.5 \text{ mH}$

Result $\underline{Z}_L = j \omega L = j 2\pi \cdot 150 \text{ kHz} \cdot 3.5 \text{ mH} = j 3.3 \text{ }\Omega$

$$\underline{U} = 0.5 \text{ V} - j \cdot 1.5 \text{ V}$$

At the cutoff frequency the absolute values of impedances $|Z_L|$ is equal to $|Z_C|$. This leads to $\omega L = \frac{1}{\omega C}$ and $\omega = \frac{1}{L C}$.
 $\omega = \frac{1}{3.5 \text{ mH} \cdot 10 \text{ nF}} = 28571.4 \text{ rad/s}$
 $f = \frac{\omega}{2\pi} = 4547.4 \text{ Hz}$



.. Calculate the impedance $\underline{Z}_L$.

Solution

$$\underline{Z}_L = j \omega L = j 2\pi \cdot 150 \text{ kHz} \cdot 3.5 \text{ mH}$$

Exercise E2 Pure Resistor Network Simplification (written test, approx. 12 % of a 60-minute written test, SS2023)

Calculate the voltage U_K , when switch S is closed.

Result

The values in the circuit are

Solution

- $R_1 = 60 \text{ }\Omega$

• $R_2 = 40 \text{ }\Omega$

$$R_3 = 40 \text{ }\Omega$$

The voltage divider for node K has the same proportionality as the voltage

divider for node A . Therefore, the potential of K is the same as for A .

There will be no current flow through R_3 . The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node K , when switch S is open.
It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch S is opened, the upper part is a parallel circuit. Therefore, R_{eq} is given as:

$$R_{\text{eq}} = (R_1 + R_2) \parallel (R_1 + R_2) + R_4 \parallel \frac{1}{2} \cdot (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (60 \, \Omega + 40 \, \Omega) + 100 \, \Omega$$

Exercise E1 Pure Resistor Network Simplification I (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below shall be given as $U_{\text{AB}} = 60 \, \text{V}$. What is the value for I_{AB} in the circuit?

Solution

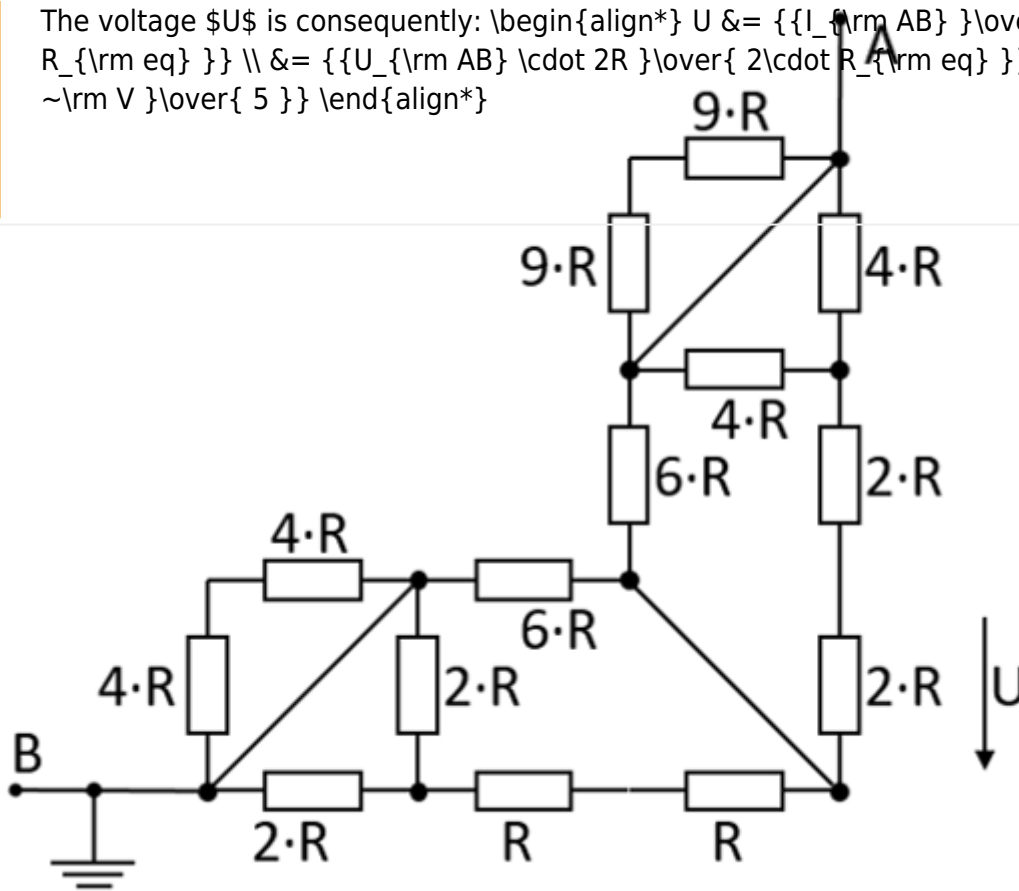
$$I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}} = \frac{60 \, \text{V}}{60 \, \Omega} = 1 \, \text{A}$$

