

Exam Winter Semester 2022

Student Group

First Name	Surname	Matrikel Nr.

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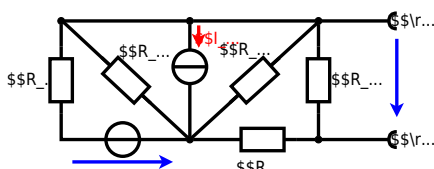
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Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

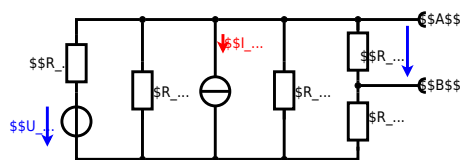
$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



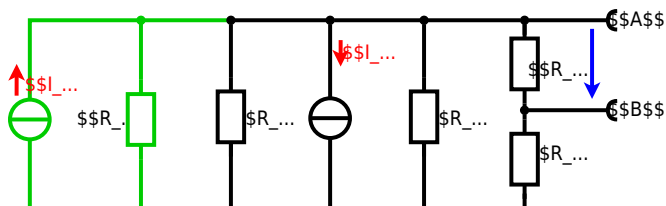
Calculated the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B .
$$\begin{aligned} R_1 &= 5.0 \text{ } \Omega, \quad U_2 = 6.0 \text{ V}, \quad R_3 = 10 \text{ } \Omega, \quad I_4 = 4.2 \text{ A}, \\ R_5 &= 10 \text{ } \Omega, \quad R_6 = 7.5 \text{ } \Omega, \quad R_7 = 15 \text{ } \Omega \end{aligned}$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{24} = R_{135} \cdot I_{24} \quad \&= \left(\frac{U_2}{R_1} - I_4 \right) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \quad \&= \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \quad R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. According to question 2, the effective resistance is not given, but can be identified. Result: $R_{eff} = 15\Omega$ at $x = 25$ is your answer.

Its temperature coefficients are: $\alpha = 0.01 \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \frac{1}{\text{K}^2}$

Result

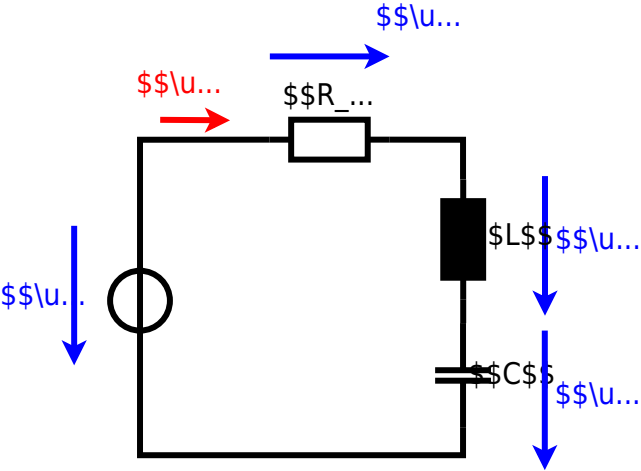
The temperature inside the refrigeration system can reach down to -40°C .

$$R = 6.5\text{ k}\Omega \quad \text{at } -40^\circ\text{C}$$

The power transfer is reduced by a factor of 10 and the heat flow is reduced by a factor of 10. Therefore, a solution is to use a heat exchanger.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \quad R = 10\text{ k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$



Exercise E1 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with $R_1 = 1.00 \text{ k}\Omega$ and a capacitor with $C_1 = 40 \text{ nF}$. The voltage across the resistor is $U_R = 10.0 \text{ V}$ at a frequency of $f = 4 \text{ MHz}$. Calculate the voltage across the capacitor U_C at the same frequency.

Solution

$$R_1 = 1.00 \text{ k}\Omega$$

$$C_1 = 40 \text{ nF}$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and C combined is given by

$$Z = \sqrt{R^2 + X_C^2}$$

Parallel circuit means that the voltage is the same on R and C . Since $U_R = 10.0 \text{ V}$ and $U_C = 10.0 \text{ V}$, the voltage across the combination is $U = 14.14 \text{ V}$.

