

# Exam Winter Semester 2022

## Student Group

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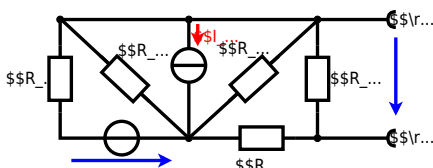
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## Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

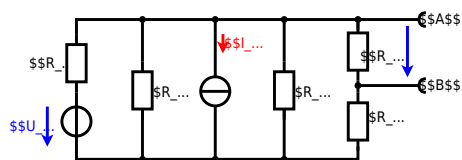
$$\begin{aligned} U_{\text{S}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



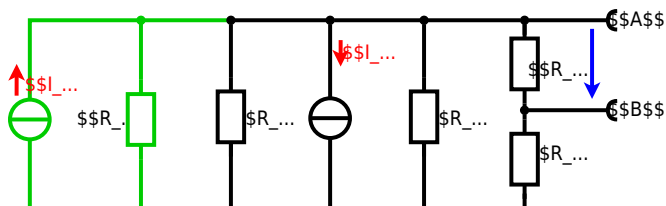
Calculated the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{S}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ .  
 $R_1 = 5.0 \text{ } \Omega$ ,  $U_2 = 6.0 \text{ V}$ ,  $R_3 = 10 \text{ } \Omega$ ,  $I_4 = 4.2 \text{ A}$ ,  $R_5 = 10 \text{ } \Omega$ ,  $R_6 = 7.5 \text{ } \Omega$ ,  $R_7 = 15 \text{ } \Omega$  Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{24} = R_{135} \cdot I_{24} \quad \&= \left( \frac{U_2}{R_1} - I_4 \right) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between \$A\$ and \$B\$ is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \quad \&= \left( \frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0\,\Omega$ , so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with  $R_1 \parallel R_3 \parallel R_5 = 5\,\Omega \parallel 10\,\Omega \parallel 10\,\Omega = 5\,\Omega \parallel 5\,\Omega = 2.5\,\Omega$ :

$$U_{AB} = \frac{6.0\,\text{V}}{5.0\,\Omega} - 4.2\,\Omega \cdot \frac{15\,\Omega \cdot 2.5\,\Omega}{7.5\,\Omega + 15\,\Omega + 2.5\,\Omega} \quad \&= 15\,\Omega \parallel (7.5\,\Omega + 2.5\,\Omega)$$

### Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. According to question 2, the effective resistance is not given, but can be identified. Result:  $R_{eff} = 10\,\Omega$  at  $x = 25$  is your answer.

Its temperature coefficients are:  $\alpha = 0.01\,\text{K}^{-1}$  and  $\beta = 71 \cdot 10^{-6}\,\text{K}^{-2}$

**Result**

The temperature inside the refrigeration system can reach down to  $-40\,\text{C}$ .

$$R = 6.5\,\text{k}\Omega \quad \text{at } -40\,\text{C}$$

The power transfer is reduced by a factor of 2 and the heat flow is halved. Therefore, a solution is to let the float up the refrigeration system.

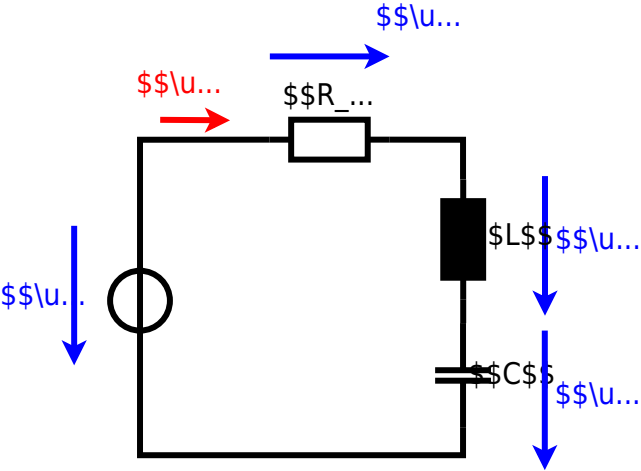
Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \\ R = 10\,\text{k}\Omega \cdot \left( 1 + 0.01\,\text{K}^{-1} \cdot (-40\,\text{C} - 25\,\text{C}) + 71 \cdot 10^{-6}\,\text{K}^{-2} \cdot (-40\,\text{C} - 25\,\text{C})^2 \right)$$









