Exam Summer Semester 2023

Student Group

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Exercise E1 Resistivity and temperature dependent Resistance
(written test, approx. 7 % of a 60-minute written test, SS2023)

The resistance of ... $R(T)$ ... can be described by the ... exponential formula. In this case, ... $B25 = 15000$ for T between $20 ~\rm °C$ and $100 ~\rm °C$)

Calculate the resistance for the dielectric material for $55 ~\rm °C$.

Result

The dielectric of the given film capacitor has a surface of $A=100 ~\rm mm^2$ and a distance between the plates of $d=0.8 ~\rm µm$.

Solution

The resistivity of the dielectric material is $\rho_{\rm PP}(20 ~\rm °C)=10^{17} ~\Omega m$.

\begin{align*} R(55 ~\rm °C) &= 80.16 \rm G\Omega \end{align*}

For the given material the temperature coefficients in the range of $20 ~\rm °C$ and $55 ~\rm °C$ are given as $\alpha =-0.048 ~\rm 1/K$ and $\beta=+0.00057 ~\rm 1/K^2$.

\begin{align*} R(55 ~\rm °C) &= R(20 ~\rm °C) \cdot (1+\alpha\cdot\Delta T + \beta\cdot T^2 + ...)\\ &= 80 ~\rm G\Omega \cdot (1-0.048 ~\rm 1/K \cdot(35 ~{\rm K}) + 0.00057 ~\rm 1/K^2\cdot\Delta (35 ~{\rm K})^2) \end{align*}

Calculate the resistance for the dielectric material for $20 ~\rm °C$.

Solution

\begin{align*} R(20 ~\rm °C) &= \rho\cdot {{d}\over{A}}\\ &= 10^{17} ~\Omega

\rm m \cdot {{0.8\cdot 10^{-6} ~\rm m}\over{1 ~\rm m^2}} \end{align*}

Exercise E1 Resistivity and temperature dependent Resistance
(written test, approx. 7 % of a 60-minute written test, SS2023)

The given film capacitor has an internal surface of $A=100 ~\rm cm^2$ and a distance between the plates of $d=0.8 ~\rm µm$.

The resistivity of the dielectric material is $\rho_{\rm PP}(20 ~\rm °C)=10^{17} ~\Omega m$.

For the given material the temperature coefficients in the range of $20 ~\rm °C$ and $55 ~\rm °C$ are given as $\alpha =-0.048 ~\rm 1/K$ and $\beta=+0.00057 ~\rm 1/K^2$.

Calculate the resistance for the dielectric material for $55 ~\rm °C$.

$B25$ value in an exponential formula. In this case, the best fit would be $B25 = 15'000$ for T between $20 ~\rm °C$ and $100 ~\rm °C$)

Result

\begin{align*} R(55 ~\rm °C) &= 1.46 \rm G\Omega \end{align*}

Solution

\begin{align*} R(55 ~\rm °C) &= R(20 ~\rm °C) \cdot (1+\alpha\cdot\Delta T + \beta\cdot T^2 + ...)\\ &= 80 ~\rm G\Omega \cdot (1-0.048 ~\rm 1/K \cdot(35 ~{\rm K}) + 0.00057 ~\rm 1/K^2\cdot\Delta (35 ~{\rm K})^2) \end{align*}

Calculate the resistance for the dielectric material for $20 ~\rm °C$.

Solution

\begin{align*} R(20 ~\rm °C) &= \rho\cdot {{d}\over{A}}\\ &= 10^{17} ~\Omega \rm m \cdot {{0.8\cdot 10^{-6} ~\rm m}\over{1 ~\rm m^2}} \end{align*}

Exercise E1 Analyzing a Scope Plot
(written test, approx. 12 % of a 60-minute written test, SS2023)

On the oscilloscope the following ... voltage and current.

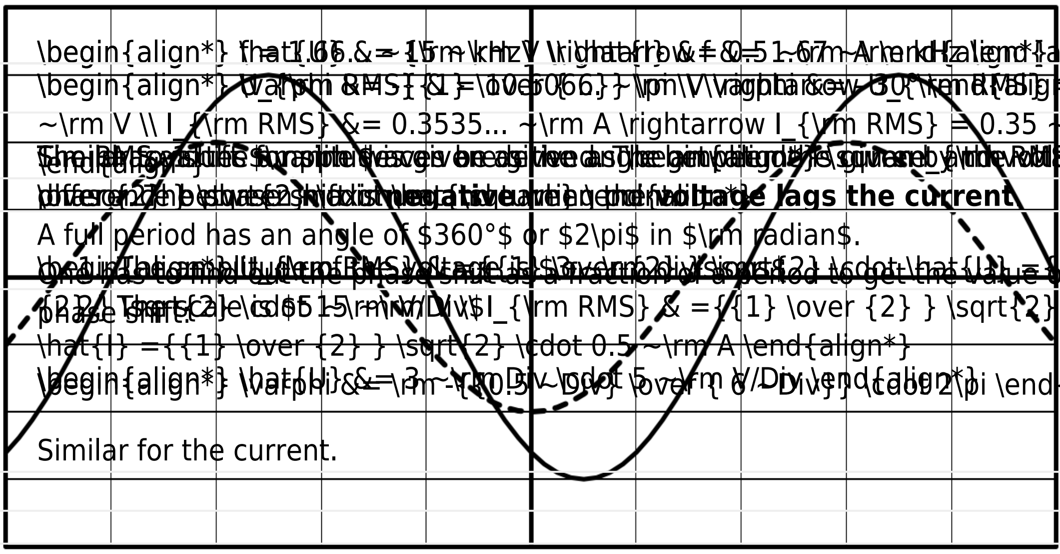
3. What is the phase shift ... (in radiant and degree)?

Result

The measured current curve shall be visible as a dashed line.

The continuous line shows the voltage.

Solution



\begin{align*} f = 1.66 ~\rm kHz \rightarrow f = 1.67 ~\rm kHz \end{align*}
\begin{align*} \hat{U} &= 15 ~\rm V \\ \hat{I} &= 0.5 ~\rm A \end{align*}
\begin{align*} \varphi &= {{1}\over{6}} \pi \\ \varphi &= 30° \end{align*}
\begin{align*} U_{\rm RMS} &= 10.6 ~\rm V \\ I_{\rm RMS} &= 0.3535... ~\rm A \rightarrow I_{\rm RMS} = 0.35 ~\rm A \end{align*}

The RMS value for sine waves ... The amplitude is given by the voltage ... {{1} \over {2}} \sqrt{2} ...

The phase shift is negative, since **the voltage lags the current**

A full period has an angle of $360°$ or 2π in $\rm radian$.