Therefore, the resulting current of the parallel circuit is given as:

$$I = \frac{U}{Z} = \frac{14.14 \text{ V}}{\sqrt{1.00^2 + 10.0^2} \text{ k}\Omega} = 1.41 \text{ mA}$$

$$U_C = I \cdot X_C = 1.41 \text{ mA} \cdot 10.0 \text{ k}\Omega = 14.1 \text{ V}$$

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Exercise E1 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A heating element made of nichrome wire with a diameter of $d = 0.5 \text{ mm}$ and a length of $L = 3 \text{ m}$ is used for heating. The power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Calculate the current I and the voltage U for heating elements.

The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 0.5 mm .

Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \cdot W}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad \& \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ 1.10 \cdot 10^{-6} \cdot \Omega \cdot \frac{4 \cdot 3 \cdot m}{(3.57 \cdot 10^{-3} \cdot m)^2 \cdot \pi} \quad \& \quad \end{aligned} \quad \text{align*}$$

Exercise E1 Charging Capacitors

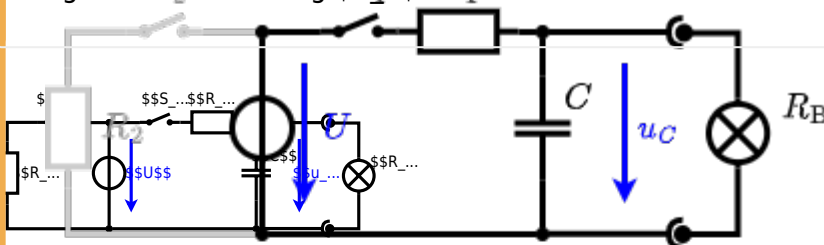
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (without the light bulb) also is the circuit of Exercise 2. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution The ideal voltage source $U = 12 \text{ V}$ is in series with the resistor $R_1 = 20 \Omega$. The voltage $u_c(t)$ is independent of this series combination.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .

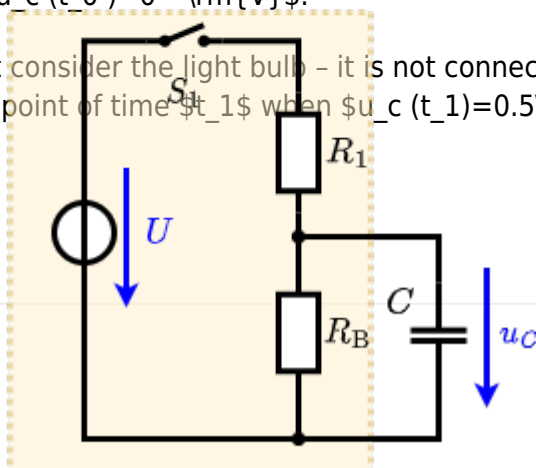


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \Omega$ and a capacitor of $C = 100 \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($= 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

$$(1 - e^{-t_2/(R_1 \cdot C)}) = \frac{1}{2} \quad (1 - e^{-t_2/(10 \Omega \cdot 100 \mu F)}) = \frac{1}{2}$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$

It has to be rearranged to t :

$$(1 - e^{-t/\tau}) = 0.5 \quad 1 - e^{-t/\tau} = 0.5 \quad e^{-t/\tau} = 0.5 \quad -t/\tau = \ln(0.5) \quad t = \tau \cdot \ln(0.5) \quad t = R_1 \cdot C \cdot \ln(0.5)$$

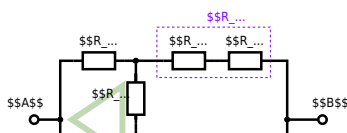
Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be simplified with $R_1 = 200 \Omega$, $R_2 = 100 \Omega$, $R_3 = 150 \Omega$, $R_4 = 100 \Omega$ and the voltage $U = 10 \text{ V}$.
Result given: R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

