

Exam Winter Semester 2022

Student Group

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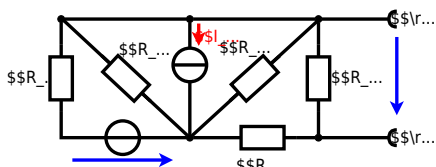
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Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

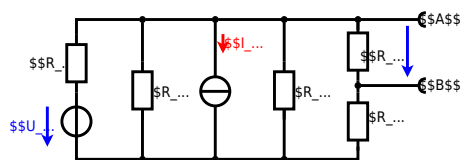
$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



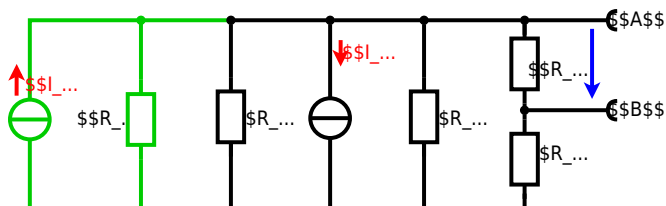
Calculate the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B .
 $R_1 = 5.0 \text{ } \Omega$, $U_2 = 6.0 \text{ V}$, $R_3 = 10 \text{ } \Omega$, $I_4 = 4.2 \text{ A}$,
 $R_5 = 10 \text{ } \Omega$, $R_6 = 7.5 \text{ } \Omega$, $R_7 = 15 \text{ } \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$\&= R_{135} \cdot I_{24} \quad \&= \left(\frac{U_2}{R_1} - I_4 \right) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \quad R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

d

Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. The given diagram explains the effect of temperature on the resistance of a thermistor. It has a resistance of $10\text{k}\Omega$ at 25°C and $6.5\text{k}\Omega$ at -40°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \frac{1}{\text{K}^2}$.

Result
The temperature inside the refrigeration system can reach down to -40°C .

$$R = 6.5\text{k}\Omega \quad \text{at } -40^\circ\text{C}$$

The power of the resistor $P = U \cdot I$ and the current $I = \frac{U}{R}$ to heat the refrigeration system. Therefore, a solution is to use a higher resistance.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad \text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \quad R = 10\text{k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

d

Exercise E1 Analyzing complex Impedances

(written test, approx. 14 % of a 60-minute written test, WS2022)

2. **Drawn** and the pool has been completed then (Rule 5 (c) m 2) \$ 3130 (underline) shall be given to the Result Components. (\$\$ and \$ \underline{X} 1\$) shall be given.

After analysis, the following dimensionless parameters have been extracted and denoted (figure) in the table (Table 1). The (2) have been (3) + 5 (4) \Omega \end{align*}

.. Calculation of the 103 values of the vector components.

$$\text{Solve} \left\{ \begin{aligned} & \text{begin} \{ \text{align}^* \} \quad \text{R} \text{und} \{ 0.24 \} \text{IV} \text{on} \{ 20.67 \} \& \text{r} \{ 2.26 \} \text{~} \backslash \text{r} \text{m} \text{a} \text{r} \text{p} \text{h} \text{i} \} \& \text{r} \{ 27.06^\circ \} \text{~} \backslash \text{e} \text{n} \text{d} \{ \text{align}^* \} \end{aligned} \right.$$

Solution

[illegible]

d

Exercise E1 Complex Impedance Circuit

(written test, approx. 15 % of a 60-minute written test, WS2022)

[illegible]

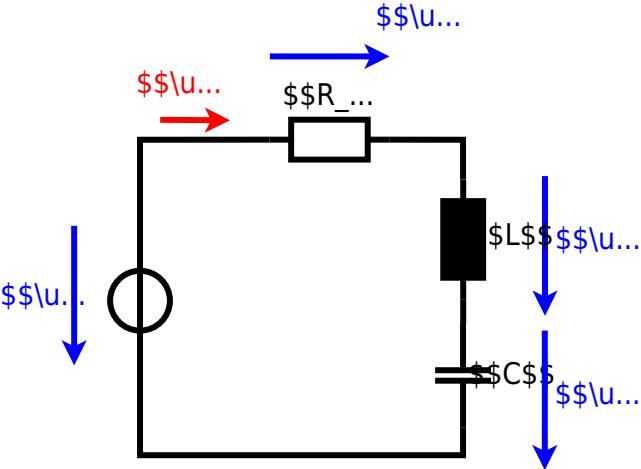
Solution A linear source is connected with an inductor of $330 \sim \{\rm \mu H\}$ and a capacitor of $0.22 \sim \{\rm \mu F\}$, all in series.