The current through the circuit is given as $I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}}$.

This current has to flow in summary through parallel branches. The voltage U in question in the upper right branch given by $(4R \parallel 4R) + 2R + 2R$. Its resistance is just the same as the upper left branch $6R$.

Therefore, half of the current flows to the left half to the right side.

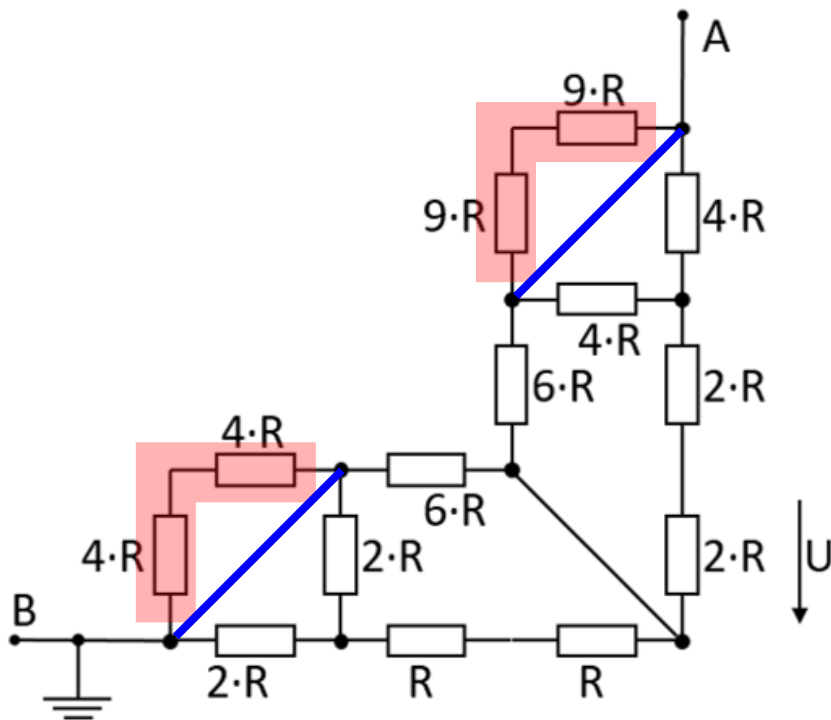
The voltage U is consequently:
$$U = \frac{I_{\text{arm AB}}}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{U_{\text{AB}} \cdot 2R}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{60 \text{ V}}{5}$$



1. What is the equivalent resistance R_{eq} ?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

Therefore, R_{eq} is given as:
$$R_{\text{eq}} = (2R \parallel 2R + R +$$

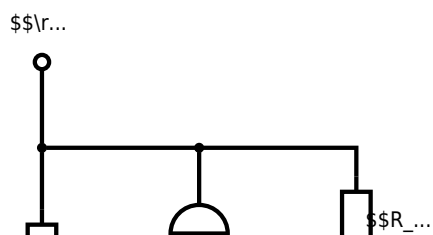
$$R) \parallel 6R \text{ \&\&+ } 6R \parallel (2R + 2R + 4R) \parallel 4R \text{ \&\&= } (R + R + R) \parallel 6R \text{ \&\&+ } 6R \parallel (2R + 2R + 2R) \text{ \&\&= } 3R \parallel 6R \text{ \&\&+ } 6R \parallel 6R \text{ \&\&= } \left\{ \frac{3R \cdot 6R}{3R + 6R} \right\} \text{ \&\&+ } 3R \text{ \&\& \end{align*}}$$

Exercise E3 Equivalent Linear Source (written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification.