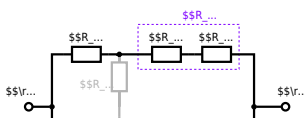
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



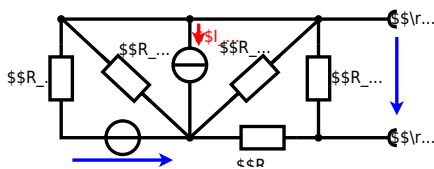
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel R_{\text{eq}} = (500 \, \Omega) \parallel (200 \, \Omega) \parallel R_{\text{eq}} = \frac{500 \, \Omega \cdot 200 \, \Omega}{500 \, \Omega + 200 \, \Omega} \parallel R_{\text{eq}}$$

Exercise E4 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

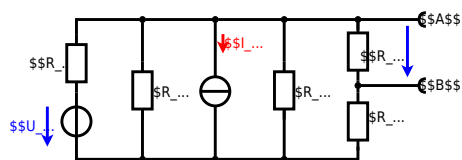
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \, \text{V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \, \Omega$$



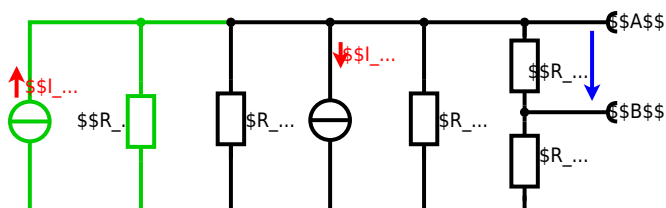
Calculate the internal resistance R_{int} and the source voltage U_{S} of an equivalent linear voltage source on the connectors A and B. $\begin{aligned} R_1 &= 5.0 \, \Omega, \quad U_2 = 6.0 \, \text{V}, \quad R_3 = 10 \, \Omega, \quad I_4 = 4.2 \, \text{A}, \\ R_5 &= 10 \, \Omega, \quad R_6 = 7.5 \, \Omega, \quad R_7 = 15 \, \Omega \end{aligned}$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_4 \cdot R_1$$

$$U_{24} = R_{135} \cdot I_{24} \quad \&= \left(\frac{U_2}{R_1} - I_4 \right) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between \$A\$ and \$B\$ is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \quad \&= \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \quad R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. According to question 2, the effect of resistance on power can be identified. The resistor of the system (R_0) is at 25°C and its resistance is 10Ω . Its temperature coefficients are: $\alpha = 0.01\text{K}^{-1}$ and $\beta = 71 \cdot 10^{-6}\text{K}^{-2}$.

Result

The temperature inside the refrigeration system can reach down to -40°C .

$$R = 10\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad \Delta T = T_{\text{end}} - T_{\text{start}} = -40^\circ\text{C} - 25^\circ\text{C} = -65^\circ\text{C}$$

The power transfer is reduced by a factor of 10 and the heat flow is reduced by a factor of 10. Therefore, a solution is to use a heat exchanger to pre-heat the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad \Delta T = T_{\text{end}} - T_{\text{start}} = -40^\circ\text{C} - 25^\circ\text{C} = -65^\circ\text{C}$$

$$R = 10\Omega \cdot (1 + 0.01\text{K}^{-1} \cdot (-65^\circ\text{C}) + 71 \cdot 10^{-6}\text{K}^{-2} \cdot (-65^\circ\text{C})^2)$$

Exercise E6 Analyzing complex Impedances

(written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance $\underline{Z}$ of the circuit shown in the figure. The components R and X_L shall be given.

After analysis, the full bridge network must be simplified to a single complex impedance $\underline{Z}$ in parallel with the voltage source $\underline{U}$. The result is $\underline{Z} = (2 + j4) \Omega$.