One has to find out the phase shift as a fraction of a period to get the value of the phase shift.

The scale is $5 ~\rm V/Div$

\begin{align*} U_{\rm RMS} &= {{1} \over {2}} \sqrt{2} \cdot \hat{U} = {{1} \over {2}} \sqrt{2} \cdot 15 ~\rm V \\ I_{\rm RMS} & = {{1} \over {2} } \sqrt{2} \cdot \hat{I} ={{1} \over {2} } \sqrt{2} \cdot 0.5 ~\rm A \end{align*}

\begin{align*} \hat{U} &= 3 ~\rm Div \cdot 5 ~\rm V/Div \end{align*}

\begin{align*} \varphi &= \rm -{0.5 ~ Div} \over { 6 ~ Div}} \cdot 2\pi \end{align*}

Similar for the current.

$5 V/Div$ $0.1 ms/Div$ $0.25 A/Div$

Use the correct symbols and units in your answers!

1. Calculate the frequency f of the periodic signals.

Solution

Frequency f is given by the period T. The period can be measured in the imagine of the scope.

1. The sine waves repeat after $6 ~\rm divisions$ (e.g. from falling turning point to falling turning point of one curve)
2. The scale is $0.1 ~\rm ms/Div$

\begin{align*} f &= {{1} \over {T}} \\ T &= 6 ~\rm Div \cdot 0.1 ~ms/Div \\

\rightarrow f &= {{1} \over {6 ~\rm Div \cdot 0.1 ~ms/Div}} \end{align*}

Exercise E1 Complex voltage dividers
(written test, approx. 16 % of a 60-minute written test, SS2023)

The circuit below is given with the following values:

3. Calculate the two impedances and the resulting phase shift between $\underline{U}_\rm O$ and $\underline{U}_\rm I$, the overall impedance

4. **Draw the two impedances and the resulting phasor diagram. Choose an appropriate scaling** factor and write it down.

Result

- $R = 1.1 ~\rm k\Omega$
- $L = 3.5 ~\rm mH$
- $\underline{U}_\rm I = 5 ~\rm V$
- $f_0 = 150 ~\rm kHz$

Solution

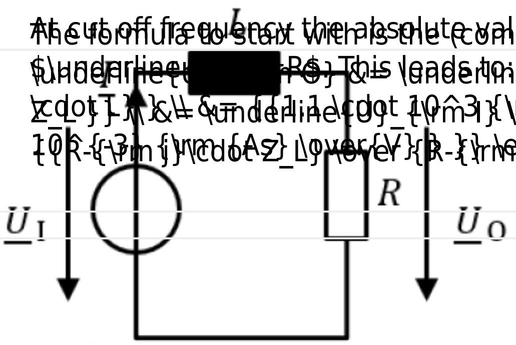
Result

\begin{align*} \underline{Z}_L &= 50{\rm j} \cdot 3.3 ~\rm k\Omega \end{align*}

\begin{align*} \underline{U}_{\rm O} &= 0.5 ~\rm V - j \cdot 1.5 ~V \end{align*}

At cut off frequency the absolute values of impedances $\underline{Z}_L$ is equal to $\underline{...} = R$. This leads to: \begin{align*} f_{\rm cutoff} &= {{R} \over {2\pi \cdot L }} \\ &= {{1.1 \cdot 10^3 {\rm {V} \over{A} } } \over {2\pi \cdot 3.5 \cdot 10^{-3} {\rm {As} \over{V} } }} \end{align*}

The formula to start with is the (complex) voltage divider: \begin{align*} \underline{U}_{\rm O} &= \underline{U}_{\rm I} \cdot {{R} \over {R+{\rm j}\cdot Z_L }} \\ &= \underline{U}_{\rm I} \cdot {{R} \over {R+{\rm j}\cdot Z_L }} \cdot {{R-{\rm j}\cdot Z_L} \over {R-{\rm j}\cdot Z_L }}\\ \end{align*}



1. Calculate the impedance $\underline{Z}_L$.

Solution

\begin{align*} \underline{Z}_L &= {\rm j} \cdot \omega \cdot L \\ &= {\rm j} \cdot 2\pi \cdot 150 ~\rm kHz \cdot 3.5 ~\rm mH \end{align*}

Exercise E2 Pure Resistor Network Simplification
(written test, approx. 12 % of a 60-minute written test, SS2023)

The circuit below shall be given.

2. Calculate the voltage at node $\rm K$, when switch $\rm S$ is closed.

Result

The values in the circuit are

Solution

- $R_1 = 60 ~\Omega$
- $R_2 = 40 ~\Omega$
- $R_3 = 40 ~\Omega$

The equivalent resistance is similar to the circuit with opened switch. \begin{align*} R_{\rm eq} &= 150 ~\Omega \end{align*}

\begin{align*} R_{\rm eq} &= 150 ~\Omega \end{align*}

- $R_4 = 100 ~\Omega$
- $U_{\rm AB} = 10 ~\rm V$

The voltage divider for node $\rm K$ has the same proportionality as the voltage divider for node $\rm K'$. Therefore, the potential of $\rm K$ is the same as for $\rm K'$. There will be no current flow through R_3. The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node $\rm K$, when switch $\rm S$ is open.
It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch $\rm S$ is opened, the upper part is a parallel circuit. Therefore, $R_{\rm eq}$ is given as:

$$\begin{align*} R_{\rm eq} &= (R_1+R_2)||(R_1+R_2)+R_4 \\ &= {{1}\over{2}}\cdot(R_1+R_2)+R_4 \\ &= {{1}\over{2}}\cdot(60~\Omega + 40~\Omega) + 100~\Omega \\ \end{align*}$$

Exercise E1 Pure Resistor Network Simplification I
(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below shall be simplified.