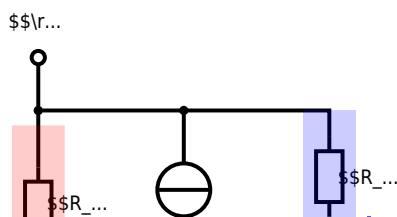
Calculate the internal resistance R_{i} and the source voltage U_{s} of an equivalent linear voltage source.

- $R_1 = 5 \text{ } \Omega$
 - $U_1 = 10 \text{ V}$
 - $R_2 = 5 \text{ } \Omega$
 - $I_3 = 0.5 \text{ A}$
 - $R_4 = 10 \text{ } \Omega$
 - $U_5 = 4 \text{ V}$
- Result:
 $U_{\text{AB}} = 1.11 \text{ V}$
 $R_{\text{i}} = 5.55 \text{ } \Omega$



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = \frac{U_x}{R_x}$

$$\begin{aligned} I_0 &= \frac{U_0}{R_1} = \frac{10 \text{ V}}{5 \text{ } \Omega} = 2 \text{ A} \\ I_5 &= \frac{U_5}{R_4} = \frac{4 \text{ V}}{10 \text{ } \Omega} = 0.4 \text{ A} \end{aligned}$$

I_3 and I_0 can be combined to $I_{03} = I_0 - I_3$ facing upwards:

$$I_{03} = 1.5 \text{ A}$$

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03} = R_1 \cdot I_{03}$ facing down.

$$U_{03} = 7.5 \text{ V}$$

Then, the resistors R_1 and R_2 can be combined to $R_{12} = R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12} . The current will be $I_{0123} = \frac{U_{03}}{R_{12}}$, facing up again.

$$I_{0123} = 0.6 \text{ A}$$

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235} = I_{0123} - I_5$ and the resistors as $R_{124} = R_{12} || R_4$.

$$I_{01235} = 0.2 \text{ A} \quad R_{124} = 5.55 \text{ } \Omega$$

The final step is the back-transformation to a linear voltage source, with $U_{\text{AB}} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.

Here, there is also a full final formula given:

$$\begin{aligned} U_{\text{AB}} &= U_{\text{AB}} = I_{01235} \cdot R_{124} \quad \&= (I_{0123} - I_5) \cdot (R_{12} \parallel R_4) \quad \&= \left(\frac{U_{03}}{R_{12}} - I_5 \right) \cdot (R_{12} \parallel R_4) \\ &= \left(\frac{R_1 \cdot I_{03}}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \quad \&= \left(\frac{R_1 \cdot \left(\frac{U_0}{R_1} - I_3 \right)}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \end{aligned}$$

Exercise E4 (Dis)Charging Capacities

(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit shown in the drawing consists of a DC voltage source $U = 10 \text{ V}$, a capacitor $C = 200 \text{ nF}$, and three resistors $R_1 = 8 \text{ k}\Omega$, $R_2 = 17 \text{ k}\Omega$, and $R_3 = 50 \text{ k}\Omega$. The switch S_1 is initially open and the capacitor is fully discharged. At $t = 0$, the switch S_1 is closed. What is the new time constant?

- $\tau = 200 \text{ nF}$

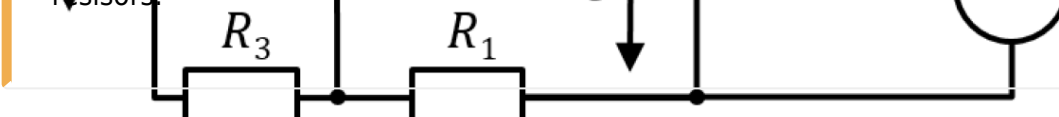
Solution: $\tau = 8.0 \text{ k}\Omega \cdot 200 \text{ nF}$

Solution: $\tau = 8.0 \text{ k}\Omega \cdot 200 \text{ nF} = 1.6 \text{ ms}$

$$\begin{aligned} U_C(t) &= U \cdot \left(1 - e^{-t/\tau} \right) \\ U_C(0.2 \text{ ms}) &= 10 \text{ V} \cdot \left(1 - e^{-0.2 \text{ ms} / 1.6 \text{ ms}} \right) \end{aligned}$$

Again, the time constant is given by the equivalent circuit consisting of C in parallel with $R_1 \parallel R_2$. Without the parallel resistors, the current source would charge the capacitor "to infinity" ($C \rightarrow \infty$). This is here limited by the parallel resistors R_1 and R_2 . When S_2 is closed, then, the source U drives the current through the series circuit given by U , C , R_1 and R_2 . Therefore, the voltage on the branch with the resistors ($R_1 \parallel R_2$) is $U_C = (8 \text{ k}\Omega + 17 \text{ k}\Omega) \cdot 0.2 \text{ ms} \cdot 10^{-3} \text{ F} = 25 \cdot 10^{-3} \text{ V} = 25 \text{ mV}$.

