task_tb6pi8dgh0m2e2pw_with_calculation

Student Group

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Charging Capacitors, dc network analysis, pure resistor network simplification, delta wye transformation, exam ee1 WS2022

Exercise E1.1 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

4. Now, a bulb with an internal resistance of $R_B = 20 ~\Omega$ is connected to the capacitor (switch S_2 is open). The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and R_B.

Result

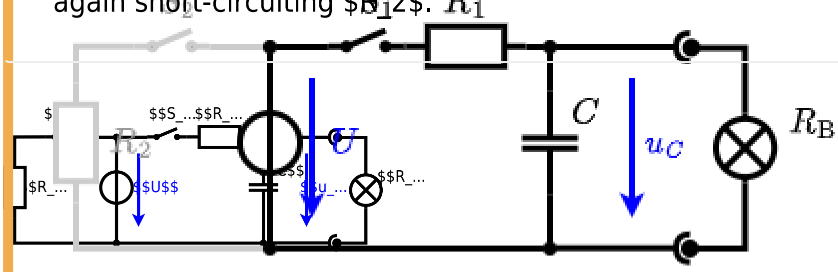
\begin{align*} t_{\Delta 2}/R_3 = 7.92 ~\rm{V} \end{align*}

Solution

\begin{align*} \Delta W_C &= {{1}\over{2}} \cdot C \cdot U^2 \\ &= {{1}\over{2}} \cdot 100 ~\rm{µF} \cdot (12 ~\rm{V})^2 \end{align*}

$U=12 ~\rm{V}$ independent of this resistor.

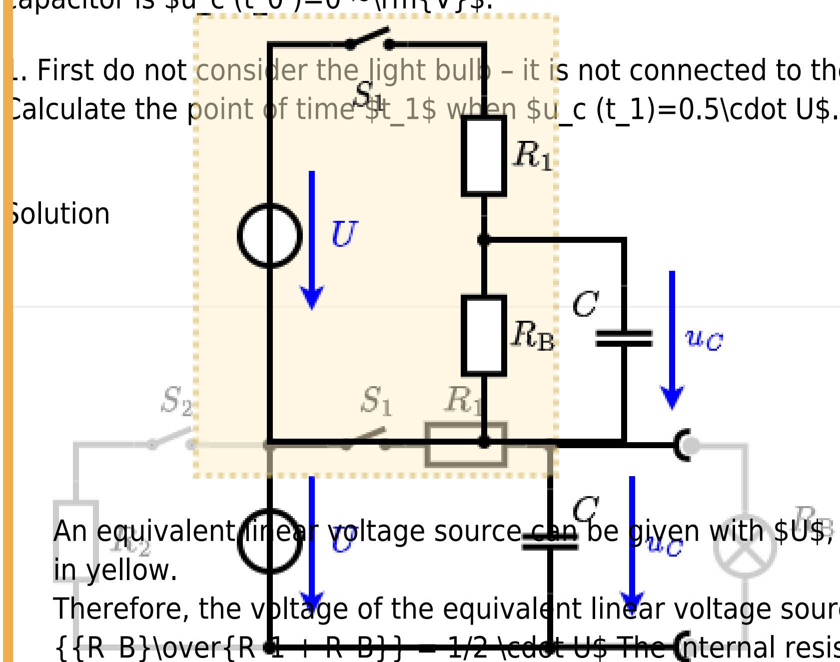
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2.



The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution



An equivalent linear voltage source can be given with U, R_1, and R_B as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_B}\over{R_1 + R_B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit).

So, here only R_1 and C give the time constant $\tau = R_1 \cdot C$
\begin{align*} R_1 &= R_1 || R_B \\ &= 10~\Omega \\ \end{align*}

The following formula describes the time course of $U_C(t)$, which has to be $U_C(t_1)=0.5\cdot U$: \begin{align*} U_C(t) &= U \cdot (1 - e^{-t/\tau}) \end{align*}
\begin{align*} U_c(t_2) &= U_c(t_1) \cdot (1 - e^{t_2/(R_1 \cdot C)}) \\ &= {{1}\over{2}} \cdot U \\ \tau &= 10~\Omega \cdot 100~\mu F = 0.5 \cdot U
\end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}