2. Now the input voltage shall be given as $U_{\rm AB} = 60 ~\rm V$. What is the value for U in the circuit?

Result

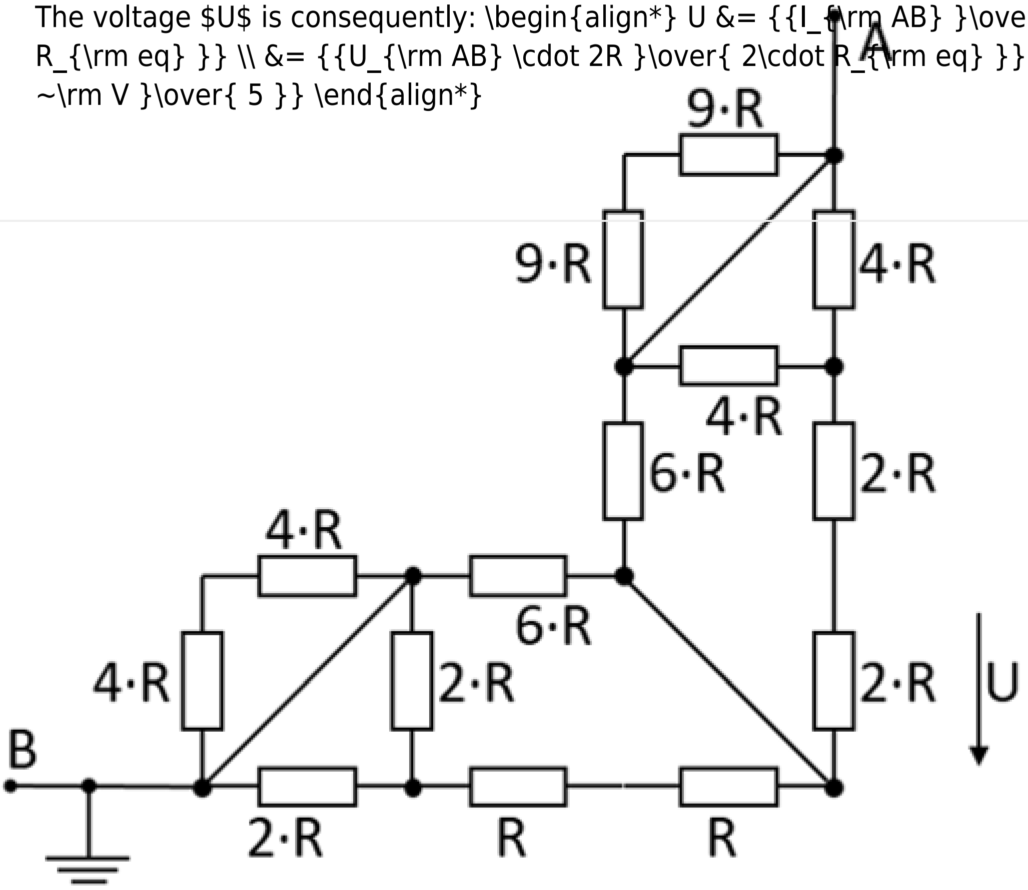
Solution

$$\begin{align*} R_{\rm eq} &= 5R \end{align*}$$

$$\begin{align*} U &= 12 ~\rm V \end{align*}$$

The current through the circuit is given as $I_{\rm AB} = U_{\rm AB} \cdot R_{\rm eq}$.
This current has to flow in summary through parallel branches. The voltage U in question in the upper right branch given by $(4R||4R)+2R+2R$. Its resistance is just the same as the upper left branch $6R$.
Therefore, half of the current flows to the left half to the right side.

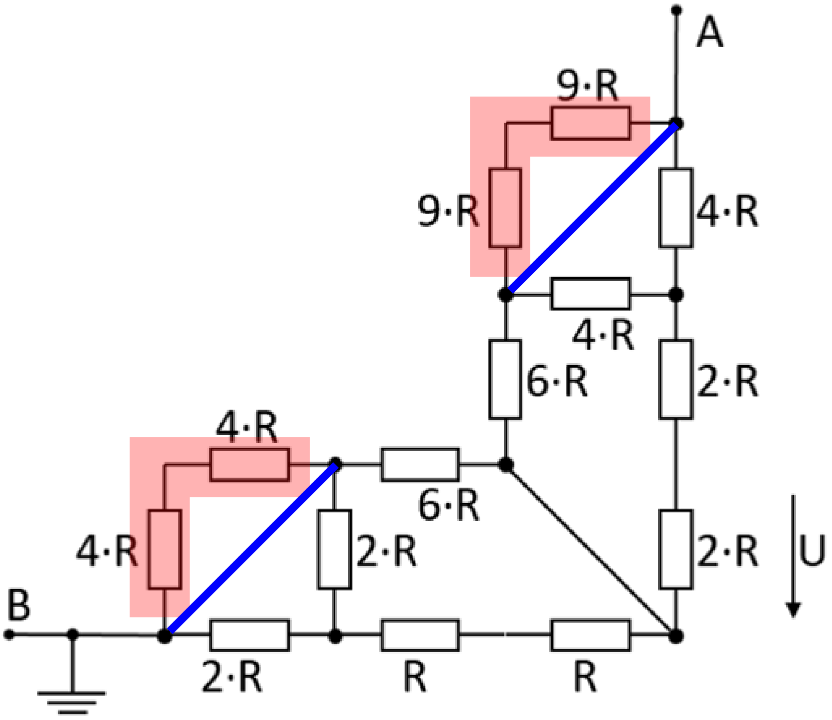
The voltage U is consequently: $$\begin{align*} U &= {{I_{\rm AB} }\over{ 2\cdot R_{\rm eq} }} \\ &= {{U_{\rm AB} \cdot 2R }\over{ 2\cdot R_{\rm eq} }} \\ &= {{60 ~\rm V }\over{ 5 }} \end{align*}$$



1. What is the equivalent resistance $R_{\rm eq}$?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

Therefore, $R_{\rm eq}$ is given as: \begin{align*} R_{\rm eq} &= (2R||2R + R +

R)||6R &&+ 6R ||(2R + 2R + 4R||4R) \\ &= (R + R + R)||6R &&+ 6R ||(2R + 2R + 2R) \\ &= 3R||6R &&+ 6R ||6R \\ &= {{3R\cdot 6R}\over{3R+6R}} &&+3R \\ \end{align*}

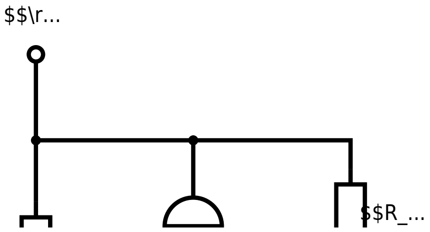
Exercise E3 Equivalent Linear Source
(written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification. Calculate the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source.