This is also the maximum voltage on the capacitor, since it is in parallel with the resistors.



Before t_0 all switches are switched as shown and the capacitor is fully discharged. At $t_0 = 0 \text{ s}$ the switch S_1 shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

Solution

The time constant is generally given as: $\tau = R \cdot C$

Once S_1 is closed and S_2 is open at t_0 , the source U drives the current through the series circuit given by S_1 , C , R_1 and R_3 .

Therefore, $R = R_1 + R_3$ $\tau_1 = (R_1 + R_3) \cdot C \quad \&= (8 \cdot 10^3 + 7 \cdot 10^3) \cdot 0.2 \cdot 10^{-6} \text{ s} = 15 \cdot 10^{-3} \cdot 0.2 \cdot 10^{-6} \text{ s} = 3 \cdot 10^{-6} \text{ s}$

⚡...

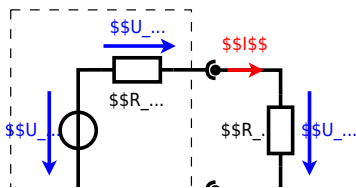
Solution

Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ($\propto (1 - e^{-x})$).

Exercise E1 Impedances at Frequencies

(written test, approx. 14 % of a 60-minute written test, SS2023)

At a high frequency with $C = 20 \text{ nF}$ following impedances Z_1 and Z_2 are given. The value of the



Exercise E5 Analyzing a Scope Plot (written test, approx. 12 % of a 60-minute written test, SS2023)

2. What is the phase difference between the voltage and current?

Result
The measured current curve shall be visible as a dashed line.

The continuous line shows the voltage.

Solution

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\begin{align*} f &= 15 \text{ kHz} \\ U_1 &= 1.67 \text{ V} \\ U_2 &= 1.06 \text{ V} \\ I_2 &= 0.35 \text{ A} \end{align*}
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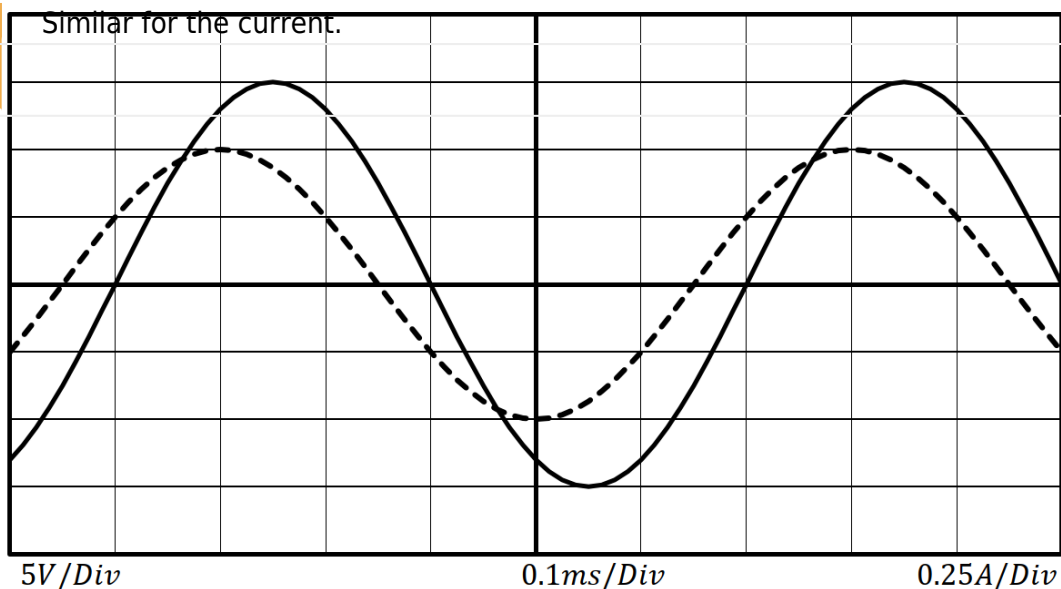
The phase difference between the voltage and current is given by the RMS value of the voltage and current.