### Exercise E1 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with  $R_1 = 1.00 \text{ k}\Omega$  and a capacitor with  $C_1 = 40 \text{ nF}$ . The voltage across the resistor is  $U_R = 10.0 \text{ V}$  at a frequency of  $f = 4 \text{ MHz}$ . Calculate the voltage across the capacitor  $U_C$  and the total impedance  $Z$  of the circuit.

Solution

$$R_1 = 1.00 \text{ k}\Omega$$

$$C_1 = 40 \text{ nF}$$

A series circuit means that the current is constant on every component.

The equivalent impedance for  $R$  and  $C$  combined is given by

$$Z = \sqrt{R^2 + X_C^2}$$

Parallel circuit means that the voltage is the same on  $R$  and  $C$ . Since  $U_R = 10.0 \text{ V}$  and  $U_C = 10.0 \text{ V}$ , the total voltage is  $U = 20.0 \text{ V}$ .

Since  $U_R = 10.0 \text{ V}$  and  $U_C = 10.0 \text{ V}$ , the total voltage is  $U = 20.0 \text{ V}$ .

Since  $U_R = 10.0 \text{ V}$  and  $U_C = 10.0 \text{ V}$ , the total voltage is  $U = 20.0 \text{ V}$ .

Therefore, the resulting current of the parallel circuit is given as:

$$I = \frac{U}{Z}$$

$$I = \frac{20.0 \text{ V}}{\sqrt{(1.00 \text{ k}\Omega)^2 + (10.0 \text{ V})^2}} = 1.96 \text{ mA}$$

$$I = 1.96 \text{ mA}$$

$$R_3 \cdot I = X_C \cdot I$$

$$R_3 = X_C$$

$$R_3 = \frac{1}{2\pi f C_3} \cdot \sqrt{I^2 - I^2}$$

$$R_3 = \frac{1}{2\pi f C_3} \cdot \sqrt{I^2 - I^2}$$

### Exercise E1 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A heating element made of nichrome wire with a diameter of  $d = 0.5 \text{ mm}$  and a length of  $L = 3 \text{ m}$  is used. The power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary.

Calculate the current  $I$  and the resistance  $R$  of the heating element.

The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .

The heating element is  $3 \text{ m}$  long and has a diameter of  $0.5 \text{ mm}$ .

Calculate the resistance  $R$  of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \cdot W}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad \& \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ 1.10 \cdot 10^{-6} \cdot \Omega \cdot m \cdot \frac{4 \cdot 3 \cdot m}{(3.57 \cdot 10^{-3} \cdot m)^2 \cdot \pi} \quad \& \quad \end{aligned} \quad \text{align*}$$

## Exercise E1 Charging Capacitors

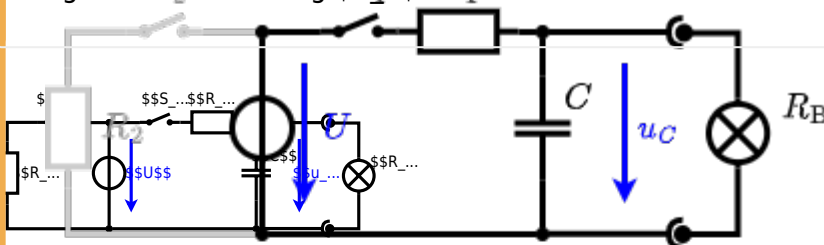
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (without the light bulb) also is the circuit of Exercise 2. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution** Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution** The ideal voltage source  $U = 12 \text{ V}$  is in series with the resistor  $R_1 = 20 \Omega$ . The voltage  $u_c(t)$  is independent of this series combination.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_2$ .