- $R_1 = 5 ~\Omega$
- $U_0 = 10 ~\rm V$
- $R_2 = 7.5 ~\Omega$
- $I_3 = 0.5 ~\rm A$
- $R_4 = 10 ~\Omega$
- $U_5 = 4 ~\rm V$

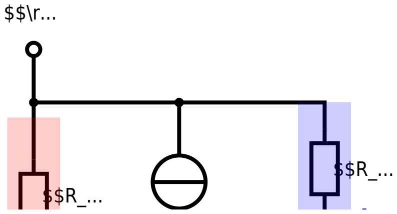
Result

\begin{align*} U_\rm s &= U_{\rm AB} =1.11...~\rm V \\ R_{\rm i} &= 5.55... ~\Omega \end{align*}



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = {{U_x}\over{R_x}}$ \begin{align*} I_0 &= {{U_0}\over{R_1}} = {{10~\rm V}\over{5 ~\Omega}} = 2 ~\rm A\\ I_5 &= {{U_5}\over{R_4}} = {{4 ~\rm V}\over{10~\Omega}} = 0.4 ~\rm A \\ \end{align*}

I_3 and I_0 can be combined to $I_{03}=I_0 - I_3$ facing upwards: \begin{align*} I_{03}=1.5~\rm A \end{align*}

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03}=R_1\cdot I_{03}$ facing down. \begin{align*} U_{03}=7.5~\rm V \end{align*}

Then, the resistors R_1 and R_2 can be combined to $R_{12}= R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12}. The current will be $I_{0123}={{U_{03}}\over{R_{12}}}$, facing up again. \begin{align*} I_{0123}=0.6~\rm A \end{align*}

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235}=I_{0123}-I_5$ and the resistors as $R_{124}=R_{12}||R_4$. \begin{align*} I_{01235} &=0.2~\rm A \\ R_{124} &=5.55... ~\Omega \end{align*}

The final step is the back-transformation to a linear voltage source, with $U_{\rm AB} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.
Here, there there is also a full final formula given:

\begin{align*} U_\rm s &= U_{\rm AB} = I_{01235} \cdot R_{124} \\ &= (I_{0123}-I_5) \cdot (R_{12}||R_4) \\ &= \left({{U_{03}}\over{R_{12}}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ &= \left({{R_1\cdot I_{03}}\over{R_1+R_2}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ &= \left({{R_1\cdot \left({{U_0}\over{R_1}} - I_3\right)}\over{R_1+R_2}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ \end{align*}

Exercise E4 (Dis)Charging Capacities
(written test, approx. 14 % of a 60-minute written test, SS2023)

2. What will be the voltage $u_C(t_1)$ ... At the point t_1 ... the switch S_1 switches to the situation shown in the drawing. What is the new time constant?

Result

Solution

- $U = 10 ~\rm V$
- $C_1 = 200 ~\rm nF$
- $R_1 = 8.0 ~\rm k\Omega$
- $R_2 = 17 ~\rm k\Omega$
- $R_3 = 9.0 ~\rm k\Omega$
- $I = 2 ~\rm mA$

Solution

Solution

\begin{align*} \tau(t_1) ... 3.36~\rm ms \end{align*}

\begin{align*} U_C &= 50~\rm V \end{align*}

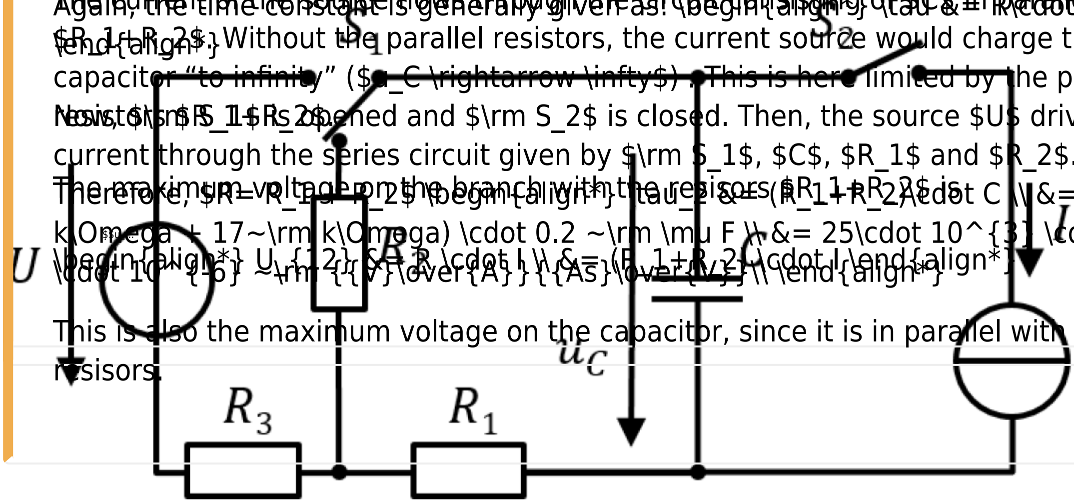
\begin{align*} u_C(t) &= U_0 \cdot(1-e^{-t/\tau}) \\ &= 10~\rm V \cdot(1-e^{-4~ms/3~ms}) \end{align*}

Again, the time constant is generally given as: \begin{align*} \tau &= R\cdot C \end{align*}

Now, $\rm S_1$ is opened and $\rm S_2$ is closed. Then, the source U drives the current through the series circuit given by $\rm S_1$, C, R_1 and R_2.
Therefore, $R = R_1 + R_2$ \begin{align*} \tau_2 &= (R_1+R_2)\cdot C \\ &= (8~\rm k\Omega + 17~\rm k\Omega) \cdot 0.2 ~\rm \mu F \\ &= 25\cdot 10^{3} \cdot 0.2 \cdot 10^{-6} ~\rm {{V}\over{A}}{{As}\over{V}} \\ \end{align*}

The current of the source flows through the circuit consisting of C in parallel with R_1+R_2. Without the parallel resistors, the current source would charge the capacitor "to infinity" ($u_C \rightarrow \infty$). This is here limited by the parallel resistors R_1+R_2.
The maximum voltage on the branch with the resisors R_1+R_2 is
\begin{align*} U_{12} &= R \cdot I \\ &= (R_1+R_2)\cdot I \end{align*}

This is also the maximum voltage on the capacitor, since it is in parallel with the resisors.



Before t_0 all switches are switched as shown and the capacitor is fully discharged.
At $t_0=0 ~\rm s$ the switch S_1 shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

Solution

The time constant is generally given as: \begin{align*} \tau &= R\cdot C \end{align*}

Once S_1 is closed and S_2 is open at t_0, the source U drives the current through the series circuit given by S_1, C, R_1 and R_3.
Therefore, $R= R_1 + R_3$ \begin{align*} \tau_1 &= (R_1+R_3)\cdot C \\ &= (8~\rm k\Omega + 7~\rm k\Omega) \cdot 0.2 ~\rm \mu F \\ &= 15\cdot 10^{3} \cdot 0.2 \cdot 10^{-6} ~\rm {{V}\over{A}}{{As}\over{V}}\\ \end{align*}

Solution

Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ($\propto (1-e^{-x})$).

Exercise E1 Impedances at Frequencies
(written test, approx. 14 % of a 60-minute written test, SS2023)

[illegible]

[illegible] and $L_1 = 15.9 ~\rm \mu H$.