A full period has an angle of 360° or 2π in radians.

One period has an angle of 360° or 2π in radians.

The phase difference between the voltage and current is given by the RMS value of the voltage and current.

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\begin{align*} \varphi &= \arctan\left(\frac{U_2}{I_2}\right) \end{align*}
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Use the correct symbols and units in your answers!

1. Calculate the frequency f of the periodic signals.

Solution

Frequency f is given by the period T . The period can be measured in the image of the scope.

1. The sine waves repeat after $6 \sim \text{rm divisions}$ (e.g. from falling turning point to falling turning point of one curve)
2. The scale is $0.1 \sim \text{rm ms/Div}$

$$\begin{aligned} f &= \frac{1}{T} \quad T = 6 \sim \text{rm Div} \cdot 0.1 \sim \text{ms/Div} \\ \rightarrow f &= \frac{1}{6 \sim \text{rm Div} \cdot 0.1 \sim \text{ms/Div}} \end{aligned}$$

Exercise E6 Complex voltage dividers

(written test, approx. 16 % of a 60-minute written test, SS2023)

1. Calculate the two simplified impedances Z_1 and Z_2 by resulting phase for the total impedance Z_{total} . Choose an appropriate scaling factor and write it down.

- $R = 1.1 \sim \text{rm k}\Omega$

Solution

Result: $\underline{U}_I = 5 \sim \text{rm V}$

$$\underline{Z}_1 = 1.1 \sim \text{rm k}\Omega \quad \underline{Z}_2 = 50 \sim \text{rm m}\Omega \quad \underline{Z}_{\text{total}} = 1.1 \sim \text{rm k}\Omega$$

$$\underline{U}_O = 0.5 \sim \text{rm V} - j \cdot 1.5 \sim \text{V}$$

At cut off frequency the absolute values of impedances $\underline{Z}_L$ is equal to $\frac{R}{\sqrt{1 - \frac{R^2}{Z_L^2}}}$. This leads to the complex voltage divider for the cut off frequency $\underline{U}_O = \underline{U}_I \cdot \frac{R}{R + j\omega L}$. The magnitude of the voltage divider is $|\underline{U}_O| = |\underline{U}_I| \cdot \frac{R}{\sqrt{R^2 + (\omega L)^2}}$. At the cut off frequency $\omega = \frac{1}{RC}$ the magnitude of the voltage divider is $|\underline{U}_O| = |\underline{U}_I| \cdot \frac{R}{\sqrt{R^2 + (\frac{1}{RC})^2}}$. The magnitude of the voltage divider is $|\underline{U}_O| = |\underline{U}_I| \cdot \frac{R}{\sqrt{R^2 + (\frac{1}{RC})^2}}$. The magnitude of the voltage divider is $|\underline{U}_O| = |\underline{U}_I| \cdot \frac{R}{\sqrt{R^2 + (\frac{1}{RC})^2}}$.



.. Calculate the impedance $\underline{Z}_L$.

Solution

$$\underline{Z}_L = j\omega L = j2\pi \cdot 150 \text{ kHz} \cdot 3.5 \text{ mH}$$

Exercise E2 Pure Resistor Network Simplification (written test, approx. 12 % of a 60-minute written test, SS2023)

Calculate the voltage across R_3 , when switch S is closed.

Result

The values in the circuit are

Solution

- $R_1 = 60 \text{ } \Omega$
- $R_2 = 40 \text{ } \Omega$
- $R_3 = 40 \text{ } \Omega$
- $R_4 = 10 \text{ } \Omega$
- The voltage divider for R_3 has the same proportionality as the voltage divider for R_1 . Therefore, the potential of R_3 is the same as for R_1 . There will be no current flow through R_3 . The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node K , when switch S is open.
It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch S is opened, the upper part is a parallel circuit. Therefore, R_{eq} is given as:

$$R_{\text{eq}} = (R_1 + R_2) \parallel (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (60 \, \Omega + 40 \, \Omega) + 100 \, \Omega$$

Exercise E3 Pure Resistor Network Simplification I (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below shall be given as $U_{\text{AB}} = 60 \, \text{V}$. What is the value for I_{AB} in the circuit?