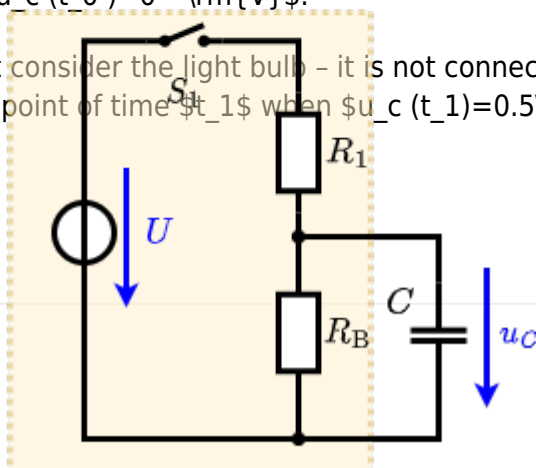


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \Omega$  and a capacitor of  $C = 100 \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $= 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U$$

So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

$$(1 - e^{-t_2/(R_1 \cdot C)}) = \frac{1}{2} \Rightarrow t_2 = \tau \cdot \ln(2) = 10 \Omega \cdot 100 \mu F \cdot \ln(2) = 0.693 \text{ ms}$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$

It has to be rearranged to  $t$ :

$$(1 - e^{-t/\tau}) = 0.5 \Rightarrow e^{-t/\tau} = 0.5 \Rightarrow -t/\tau = \ln(0.5) \Rightarrow t = \tau \cdot \ln(2) = R_1 \cdot C \cdot \ln(2)$$

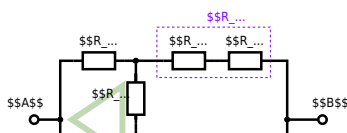
### Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be simplified with  $R_1 = 200 \Omega$ ,  $R_2 = 100 \Omega$ ,  $R_3 = 150 \Omega$ ,  $R_4 = 100 \Omega$ , and the voltage source  $U = 10 \text{ V}$ .  
Result given:  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

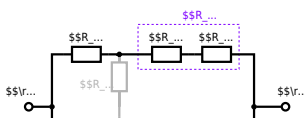
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



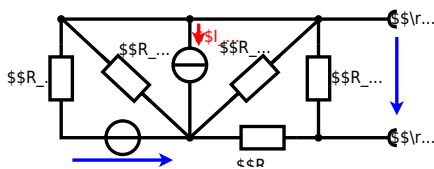
The equivalent resistor is given by a parallel configuration of resistors in series:

$$\begin{aligned} R_{\text{eq}} &= (R_2 + R_1 + R_3) \parallel (R_2 + R_3) \parallel R_4 \\ R_{\text{eq}} &= (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel \\ R_{\text{eq}} &= (500 \, \Omega) \parallel (200 \, \Omega) \parallel R_{\text{eq}} = \frac{500 \, \Omega \cdot 200 \, \Omega}{500 \, \Omega + 200 \, \Omega} \end{aligned}$$

### Exercise E4 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

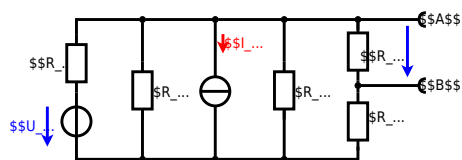
$$\begin{aligned} U_s &= U_{\text{AB}} = 4.5 \, \text{V} \\ R_i &= R_{\text{AB}} = 6 \, \Omega \end{aligned}$$



Calculate the internal resistance  $R_{\text{int}}$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors A and B.  $\begin{aligned} R_1 &= 5.0 \, \Omega, \quad U_2 = 6.0 \, \text{V}, \quad R_3 = 10 \, \Omega, \quad I_4 = 4.2 \, \text{A}, \\ R_5 &= 10 \, \Omega, \quad R_6 = 7.5 \, \Omega, \quad R_7 = 15 \, \Omega \end{aligned}$  Use equivalent sources in order to simplify the circuit!