Solution

\begin{align*} f_2 = [illegible] \end{align*}
kHz}\end{align*}

\begin{align*} X_{C3} &= [illegible]
[illegible]
[illegible]
[illegible] {{1}\over\rm {V/As
\cdot A/Vs}} } \\ \end{align*}

Exercise E1 Efficiency
(written test, approx. 14 % of a 60-minute written test, SS2023)

[illegible] V$. The battery shall provide energy for a
[illegible]
[illegible] battery datasheet.

Solution

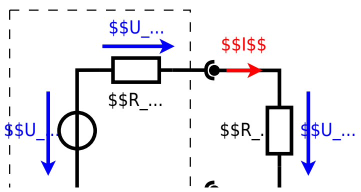
[illegible]
[illegible] \\ &= {{2
[illegible] \end{align*}
[illegible]

1. Draw an **equivalent circuit diagram** with the internal resistance and an external load. Label all voltages and currents.

\begin{align*} n_{\rm Li^+}={{Q}\over{q_\rm e}} \\ \\ Q &= 2.6 {~\rm Ah} \\ &=
2.6 \cdot 3600 {~\rm As} \\ &= 9360 {~\rm As} = 9360 {~\rm C} \end{align*}

\begin{align*} \eta &= {{U_\rm S - R_\rm i \cdot I_{\rm Dis max}}\over{U_\rm S}} \\
&= 1 - R_\rm i \cdot {{I_{\rm Dis max}}\over{U_\rm S}} \\ &= 1 - 0.05 {~\rm
\Omega} \cdot {{3~\rm A}\over{3.5 ~\rm V}} \\ \end{align*}

Result



Exercise E5 Analyzing a Scope Plot
(written test, approx. 12 % of a 60-minute written test, SS2023)

... (in radiant and degree)?

Result

The measured current curve shall be visible as a dashed line.

The continuous line shows the voltage.

Solution

$\hat{U} = 15 ~\rm V$, $\hat{I} = 0.5 ~\rm A$

$f = 1.66 ~\rm kHz \rightarrow f = 1.67 ~\rm kHz$

$U_{\rm RMS} = 10.606... ~\rm V \rightarrow U_{\rm RMS} = 10.6 ~\rm V$, $I_{\rm RMS} = 0.3535... ~\rm A \rightarrow I_{\rm RMS} = 0.35 ~\rm A$

$\varphi = -{1 \over 6} \pi$, $\varphi = -30°$

The RMS value for a sine wave is given by the amplitude divided by $\sqrt{2}$.

The phase shift is **negative**, the **voltage lags the current**.

A full period has an angle of $360°$ or 2π in $\rm radian$.

One has to find out the phase shift as a fraction of a period to get the value of the phase shift.

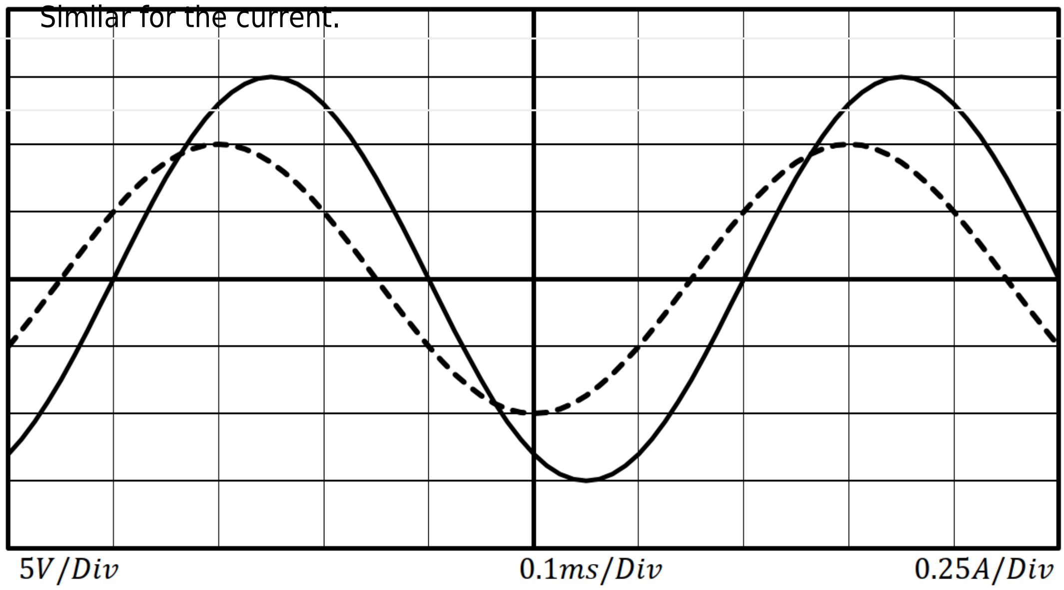
$U_{\rm RMS} = {1 \over 2} \sqrt{2} \cdot \hat{U} = {1 \over 2} \sqrt{2} \cdot 15 ~\rm V$, $I_{\rm RMS} = {1 \over 2} \sqrt{2} \cdot \hat{I} = {1 \over 2} \sqrt{2} \cdot 0.5 ~\rm A$

The amplitude of the voltage is 3 Div, the vertical scale is 5 V/Div.

$\hat{U} = 3 ~\rm Div \cdot 5 ~\rm V/Div$

$\varphi = -{0.5 ~\rm Div \over 6 ~\rm Div} \cdot 2\pi$

Similar for the current.



Use the correct symbols and units in your answers!

1. Calculate the frequency f of the periodic signals.

Solution

Frequency f is given by the period T. The period can be measured in the imagine of the scope.

1. The sine waves repeat after $6 ~\rm divisions$ (e.g. from falling turning point to falling turning point of one curve)
2. The scale is $0.1 ~\rm ms/Div$

$$\begin{align*} f &= {{1} \over {T}} \\ T &= 6 ~\rm Div \cdot 0.1 ~ms/Div \\ \rightarrow f &= {{1} \over {6 ~\rm Div \cdot 0.1 ~ms/Div}} \end{align*}$$

Exercise E6 Complex voltage dividers
(written test, approx. 16 % of a 60-minute written test, SS2023)

4. **Draw the two impedances and the resulting phasor** for the overall impedance in a diagram. **Choose an appropriate scaling** factor and write it down.