Solution

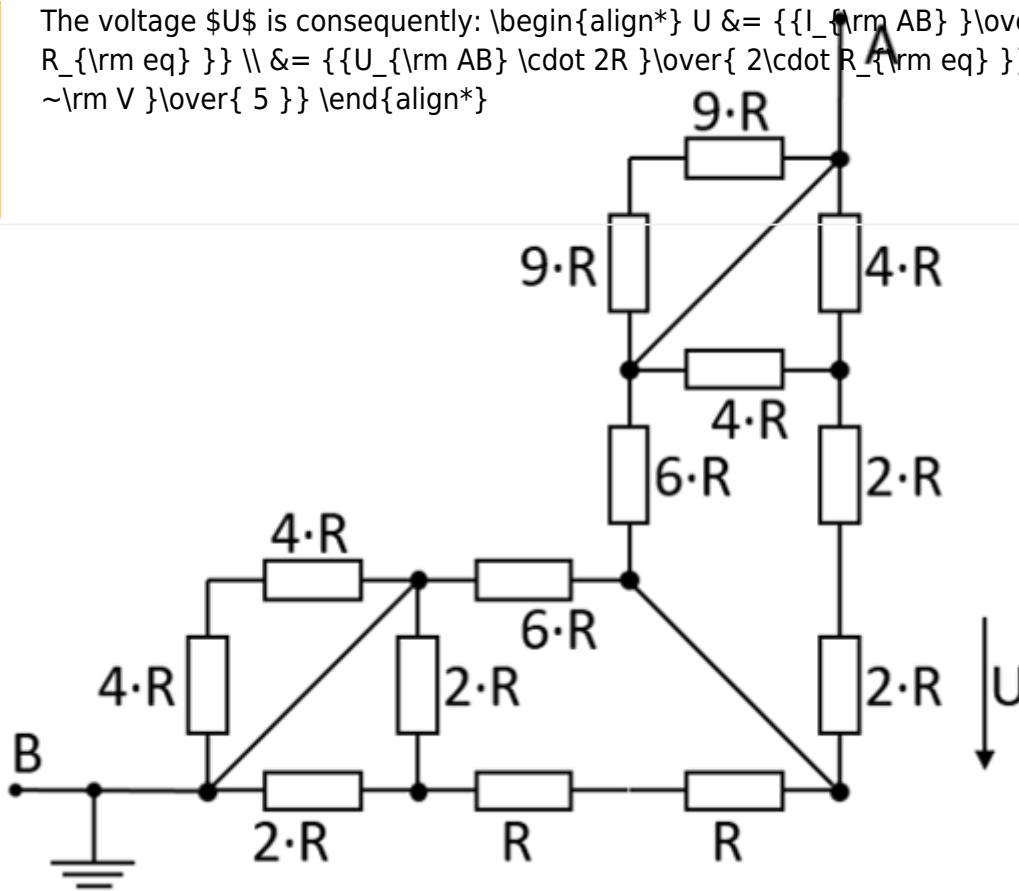
$$I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}} = \frac{60 \, \text{V}}{60 \, \Omega} = 1 \, \text{A}$$

The current through the circuit is given as $I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}}$.

This current has to flow in summary through parallel branches. The voltage U in question in the upper right branch given by $(4R \parallel 4R) + 2R + 2R$. Its resistance is just the same as the upper left branch $6R$.

Therefore, half of the current flows to the left half to the right side.

