task_jti0uzudcmg4u22t_with_calculation

Student Group

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complex impedance, exam ee1 WS2022

Exercise 1.1 : Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

A circuit with an ideal voltage source ($U=50 ~V$, $f=330 ~Hz$) and two components (R and $\underline{X}_1$) shall be given.
After analysis, the following formula for the impedance was extracted:

$$\begin{align*} \underline{Z} = \left({{2}\over{3+4j}}+5j \right) \Omega \end{align*}$$

1. Calculate the physical values of the two components.

Solution

$$\begin{align*} \underline{Z} &= \left({{2}\over{3+4j}} + 5j \right) ~\Omega \\ &= \left({{2}\over{3+4j}} \cdot {{3-4j}\over{3-4j}} + 5j \right) ~\Omega \\ &= \left({{2}\over{9+16}} \cdot (3-4j) + 5j \right) ~\Omega \\ &= \left(0.24 - 0.32j + 5j \right) ~\Omega \\ &= 0.24 ~\Omega + j \cdot 4.68 ~\Omega \\ &= R + j X_L \\ \end{align*}$$

With the complex part comes the physical value:

$$\begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \\ \end{align*}$$

Final result

$$\begin{align*} R &= 0.24 ~\Omega \\ L &= 2.26 ~\rm{mH} \end{align*}$$

2. Calculate the phase and absolute value of complex current $\underline{I}$ through the circuit.

Solution

\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + j \cdot 4.68 ~\Omega }} \\ &= {{50

> ~\rm{V}}\over{ 0.24 ~\Omega + j \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - j \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - j \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }} \cdot (0.24 ~\Omega - j \cdot 4.68 ~\Omega) \\ \end{align*}
>
> The absolute value $|\underline{I}|$ can be calculated as: \begin{align*} |\underline{I}| &= {|{\underline{U}|}\over{|\underline{Z}|}} \\ &= {{50 ~\rm{V}}\over{| 0.24 ~\Omega + j \cdot 4.68 ~\Omega |}} \\ &= {{50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}} \end{align*}
>
> The phase φ_i can be calculated as \begin{align*} \varphi_i &= arctan \left({{Im()}\over{Re()}} \right) \\ &= arctan \left({{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

Final result

> \begin{align*} |\underline{I}| &= 10.67 ~\rm{A} \\ \varphi_i &= -87.06° \end{align*}

3. Now an additional component $\underline{X}_2$ shall be added in series to the two components.
This component shall be dimensioned in such a way that the current and voltage are in phase. Calculate these component value!

Solution

> The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.
> Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L| $ \begin{align*} X_C &= {{1}\over{\omega \cdot C}} = X_L \\ C &= {{1}\over{\omega \cdot X_L}} \\ &= {{1}\over{2\pi \cdot 300 ~\rm{Hz} \cdot 4.68 ~\Omega}} \\ \end{align*}

Final result

> \begin{align*} C = 103 ~\rm{µF} \end{align*}