- $R = 1.1 ~\rm k\Omega$
- $L = 3.5 ~\rm mH$
- $\underline{U}_\rm I = 5 ~\rm V$

Solution

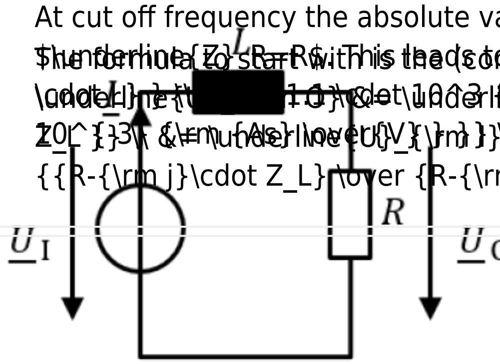
Result

$$\begin{align*} \underline{Z}_L &= j \cdot 3.3 ~\rm k\Omega \end{align*}$$

$$\begin{align*} \underline{U}_{\rm O} &= 0.5 ~\rm V - j \cdot 1.5 ~V \end{align*}$$

At cut off frequency the absolute values of impedances $\underline{Z}_L$ is equal to $\underline{Z}_R=R$. This leads to: \begin{align*} f_{\rm cutoff} &= {R} \over {2\pi \cdot L} \\ &= {1 \cdot 10^3 {\rm V} \over{A}} } \over {2\pi \cdot 3.5 \cdot 10^{-3} {\rm {As} \over{V}} } \end{align*}

The formula to start with is the complex voltage divider: \begin{align*} \underline{U}_{\rm O} &= \underline{U}_{\rm I} \cdot {R} \over {R+{\rm j}\cdot Z_L }} \\ &= \underline{U}_{\rm I} \cdot {{R}\over {R+{\rm j}\cdot Z_L }} \cdot {{R-{\rm j}\cdot Z_L} \over {R-{\rm j}\cdot Z_L }}\\ \end{align*}



1. Calculate the impedance $\underline{Z}_L$.

Solution

\begin{align*} \underline{Z}_L &= {\rm j} \cdot \omega \cdot L \\ &= {\rm j} \cdot 2\pi \cdot 150 ~\rm kHz \cdot 3.5 ~\rm mH \end{align*}

Exercise E2 Pure Resistor Network Simplification
(written test, approx. 12 % of a 60-minute written test, SS2023)

The circuit below shall be given.

2. Calculate the voltage at node $\rm K$, when switch $\rm S$ is closed.

Result

The values in the circuit are

Solution

- $R_1 = 60 ~\Omega$
- $R_2 = 40 ~\Omega$
- $R_3 = 40 ~\Omega$
- $R_4 = 100 ~\Omega$
- $U_{\rm AB} = 10 ~\rm V$

The equivalent Resistance is similar to the circuit with opened switch. \begin{align*} R_{\rm eq} &= 150 ~\Omega \end{align*}

The voltage divider for node $\rm K$ has the same proportionality as the voltage divider for node $\rm K'$. Therefore, the potential of $\rm K$ is the same as for $\rm K'$. There will be no current flow through R_3. The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node $\rm K$, when switch $\rm S$ is open.
It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch $\rm S$ is opened, the upper part is a parallel circuit. Therefore, $R_{\rm eq}$ is given as:

$$\begin{align*} R_{\rm eq} &= (R_1+R_2)||(R_1+R_2)+R_4 \\ &= {{1}\over{2}}\cdot(R_1+R_2)+R_4 \\ &= {{1}\over{2}}\cdot(60~\Omega + 40~\Omega) + 100~\Omega \\ \end{align*}$$

Exercise E3 Pure Resistor Network Simplification I
(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below shall be simplified.

2. Now the input voltage shall be given as $U_{\rm AB} = 60 ~\rm V$. What is the value for U in the circuit?

Result

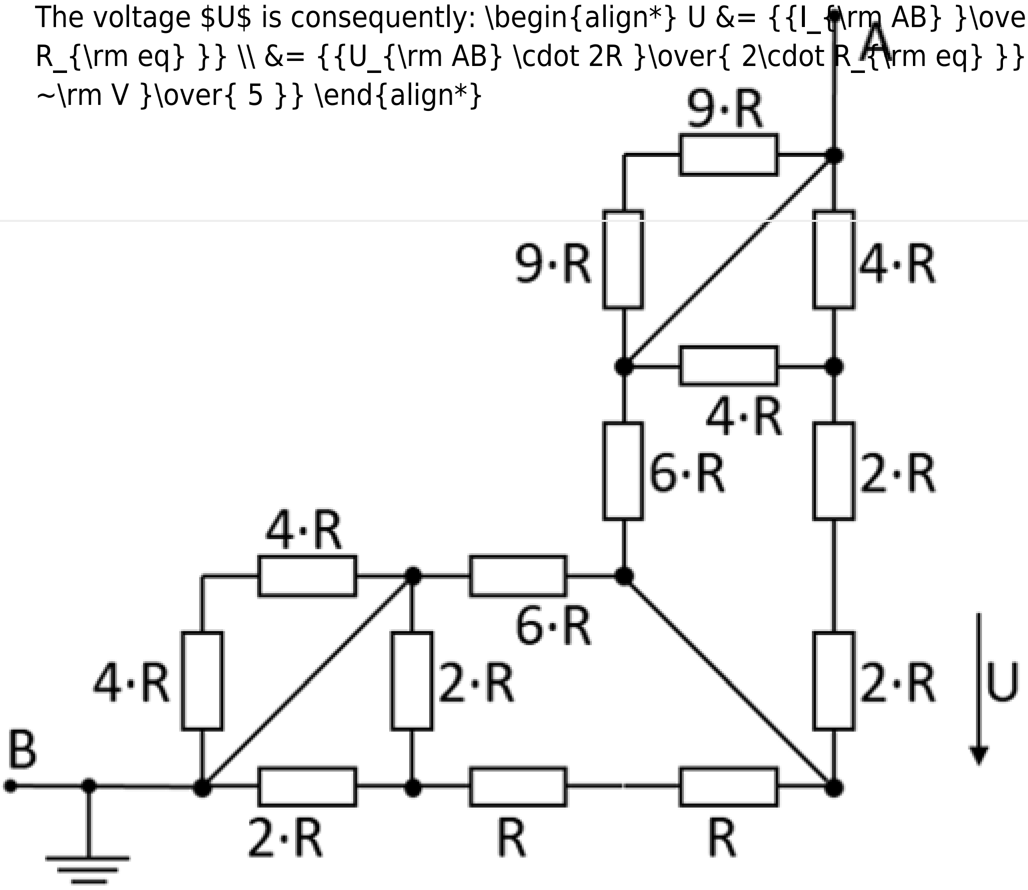
Solution

$$\begin{align*} R_{\rm eq} &= 5R \end{align*}$$

$$\begin{align*} U &= 12 ~\rm V \end{align*}$$

The current through the circuit is given as $I_{\rm AB} = U_{\rm AB} \cdot R_{\rm eq}$.
This current has to flow in summary through parallel branches. The voltage U in question in the upper right branch given by $(4R||4R)+2R+2R$. Its resistance is just the same as the upper left branch $6R$.
Therefore, half of the current flows to the left half to the right side.