Result

.. Draw the circuit diagram of the given circuit. $48.2 \sim \Omega$ $\parallel Z = 19.8 \sim \Omega$

Label all components, voltages, and currents.

$$\begin{aligned} \text{\texttt{\textbackslash begin{align*}}} Z &= \frac{\hat{U}}{\hat{I}} \quad \text{\texttt{\textbackslash \&}} \quad \frac{\hat{I}}{\hat{Z}} = \frac{\hat{U}}{Z} \quad \text{\texttt{\textbackslash \&}} \\ \text{\texttt{\textbackslash end{align*}}} \\ \text{\texttt{\textbackslash begin{align*}}} Z_C &= \frac{1}{2\pi f C} \quad \text{\texttt{\textbackslash \&}} \quad \frac{1}{2\pi \cdot 15} \\ \text{With } f &= 1500 \text{ Hz} \quad \text{\texttt{\textbackslash end{align*}}} \text{\texttt{\textbackslash begin{align*}}} I &= \\ \frac{U}{Z} &= \frac{200}{\sqrt{Z_L^2 + Z_C^2}} = \frac{200}{\sqrt{19.28^2 + 0.22^2}} \\ &= 10.2 \text{ A} \quad \text{\texttt{\textbackslash end{align*}}} \\ \text{\texttt{\textbackslash begin{align*}}} Z_C &= \frac{1}{2\pi f C} \quad \text{\texttt{\textbackslash \&}} \quad \frac{1}{2\pi \cdot 15} \\ &\sim 330 \text{ }\mu\text{H} \quad \text{\texttt{\textbackslash end{align*}}} \\ \text{\texttt{\textbackslash begin{align*}}} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \quad \text{\texttt{\textbackslash \&}} \quad R + j \\ &\cdot Z_L - j \cdot Z_C \quad \text{\texttt{\textbackslash \&}} \quad R + j \cdot (Z_L - Z_C) \quad \text{\texttt{\textbackslash \&}} \\ &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2} \quad \text{\texttt{\textbackslash end{align*}}} \end{aligned}$$



d

Exercise E1 Impedances at different Frequencies**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with $R_1 = 1.00 \text{ k}\Omega$, a capacitor with $C_1 = 40 \text{ nF}$, and an AC voltage source with $U_0 = 230 \text{ V}$ and $f = 50 \text{ Hz}$. The current I through the resistor R_1 shall have the same absolute value of the impedance as a capacitor C_1 at $f_1 = 4 \text{ MHz}$.

Solution

$$\begin{aligned} R_1 &= 1.00 \text{ k}\Omega \end{aligned}$$

$$C_1 = 40 \text{ nF}$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by

$$Z_{\text{parallel}} = \frac{1}{\frac{1}{R_1} + \frac{1}{j\omega L_1}} = \frac{R_1 j\omega L_1}{1 + j\omega L_1 R_1}$$

Since R_1 and L_1 are in parallel, the voltage across both is the same. The voltage across R_1 is $U_R = I R_1$ and the voltage across L_1 is $U_L = I j\omega L_1$. The voltage across the parallel combination is $U_{\text{parallel}} = U_R + U_L = I (R_1 + j\omega L_1)$. The current through the parallel combination is $I_{\text{parallel}} = \frac{U_{\text{parallel}}}{Z_{\text{parallel}}} = \frac{I (R_1 + j\omega L_1)}{\frac{R_1 j\omega L_1}{1 + j\omega L_1 R_1}} = I (1 + j\omega L_1 R_1)$.

Therefore, the resulting current of the parallel circuit is given as:

$$I_{\text{parallel}} = I (1 + j\omega L_1 R_1)$$

$$I_{\text{parallel}}^2 = I^2 (1 + \omega^2 L_1^2 R_1^2)$$

$$I_{\text{parallel}}^2 = I^2 (1 + (2\pi \cdot 450 \text{ kHz} \cdot 4.7 \text{ }\mu\text{H})^2)$$

$$I_{\text{parallel}}^2 = I^2 (1 + (2\pi \cdot 450 \cdot 10^3 \cdot 4.7 \cdot 10^{-6})^2)$$

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d

Exercise E1 Resistance of a Wire by Resistivity**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. The heating element is used to heat the wire with a temperature of 1800 K . The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Calculate the current I needed to operate the heating element.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \& \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

d

Exercise E1 Charging Capacitors

(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (without the light bulb) also takes the form of an RC circuit. The voltage across the capacitor is again U at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