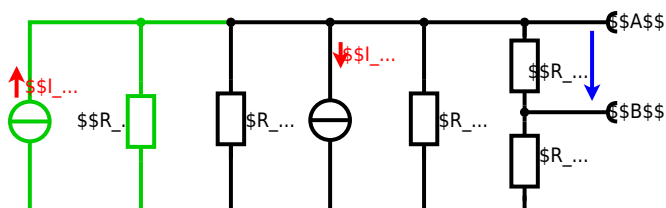
### Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :





Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{24} = R_{135} \cdot I_{24} \quad \&= \left( \frac{U_2}{R_1} - I_4 \right) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between \$A\$ and \$B\$ is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \quad \&= \left( \frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0\Omega$ , so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with  $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$ :

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \quad R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

## Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. According to question 2, the effect of temperature on the resistance of the thermistor can be described by the equation:

Resistance of the thermistor  $R$  in  $\Omega$  as a function of the temperature  $T$  in  $^\circ\text{C}$  is given by:

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

**Result**  
The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

$$R = 6.5 \text{ k}\Omega \quad \text{at } T = -40^\circ\text{C}$$

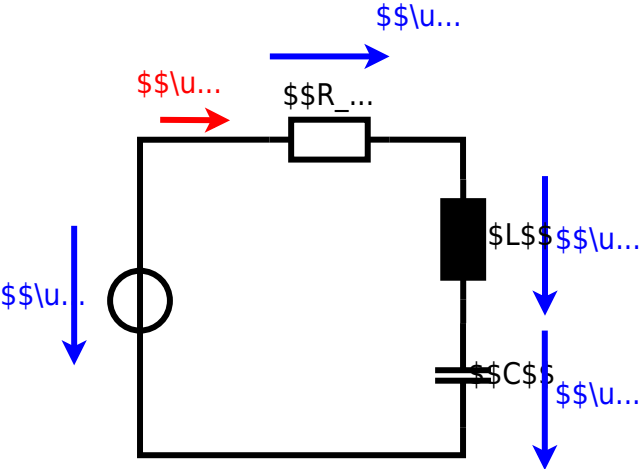
The power transfered to the thermistor is  $P = \frac{U^2}{R}$  and  $Q = P \cdot t$ . Therefore, a solution is to heat the thermistor up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad \& \quad \Delta T = T_{\text{end}} - T_{\text{start}} \quad R = 10 \text{ k}\Omega \cdot \left( 1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2 \right)$$







### Exercise E7 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit with a resistor with  $R_1 = 1.00 \text{ k}\Omega$  and a capacitor with  $C_1 = 40 \text{ nF}$  is connected to an AC voltage source with  $U = 10 \text{ V}$  and  $f = 4 \text{ MHz}$ . The high-frequency source is a capacitor with  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ . The resistor  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ .

Solution

$$\begin{aligned} R_1 &= 1.00 \text{ k}\Omega \\ R_2 &= 10.0 \text{ k}\Omega \end{aligned}$$

$$\begin{aligned} R_2 &= 10.0 \text{ k}\Omega \\ R_3 &= 10.0 \text{ k}\Omega \end{aligned}$$

A series circuit means that the current is constant on every component.

The equivalent impedance for  $R_1$  and  $C_1$  combined is given by

$$\begin{aligned} \underline{Z}_{\text{eq}} &= R_1 + \frac{1}{j\omega C_1} \\ \underline{Z}_{\text{eq}} &= R_1 + \frac{1}{j\omega C_1} \end{aligned}$$

$$\begin{aligned} \underline{Z}_{\text{eq}} &= R_1 + \frac{1}{j\omega C_1} \\ \underline{Z}_{\text{eq}} &= R_1 + \frac{1}{j\omega C_1} \end{aligned}$$

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### Exercise E1 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A heating element made of nichrome wire with a diameter of  $d = 1.80 \text{ mm}$  and a length of  $L = 3.57 \text{ m}$  is connected to a power supply with  $U = 10 \text{ V}$ . The power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary.