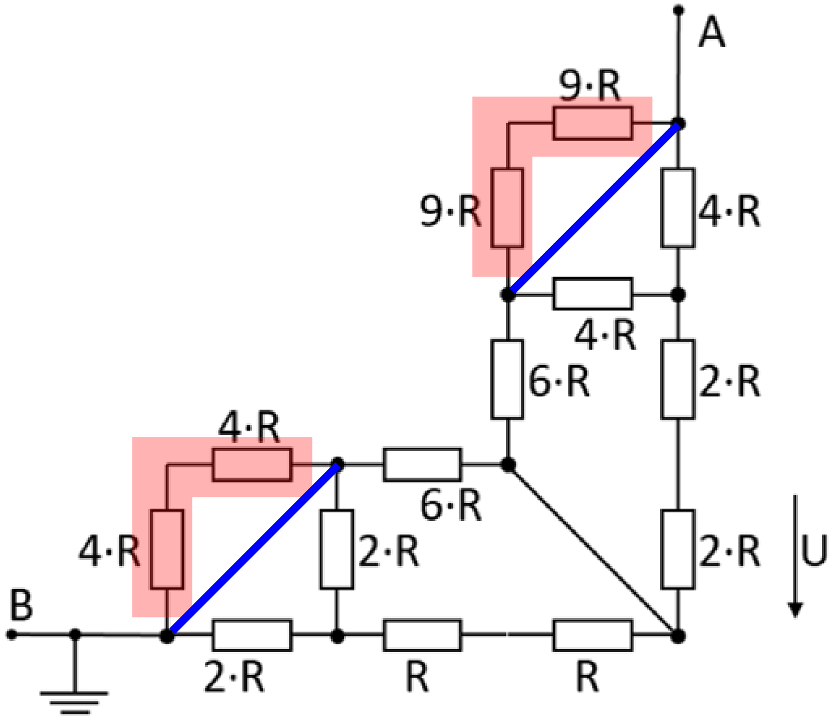
The voltage U is consequently: \begin{align*} U &= {{I_{\rm AB} }\over{ 2\cdot R_{\rm eq} }} \\ &= {{U_{\rm AB} \cdot 2R }\over{ 2\cdot R_{\rm eq} }} \\ &= {{60 ~\rm V }\over{ 5 }} \end{align*}



1. What is the equivalent resistance $R_{\rm eq}$?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

Therefore, $R_{\rm eq}$ is given as: \begin{align*} R_{\rm eq} &= (2R||2R + R +

R)||6R &&+ 6R ||(2R + 2R + 4R||4R) \\ &= (R + R + R)||6R &&+ 6R ||(2R + 2R + 2R) \\ &= 3R||6R &&+ 6R ||6R \\ &= {{3R\cdot 6R}\over{3R+6R}} &&+3R \\ \end{align*}

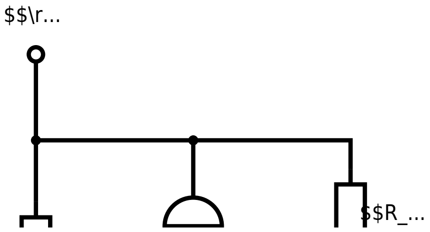
Exercise E4 Equivalent Linear Source
(written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification. Calculate the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source.

Result

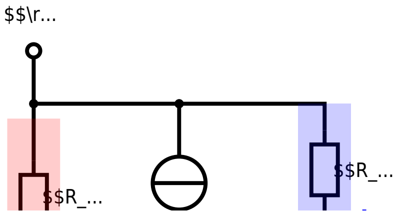
\begin{align*} U_\rm s &= U_{\rm AB} =1.11...~\rm V \\ R_{\rm i} &= 5.55... ~\Omega \end{align*}

- $R_1 = 5 ~\Omega$
- $U_0 = 10 ~\rm V$
- $R_2 = 7.5 ~\Omega$
- $I_3 = 0.5 ~\rm A$
- $R_4 = 10 ~\Omega$
- $U_5 = 4 ~\rm V$



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = {{U_x}\over{R_x}}$ \begin{align*} I_0 &= {{U_0}\over{R_1}} = {{10~\rm V}\over{5 ~\Omega}} = 2 ~\rm A\\ I_5 &= {{U_5}\over{R_4}} = {{4 ~\rm V}\over{10~\Omega}} = 0.4 ~\rm A \\ \end{align*}

I_3 and I_0 can be combined to $I_{03}=I_0 - I_3$ facing upwards: \begin{align*} I_{03}=1.5~\rm A \end{align*}

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03}=R_1\cdot I_{03}$ facing down. \begin{align*} U_{03}=7.5~\rm V \end{align*}

Then, the resistors R_1 and R_2 can be combined to $R_{12}= R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12}. The current will be $I_{0123}={{U_{03}}\over{R_{12}}}$, facing up again. \begin{align*} I_{0123}=0.6~\rm A \end{align*}

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235}=I_{0123}-I_5$ and the resistors as $R_{124}=R_{12}||R_4$. \begin{align*} I_{01235} &=0.2~\rm A \\ R_{124} &=5.55... ~\Omega \end{align*}

The final step is the back-transformation to a linear voltage source, with $U_{\rm AB} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.
Here, there there is also a full final formula given:

$$\begin{align*} U_\rm s &= U_{\rm AB} = I_{01235} \cdot R_{124} \\ &= (I_{0123}-I_5) \cdot (R_{12}||R_4) \\ &= \left({{U_{03}}\over{R_{12}}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ &= \left({{R_1\cdot I_{03}}\over{R_1+R_2}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ &= \left({{R_1\cdot \left({{U_0}\over{R_1}} - I_3\right)}\over{R_1+R_2}}-I_5\right) \cdot \left((R_1 + R_2)||R_4\right) \\ \end{align*}$$

Exercise E7 (Dis)Charging Capacities
(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below is given, with the values given in the drawing. What will be the

... S_1 switches to the situation shown in the drawing. What is the new time constant?

Result

Solution

- $U = 10 ~\rm V$
- $C_1 = 200 ~\rm nF$
- $R_1 = 8.0 ~\rm k\Omega$
- $R_2 = 17 ~\rm k\Omega$
- $R_3 = 7.0 ~\rm k\Omega$
- $I = 2 ~\rm mA$

Solution

$\begin{align*} \tau = C (R_1 + R_2) = 3.36 ~\rm ms \end{align*}$

$\begin{align*} U_C &= 50 ~\rm V \end{align*}$

$\begin{align*} u_C(t) &= U_0 \cdot(1-e^{-t/\tau}) \\ &= 10~\rm V \cdot(1-e^{-4~ms/3~ms}) \end{align*}$

Again, the time constant is generally given as: $\begin{align*} \tau &= R\cdot C \end{align*}$

The current of the source flows through the circuit consisting of C in parallel with R_1+R_2. Without the parallel resistors, the current source would charge the capacitor "to infinity" ($U_C \rightarrow \infty$). This is here limited by the parallel resistors R_1+R_2.