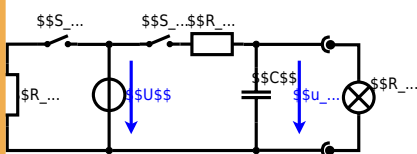
Solution

Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

$$\Delta U = U - \frac{R_2}{R_1 + R_2} U$$

Solution: The ideal voltage source U is in series with the resistor R_1 . The voltage across the capacitor is $u_c(t) = U \cdot (1 - e^{-t/\tau})$, where $\tau = R_1 C$.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



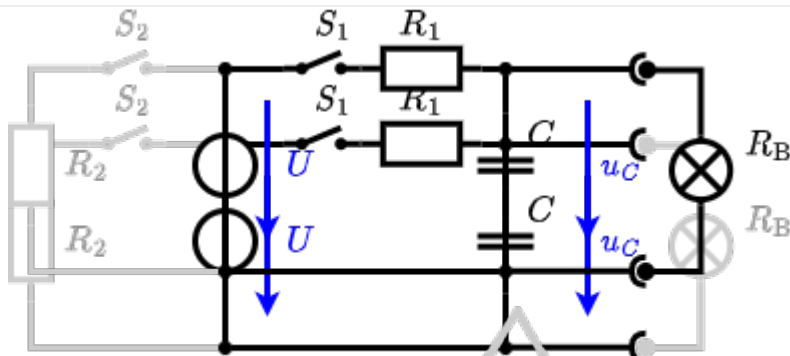
The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution

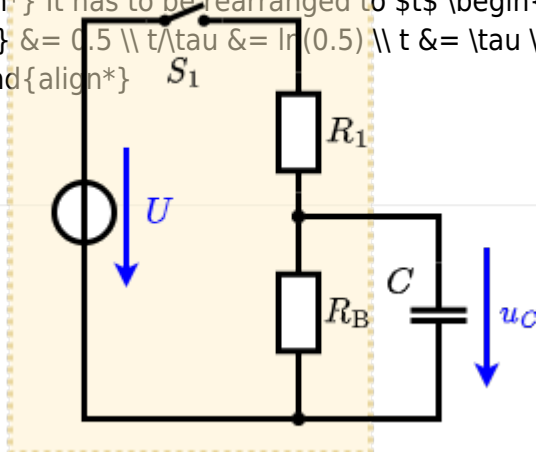


So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$

It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$

$$e^{-t/\tau} = 0.5 \quad t/\tau = \ln(0.5) \quad t = \tau \cdot \ln(0.5) \quad t = R_1 \cdot C \cdot \ln(0.5)$$


An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($= 0 \, \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \, \Omega$$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \, \Omega \cdot 100 \, \mu\text{F})})$$

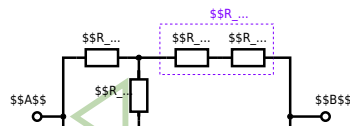
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Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0 temperature, $R_1 = R_2 = 10 \, \Omega$, $R_3 = 10 \, \Omega$, $R_4 = 10 \, \Omega$, and the voltage source $U = 10 \, \text{V}$. The result is given in Ω .

Solution

R_{eq} is given as:



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

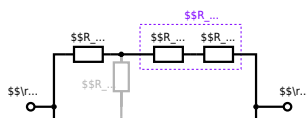
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_3) \parallel (R_2 + R_3) \parallel R_4$$

$$R_{\text{eq}} = (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel R_4$$

$$R_{\text{eq}} = (500 \, \Omega) \parallel (200 \, \Omega) \parallel R_4$$

$$R_{\text{eq}} = \frac{500 \, \Omega \cdot 200 \, \Omega}{500 \, \Omega + 200 \, \Omega} \parallel R_4$$

d

Exercise E4 Equivalent linear Source

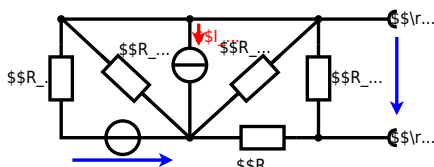
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.