3. Calculate the physical values of the two components.

Solution: $R = 2 \Omega$, $X_L = 4 \Omega$ (inductor) and $X_C = -4 \Omega$ (capacitor).

Solution

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle 0^\circ}{10 \angle -45^\circ} = 5 \angle 45^\circ = 5 \cos 45^\circ + j 5 \sin 45^\circ = 3.54 + j 3.54 \Omega$$

The voltage $\underline{U}$ is $50 \angle 0^\circ$ V and the current $\underline{I}$ is $10 \angle -45^\circ$ A. The impedance $\underline{Z}$ is the ratio of voltage to current. The real part is 3.54Ω and the imaginary part is $j 3.54 \Omega$. The real part is the resistance R and the imaginary part is the inductive reactance X_L . The value of X_L is 3.54Ω .

With the complex part $3.54 + j 3.54 \Omega$, the physical values are $R = 3.54 \Omega$ and $X_L = 3.54 \Omega$. The phase φ can be calculated as $\varphi = \arctan \left(\frac{X_L}{R} \right) = \arctan \left(\frac{3.54}{3.54} \right) = 45^\circ$.

Exercise E8 Complex Impedance Circuit

(written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance $\underline{Z}$ of the circuit shown in the figure. The components R and X_L shall be given. The voltage source $\underline{U}$ is $3.0 \angle 0^\circ$ V and the current $\underline{I}$ is $1.0 \angle 0^\circ$ A.

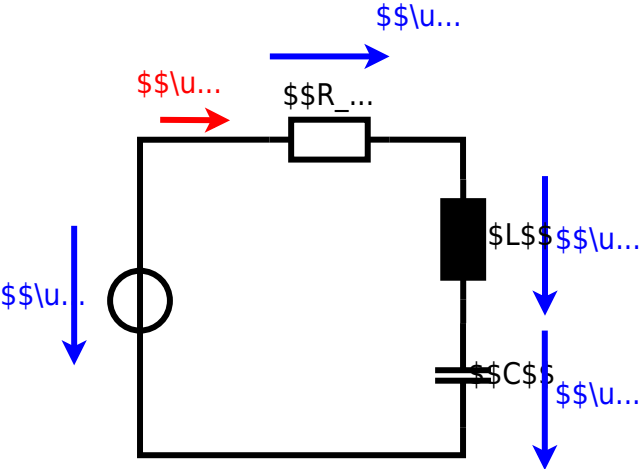
Solution: The linear source is connected with an inductor of $330 \mu\text{H}$ and a capacitor of $0.22 \mu\text{F}$, all in series.

Result

3. Draw the circuit diagram of the bridge network. Label all components, voltages, and currents.

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0 \angle 0^\circ}{1.0 \angle 0^\circ} = 3 \angle 0^\circ = 3 \cos 0^\circ + j 3 \sin 0^\circ = 3 \angle 0^\circ = 3 \Omega$$

With $\underline{Z} = 3 \angle 0^\circ \Omega$, the physical values are $R = 3 \Omega$ and $X_L = 0 \Omega$. The phase φ is 0° .



Exercise E7 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with $R_1 = 1.00 \text{ k}\Omega$ and a capacitor with $C_1 = 40 \text{ nF}$. The voltage across the resistor is $U_R = 10.0 \text{ V}$ at a frequency of $f = 4 \text{ MHz}$. Calculate the voltage across the capacitor U_C and the total impedance Z of the circuit.