Calculate the current  $I$  and the resistance  $R$  of the heating element.

The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .

The heating element is  $L = 3.57 \text{ m}$  long and has a diameter of  $d = 1.80 \text{ mm}$ .

Calculate the resistance  $R$  of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \cdot W}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad \& \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ 1.10 \cdot 10^{-6} \cdot \Omega \cdot \frac{4 \cdot 3 \cdot m}{(3.57 \cdot 10^{-3} \cdot m)^2 \cdot \pi} \quad \& \quad \end{aligned} \quad \text{align*}$$

### Exercise E5 Charging Capacitors

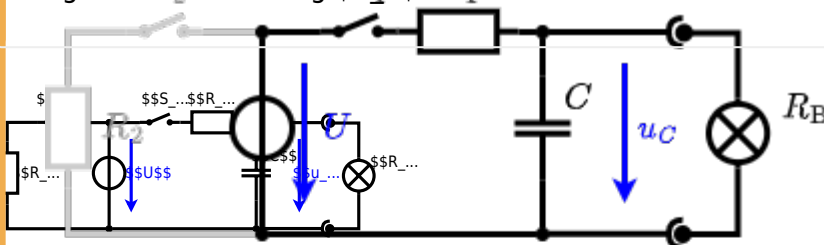
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (without the light bulb) also is the circuit of Exercise 2. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution** Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution** The ideal voltage source  $U = 12 \text{ V}$  is in series with the resistor  $R_1 = 20 \Omega$ . The voltage  $u_c(t)$  is independent of this series combination.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_2$ .

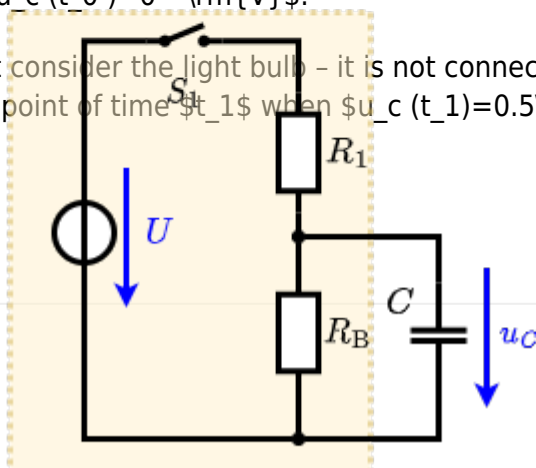


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \Omega$  and a capacitor of  $C = 100 \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $= 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms}/(10 \Omega \cdot 100 \mu\text{F})})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ : 
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to 
$$(1 - e^{-t/\tau}) = 0.5$$
 
$$e^{-t/\tau} = 0.5 \quad \ln(0.5) = -t/\tau \quad t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$

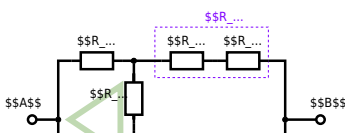
### Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with  $R_1 = 200 \Omega$ ,  $R_2 = 100 \Omega$ ,  $R_3 = 150 \Omega$ ,  $R_4 = 100 \Omega$  and the voltage  $U = 10 \text{ V}$ .  
Result given:  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.





Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

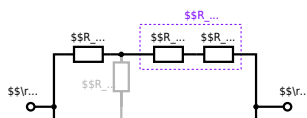
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \{ \{ 500 \sim \Omega \cdot 200 \sim \Omega \} \over {500 \sim \Omega + 200 \sim \Omega} \}$$

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Last update: **2023/03/29 22:54**