Result

$$U_s = U_{AB} = 4.5 \, \text{V}$$

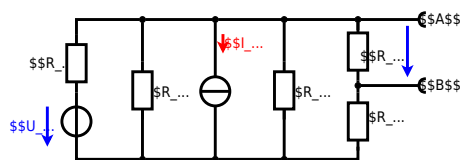
$$R_i = R_{AB} = 6 \, \Omega$$



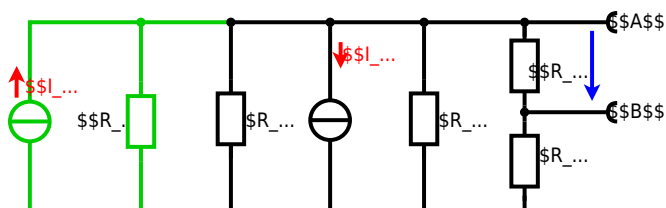
Calculate the internal resistance R_{int} and the source voltage U_s of an equivalent linear voltage source on the connectors A and B. $\begin{aligned} R_1 &= 5.0 \, \Omega, \quad U_2 = 6.0 \, \text{V}, \quad R_3 = 10 \, \Omega, \quad I_4 = 4.2 \, \text{A}, \\ R_5 &= 10 \, \Omega, \quad R_6 = 7.5 \, \Omega, \quad R_7 = 15 \, \Omega \end{aligned}$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$I_{24} = \frac{U_{24}}{R_6 + R_1 \parallel R_3 \parallel R_5} = \frac{(U_2 - I_4 R_1)}{R_6 + R_1 \parallel R_3 \parallel R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} = \frac{(U_2 - I_4 R_1) \cdot R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0V}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \quad R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

d

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. The given diagram explains a temperature-dependent resistance in a refrigeration system. The circuit has a resistance of 10Ω at 25°C and an explanation of your answer.

Its temperature coefficients are: $\alpha = 0.01 \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \frac{1}{\text{K}^2}$.

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R = 10\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) = 10\Omega \cdot (1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2)$$

The power of the resistor $P = U^2/R$ and the heat $Q = P \cdot t$ to heat the refrigerant. Therefore, a solution is to use a heat pump up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad \text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \quad R = 10\Omega \cdot (1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2)$$

d

Exercise E6 Analyzing complex Impedances**(written test, approx. 14 % of a 60-minute written test, WS2022)**

2. Calculate the phasor voltage $\underline{U}$ across the $10 \mu\text{F}$ capacitor in the circuit shown in the figure. The current $\underline{i}$ is given by $i(t) = 0.24 \cos(300t - 4.68^\circ) \text{ A}$. Both the voltage $\underline{U}$ and the current $\underline{i}$ shall be given.

After analysis, the full low-dimensional time-dependent voltage $u(t)$ shall be extracted and be given in phase notation $u(t) = \hat{U} \cos(300t - \varphi_u)$ with $\hat{U}$ in V and φ_u in degrees.

.. Calculate the physical values of the voltage $u(t)$ and the current $i(t)$.

Solution: $\underline{U} = 19.8 \angle -87.06^\circ \text{ V}$ and $u(t) = 19.8 \cos(300t - 87.06^\circ) \text{ V}$

Solution

```
\begin{align*} \underline{U} &= \frac{\underline{U}}{\underline{Z}} \parallel \underline{U} = \{50 \text{ V} \} \parallel \{ \underline{Z} \} \\ \text{The current and voltage are in phase and the voltage is pure real} \\ \text{resulting in } \underline{U} &= \underline{Z} \cdot \underline{i} = \{ 0.24 \angle -4.68^\circ \} \cdot \{ 4.68 \angle 90^\circ \} \\ \text{Therefore, the component of the voltage across the capacitor is } & \\ \underline{U} &= \{ 0.24 \angle -4.68^\circ \} \cdot \{ 4.68 \angle 90^\circ \} = \{ 1.1232 \angle 85.32^\circ \} \\ \text{Impedance } \underline{Z} &= \frac{1}{j\omega C} = \frac{1}{j \cdot 300 \cdot 10 \cdot 10^{-6}} = \{ -j4.68 \} \\ \underline{U} &= \{ 1.1232 \angle 85.32^\circ \} \cdot \{ -j4.68 \} = \{ -5.258 \angle 1.68^\circ \} \\ \text{The absolute value of the voltage is } & \\ \underline{U} &= 5.258 \text{ V} \end{align*}
```

With the complex part comes the physical value $u(t) = 5.258 \cos(300t - 1.68^\circ) \text{ V}$