Solution

$$R_1 = 1.00 \text{ k}\Omega$$

$$C_1 = 40 \text{ nF}$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and C combined is given by

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + \left(\frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 40 \text{ nF}}\right)^2}$$

$$Z = \sqrt{(1.00 \text{ k}\Omega)^2 + (0.995 \text{ k}\Omega)^2} = 1.41 \text{ k}\Omega$$

$$I = \frac{U_R}{R_1} = \frac{10.0 \text{ V}}{1.00 \text{ k}\Omega} = 10.0 \text{ mA}$$

$$U_C = I \cdot X_C = 10.0 \text{ mA} \cdot 0.995 \text{ k}\Omega = 9.95 \text{ V}$$

Exercise E1 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A heating element made of nichrome wire with a diameter of $d = 0.5 \text{ mm}$ and a length of $L = 3 \text{ m}$ is used for heating. The power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Calculate the current I and the voltage U for heating elements.

The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 0.5 mm .

Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \cdot W}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad \& \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ 1.10 \cdot 10^{-6} \cdot \Omega \cdot \frac{4 \cdot 3 \cdot m}{(3.57 \cdot 10^{-3} \cdot m)^2 \cdot \pi} \quad \& \quad \end{aligned} \quad \text{align*}$$

Exercise E5 Charging Capacitors

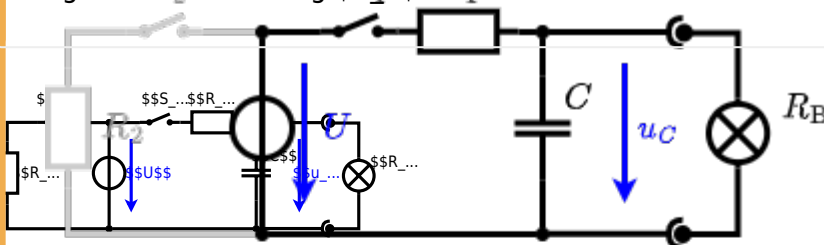
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (without the light bulb) also is the circuit of Exercise 2. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution The ideal voltage source $U = 12 \text{ V}$ is in series with the resistor $R_1 = 20 \Omega$. The voltage $u_c(t)$ is independent of this series combination.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .

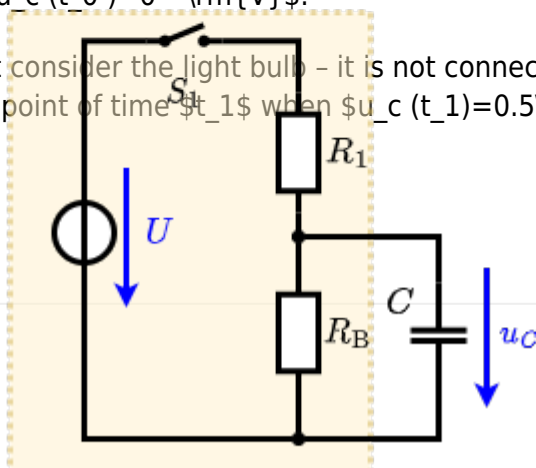


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \Omega$ and a capacitor of $C = 100 \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($= 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to
$$(1 - e^{-t/\tau}) = 0.5$$

$$e^{-t/\tau} = 0.5 \quad \ln(0.5) = -t/\tau \quad t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$

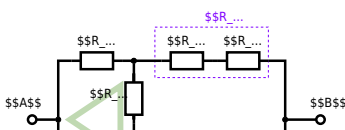
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be simplified with $R_1 = 200 \Omega$, $R_2 = 100 \Omega$, $R_3 = 150 \Omega$, $R_4 = 100 \Omega$ and the voltage $U = 10 \text{ V}$.
Result given: R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

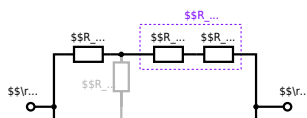
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = (R_2 + R_1 + R_1) || (R_2 + R_2) || (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) || (100 \sim \Omega + 100 \sim \Omega) || (500 \sim \Omega) || (200 \sim \Omega) || (500 \sim \Omega \cdot 200 \sim \Omega) / (500 \sim \Omega + 200 \sim \Omega)$$

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Last update: **2023/03/29 23:19**