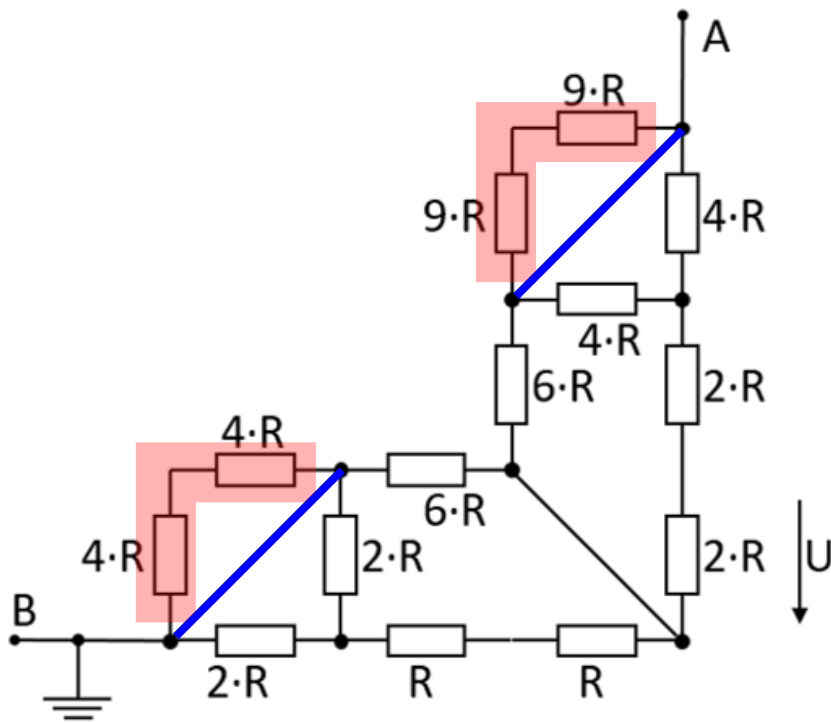
The voltage U is consequently:
$$U = \frac{I_{\text{arm AB}}}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{U_{\text{AB}} \cdot 2R}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{60 \text{ V}}{5}$$



1. What is the equivalent resistance R_{eq} ?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

Therefore, R_{eq} is given as:
$$R_{\text{eq}} = (2R \parallel 2R + R +$$

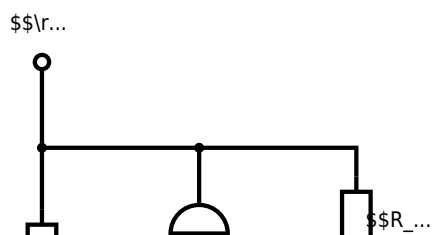
$$R) \parallel 6R \text{ \&\&+ } 6R \parallel (2R + 2R + 4R) \parallel 4R \text{ \&\&= } (R + R + R) \parallel 6R \text{ \&\&+ } 6R \parallel (2R + 2R + 2R) \text{ \&\&= } 3R \parallel 6R \text{ \&\&+ } 6R \parallel 6R \text{ \&\&= } \left\{ \frac{3R \cdot 6R}{3R + 6R} \right\} \text{ \&\&+ } 3R \text{ \&\& \end{align*}}$$

Exercise E4 Equivalent Linear Source (written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification.

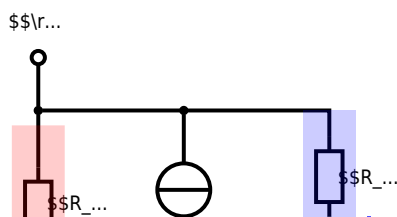
Calculate the internal resistance R_{i} and the source voltage U_{s} of an equivalent linear voltage source.

- $R_1 = 5 \text{ } \Omega$
 - $U_1 = 10 \text{ V}$
 - $R_2 = 5 \text{ } \Omega$
 - $I_3 = 0.5 \text{ A}$
 - $R_4 = 10 \text{ } \Omega$
 - $U_5 = 4 \text{ V}$
- Result:
 $U_{\text{AB}} = 1.11 \text{ V}$
 $R_{\text{i}} = 5.55 \text{ } \Omega$



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = \frac{U_x}{R_x}$

$$\begin{aligned} I_0 &= \frac{U_0}{R_1} = \frac{10 \text{ V}}{5 \text{ } \Omega} = 2 \text{ A} \\ I_5 &= \frac{U_5}{R_4} = \frac{4 \text{ V}}{10 \text{ } \Omega} = 0.4 \text{ A} \end{aligned}$$

I_3 and I_0 can be combined to $I_{03} = I_0 - I_3$ facing upwards:

$$I_{03} = 1.5 \text{ A}$$

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03} = R_1 \cdot I_{03}$ facing down.

$$U_{03} = 7.5 \text{ V}$$

Then, the resistors R_1 and R_2 can be combined to $R_{12} = R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12} . The current will be $I_{0123} = \frac{U_{03}}{R_{12}}$, facing up again.

$$I_{0123} = 0.6 \text{ A}$$

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235} = I_{0123} - I_5$ and the resistors as $R_{124} = R_{12} || R_4$.

$$I_{01235} = 0.2 \text{ A} \quad R_{124} = 5.55 \text{ } \Omega$$

The final step is the back-transformation to a linear voltage source, with $U_{\text{AB}} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.