The phase φ_u can be calculated as $\varphi_u = \arctan \left(\frac{\text{Im}(\underline{U})}{\text{Re}(\underline{U})} \right) = \arctan \left(\frac{-5.258}{0} \right) = -90^\circ$

d

Exercise E8 Complex Impedance Circuit**(written test, approx. 15 % of a 60-minute written test, WS2022)**

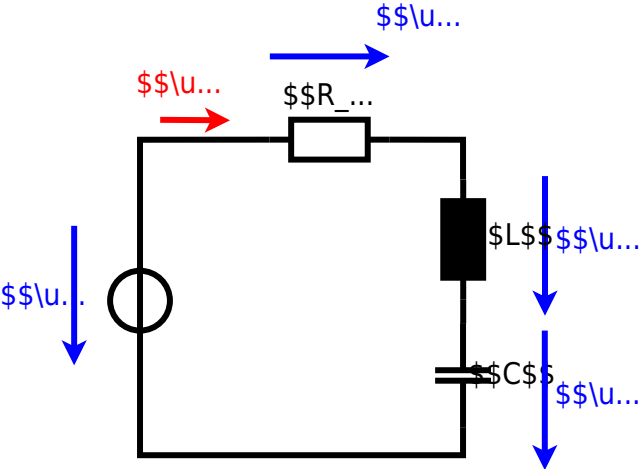
2. Calculate the time-averaged power P_{avg} in the circuit shown in the figure. The voltage source $u(t) = 3.0 \cos(2\pi \cdot 15 \cdot t) \text{ V}$ and the current $i(t) = 0.24 \cos(2\pi \cdot 15 \cdot t - 4.68^\circ) \text{ A}$ are given. The circuit consists of a resistor $R = 10 \Omega$, an inductor $L = 10 \text{ mH}$, and a capacitor $C = 10 \mu\text{F}$.

Solution: $P_{\text{avg}} = 0.22 \text{ W}$

Solution: The linear source is connected with an inductor of $330 \mu\text{H}$ and a capacitor of $30.22 \mu\text{F}$, all in series.

Result: $\underline{Z} = 19.8 \angle -87.06^\circ \Omega$ and $\underline{U} = 19.8 \angle -87.06^\circ \text{ V}$

.. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.



d

Exercise E7 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with $R_1 = 1.00 \text{ k}\Omega$, a capacitor with $C_1 = 40 \text{ nF}$, and an AC voltage source with $U_0 = 20 \text{ V}$ and $f = 4 \text{ MHz}$. The current I through the resistor R_1 shall have the same absolute value of the impedance as a capacitor C_1 at $f_1 = 4 \text{ MHz}$.

Solution

$$\begin{aligned} R_1 &= 1.00 \text{ k}\Omega \end{aligned}$$

$$R_2 = 10.0 \text{ k}\Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by

$$\begin{aligned} \underline{Z} &= R + jX_L \\ \underline{Z} &= R + j\omega L \end{aligned}$$

Since R and X_L are perpendicular, the magnitude of the impedance is given by

$$|\underline{Z}| = \sqrt{R^2 + X_L^2}$$

Therefore, the resulting current of the parallel circuit is given as:

$$\begin{aligned} I &= \frac{U_0}{|\underline{Z}|} \\ I &= \frac{U_0}{\sqrt{R^2 + X_L^2}} \end{aligned}$$

Back to the first formula:

$$\begin{aligned} R &= \sqrt{X_L^2 - R^2} \\ R &= \sqrt{(2\pi f C)^2 - R^2} \end{aligned}$$

d

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

2. The heating element is used to heat the wire with a temperature of 180°C . The power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Calculate the current I needed to operate the heating element.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ }\Omega\text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \text{with } R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

d

Exercise E5 Charging Capacitors

(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (without the light bulb) also takes the form of an RC circuit. The voltage across the capacitor is again $U_c(t_0) = 0 \text{ V}$ at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