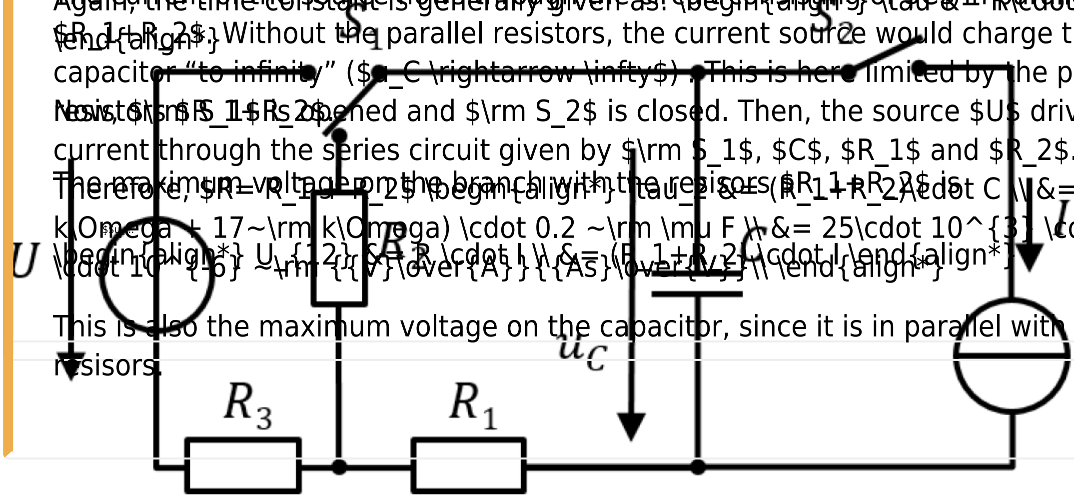
Now, $\rm S_1$ is opened and $\rm S_2$ is closed. Then, the source U drives the current through the series circuit given by $\rm S_1$, C, R_1 and R_2.

The maximum voltage on the branch with the resistors R_1+R_2 is

Therefore, $R = R_1 + R_2$ $\begin{align*} \tau_2 &= (R_1+R_2)\cdot C \\ &= (8~\rm k\Omega + 17~\rm k\Omega) \cdot 0.2 ~\rm \mu F \\ &= 25\cdot 10^{3} \cdot 0.2 \cdot 10^{-6} ~\rm {{V}\over{A}}{{As}\over{V}} \\ \end{align*}$

$\begin{align*} U_{12} &= R \cdot I \\ &= (R_1+R_2)\cdot I \end{align*}$

This is also the maximum voltage on the capacitor, since it is in parallel with the resisors.



Before t_0 all switches are switched as shown and the capacitor is fully discharged.
At $t_0=0 ~\rm s$ the switch S_1 shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

Solution

> The time constant is generally given as: \begin{align*} \tau &= R\cdot C \end{align*}
>
> Once S_1 is closed and S_2 is open at t_0, the source U drives the current through the series circuit given by S_1, C, R_1 and R_3.
> Therefore, $R= R_1 + R_3$ \begin{align*} \tau_1 &= (R_1+R_3)\cdot C \\ &= (8~\rm k\Omega + 7~\rm k\Omega) \cdot 0.2 ~\rm \mu F \\ &= 15\cdot 10^{3} \cdot 0.2 \cdot 10^{-6} ~\rm {{V}\over{A}}{{As}\over{V}}\\ \end{align*}

Solution

> Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ($\propto (1-e^{-x})$).

Exercise E8 Impedances at Frequencies
(written test, approx. 14 % of a 60-minute written test, SS2023)

2. A capacitor with $C_2 = 50 ~\rm µF$, where an AC voltage of $U_2= 6.8 ~\rm V$ [illegible]

Resonance with $X_{L1} = 60 ~\rm \Omega$ and $L_1 = 15.9 ~\rm \mu H$.

Solution

\begin{align*} f_2 = ... ~{\rm kHz}\end{align*}

\begin{align*} X_{C2} &= ... \\ \end{align*}

\sqrt{{1}\over{A/Vs \cdot A/Vs}} \cdot {{1}\over\rm {V/As \cdot A/Vs}} } \\ \end{align*}

Exercise E9 Efficiency
(written test, approx. 14 % of a 60-minute written test, SS2023)

A lithium-ion battery ... has an internal resistance ... and a nominal voltage of $3.5 ~\rm V$. The battery shall provide energy for a mobile device with a load resistance of $R_\rm L = 2 ~\rm \Omega$. The following values are from the lithium-ion battery datasheet.

- Internal impedance: ...
- Maximum discharge current: $I_{\rm Dis max} = 3 ~\rm A$
- Typical capacity: $2600 ~\rm mAh$

1. Draw an **equivalent circuit diagram** with the internal resistance and an external load. Label all voltages and currents.

2. ... **lowest possible efficiency**?

Solution

Result

Solution

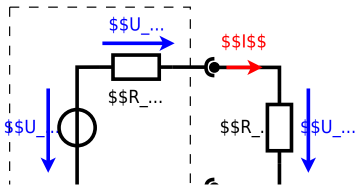
\begin{align*} R_\rm i &= ... \\ &= {{2 ~\Omega}\over{2 ~\Omega + 0.05 ~\Omega}} \end{align*}

Lowest efficiency for the highest current, so for $I_{\rm Dis max}$. In this case the efficiency is:

\begin{align*} \eta &= {{U_\rm S - R_\rm i \cdot I_{\rm Dis max}}\over{U_\rm S}} \\ &= 1 - R_\rm i \cdot {{I_{\rm Dis max}}\over{U_\rm S}} \\ &= 1 - 0.05 {~\rm \Omega} \cdot {{3~\rm A}\over{3.5 ~\rm V}} \\ \end{align*}

\begin{align*} n_{\rm Li^+} ={{Q}\over{q_\rm e}} \\ \\ Q &= 2.6 {~\rm Ah} \\ &= 2.6 \cdot 3600 {~\rm As} \\ &= 9360 {~\rm As} = 9360 {~\rm C} \end{align*}

Result


$$U_...
$$I$$
$$R_...
$$U_...
$$R_...
$$U_...