Here, there is also a full final formula given:

$$\begin{aligned} U_{\text{AB}} &= U_{\text{AB}} = I_{01235} \cdot R_{124} \quad \&= (I_{0123} - I_5) \cdot (R_{12} \parallel R_4) \quad \&= \left(\frac{U_{03}}{R_{12}} - I_5 \right) \cdot (R_{12} \parallel R_4) \\ &= \left(\frac{R_1 \cdot I_{03}}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \quad \&= \left(\frac{R_1 \cdot \left(\frac{U_0}{R_1} - I_3 \right)}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \end{aligned}$$

Exercise E7 (Dis)Charging Capacities

(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit shown in the drawing consists of a DC voltage source $U = 10 \text{ V}$, a capacitor $C = 200 \text{ nF}$, and three resistors $R_1 = 8.0 \text{ k}\Omega$, $R_2 = 17 \text{ k}\Omega$, and $R_3 = 50 \text{ k}\Omega$. The switch S_1 is initially open and the capacitor is fully discharged. At $t = 0$, the switch S_1 is closed. What is the new time constant?

- $\tau = 10 \text{ ns}$

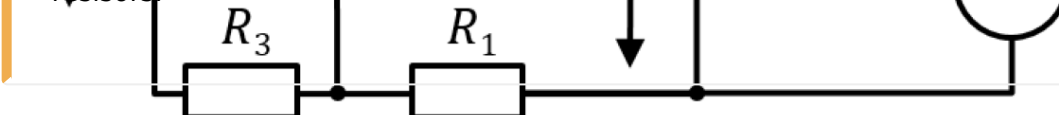
Solution: $\tau = 8.0 \text{ ns}$

Solution: $\tau = 8.0 \text{ ns}$

$$\begin{aligned} U_{\text{C}}(t) &= U_0 \cdot (1 - e^{-t/\tau}) \\ U_{\text{C}}(t) &= 10 \text{ V} \cdot (1 - e^{-t/8.0 \text{ ns}}) \end{aligned}$$

Again, the time constant is given by the equivalent circuit consisting of C in parallel with $R_1 \parallel R_2$. Without the parallel resistors, the current source would charge the capacitor "to infinity" ($C \rightarrow \infty$). This is here limited by the parallel resistors R_1 and R_2 . When S_2 is closed, then, the source U drives the current through the series circuit given by U , C , R_1 and R_2 . Therefore, the voltage on the branch with the resistors ($R_1 \parallel R_2$) is $U_{\text{C}} = (8 \text{ k}\Omega + 17 \text{ k}\Omega) \cdot 0.2 \text{ }\mu\text{F} = 25 \cdot 10^{-3} \cdot 0.2 \cdot 10^{-6} \text{ s} = 5 \text{ ns}$.

This is also the maximum voltage on the capacitor, since it is in parallel with the resistors.



Before t_0 all switches are switched as shown and the capacitor is fully discharged. At $t_0 = 0 \text{ s}$ the switch S_1 shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

Solution

The time constant is generally given as: $\tau = R \cdot C$

Once S_1 is closed and S_2 is open at t_0 , the source U drives the current through the series circuit given by S_1 , C , R_1 and R_3 .

Therefore, $R = R_1 + R_3$ $\tau_1 = (R_1 + R_3) \cdot C \quad \&= (8 \cdot 10^3 + 7 \cdot 10^3) \cdot 0.2 \cdot 10^{-6} \text{ s} = 15 \cdot 10^{-3} \cdot 0.2 \cdot 10^{-6} \text{ s} = 3 \cdot 10^{-6} \text{ s}$

⚡...

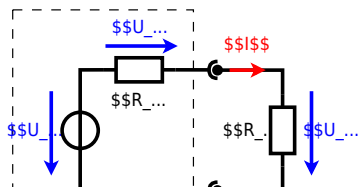
Solution

Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ($\propto (1 - e^{-x})$).

Exercise E8 Impedances at Frequencies

(written test, approx. 14 % of a 60-minute written test, SS2023)

At a high frequency with $C = 20 \text{ nF}$ following impedances Z_C and Z_L are given. Calculate the value of the



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