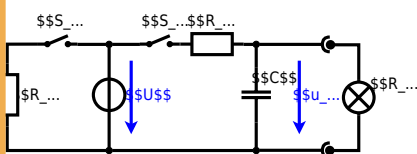
Solution

Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

$$\Delta U = U - \frac{R_2}{R_1 + R_2} U$$

Solution **Part 1** **Part 2** **Part 3** **Part 4** **Part 5** **Part 6** **Part 7** **Part 8** **Part 9** **Part 10** **Part 11** **Part 12** **Part 13** **Part 14** **Part 15** **Part 16** **Part 17** **Part 18** **Part 19** **Part 20** **Part 21** **Part 22** **Part 23** **Part 24** **Part 25** **Part 26** **Part 27** **Part 28** **Part 29** **Part 30** **Part 31** **Part 32** **Part 33** **Part 34** **Part 35** **Part 36** **Part 37** **Part 38** **Part 39** **Part 40** **Part 41** **Part 42** **Part 43** **Part 44** **Part 45** **Part 46** **Part 47** **Part 48** **Part 49** **Part 50** **Part 51** **Part 52** **Part 53** **Part 54** **Part 55** **Part 56** **Part 57** **Part 58** **Part 59** **Part 60** **Part 61** **Part 62** **Part 63** **Part 64** **Part 65** **Part 66** **Part 67** **Part 68** **Part 69** **Part 70** **Part 71** **Part 72** **Part 73** **Part 74** **Part 75** **Part 76** **Part 77** **Part 78** **Part 79** **Part 80** **Part 81** **Part 82** **Part 83** 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On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

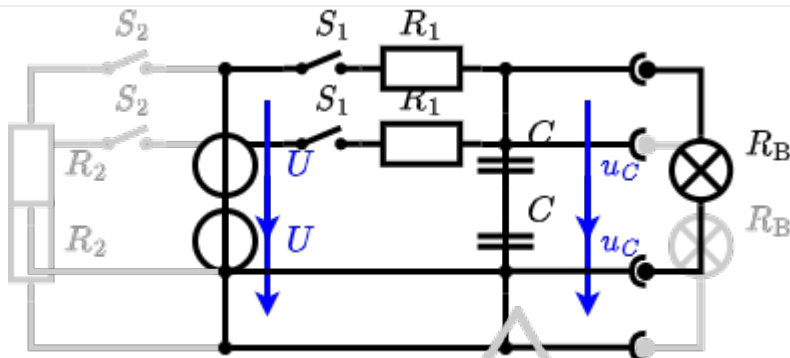


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_c(t_1) = 0.5$



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

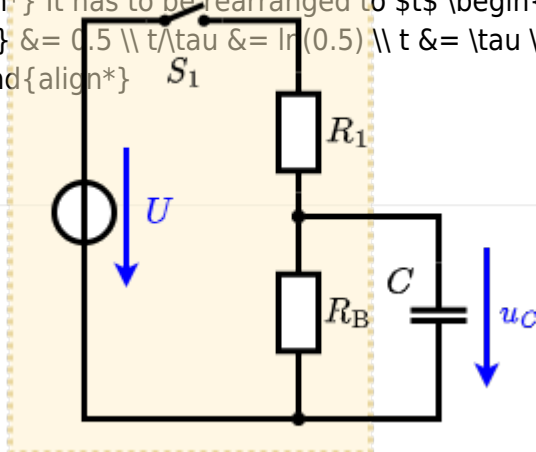
The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$

It has to be rearranged to t :

$$(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$

S_1



An equivalent linear voltage source can be given with U , R_1 , and R_{B} as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 \sim \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \sim \Omega$$

$$\begin{aligned} u_c(t_2) &= U_s \cdot (1 - e^{t_2/(R_i \cdot C)}) \parallel \&= \{1\over 2\} \\ &\cdot U \cdot (1 - e^{1 \sim \rm{ms}/(10 \sim \Omega \cdot 100 \sim \rm{\mu F})}) \end{aligned}$$

d

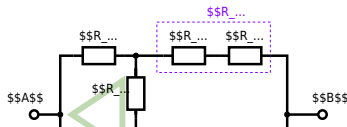
Exercise E3 Pure Resistor Network Simplification

(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall in with \$ Rise 200 at 0 ntegrate, \$ Qu 2 a R e 1 1 5 6 t a n O e s a f a n d u p s w e l l e n
B e u l A o i e d . \$ \rm B\$.

Solution

$$R_Y = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{100 \cdot 100}{100 + 100} = 50 \Omega$$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

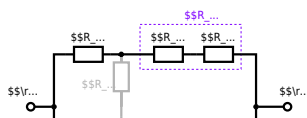
$$R_Y = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{100 \cdot 100}{100 + 100} = 50 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 50 \Omega + (50 \Omega + 400 \Omega) \parallel (50 \Omega + 100 \Omega) \parallel R_{eq}$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$\begin{aligned} R_{\text{eq}} &= (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel \\ R_{\text{eq}} &= (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \{ \{ 500 \sim \Omega \cdot 200 \sim \Omega \} \over {500 \sim \Omega + 200 \sim \Omega} \} \parallel \\ \end{aligned}$$

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