Exam Winter Semester 2022

Student Group

First Name	Surname	Matrikel Nr.

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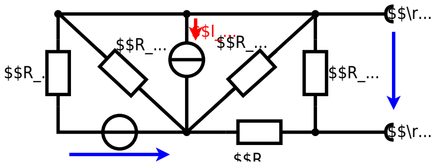
Exercise E1 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

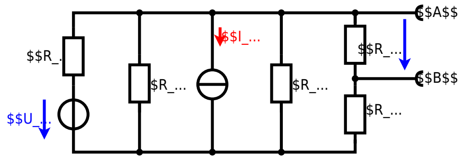
\begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}



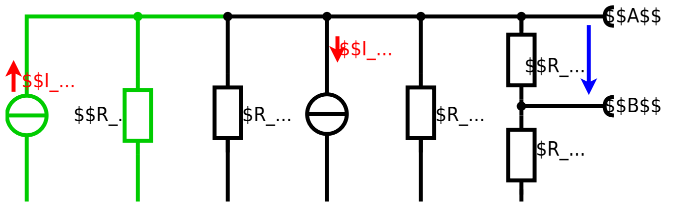
Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$. \begin{align*} R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

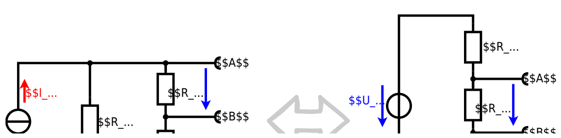
The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and R_1:



Now a lot of them can be combined. The resistors R_1, R_3, R_5 are in parallel, like also I_2 and I_4: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following: \begin{align*} U_{24}

&= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit, there is a voltage divider given by R_{135}, R_6, and R_7.
Therefore the voltage between A and B is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || (R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| (7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

Exercise E1 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

A thermistor is used as a temperature sensor in a refrigeration system. The thermistor has a resistance of $10 ~\rm{k}\Omega$ at $+25 ~°\rm{C}$.
Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$
The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

2. Additionally, explain [illegible] Give an explanation for your answer.

Result

\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}

Result

Resistors transfer electrical energy out of the circuit and generate heat. Therefore, a resistive sensor might heat up the refrigeration system.

The power of the resistor $P = U \cdot I = R \cdot I^2 = {U^2}\over{R}$ is equivalent to the heat flow.
Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

Solution

\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C}) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}

Exercise E1 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

A circuit with the complex impedance $\underline{Z}$ shall be analyzed. ... components. (R and $\underline{X}_1$) shall be given.

After analysis, the following formula for the impedance was extracted \begin{align*} \underline{Z} = \left({{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega \end{align*}

1. Calculate the physical values of the two components.

Solution

\begin{align*} C = 103 ~\rm{µF} \end{align*}

\begin{align*} R = 0.24 ~\Omega \\ L = 2.26 ~\rm{mH} \end{align*}

Solution

\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }}

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.

Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$ \begin{align*} X_C &= {{1}\over{\omega \cdot C}} = X_L \\ C &= {{1}\over{\omega \cdot X_L}} \\ &= {{1}\over{2\pi \cdot 300 ~\rm{Hz} \cdot 4.68 ~\Omega}} \\ \end{align*}

\begin{align*} \underline{Z} &= \left({{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{9+16}} \cdot (3-4{\rm j}) + 5{\rm j} \right) ~\Omega \\ &= 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega \\ &= R + {\rm j} X_L \\ \end{align*}

The absolute value $|\underline{I}|$ can be calculated as \begin{align*} |\underline{I}| &= {{50 ~\rm{V}}\over{| 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega |}} \\ &= {{50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}} \end{align*}

With the complex part comes the physical value: \begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \\ \end{align*}

The phase φ_i can be calculated as \begin{align*} \varphi_i &= \arctan \left({{\Im()}\over{\Re()}} \right) \\ &= \arctan \left({{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

Result

\begin{align*} |\underline{I}| &= 10.67 ~\rm{A} \\ \varphi_i &= -87.06° \end{align*}

Exercise E1 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

A circuit is driven by the sinusoidal voltage source $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ with an internal resistance of $10 ~\Omega$.

This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

1. Draw the circuit diagram of the given circuit.

Label all components, voltages, and currents.

2. Calculate the impedances $\underline{Z}_C$, $\underline{Z}_L$, and $|\underline{Z}|$ of the overall circuit.

3. Calculate the current I (rms value). Choose an appropriate scaling factor and write it down.

Solution

Solution

Result

\begin{align*} Z_L &= 31.1 ~\Omega \\ Z_C &= 48.2 ~\Omega \\ Z &= 19.8 ~\Omega \end{align*}

\begin{align*} I = 107 ~{\rm mA} \end{align*}

\begin{align*} Z &= {{\hat{U}}\over{\hat{I}}} \\ \hat{I} &= {{\hat{U}}\over{Z}} \\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}\\ \end{align*}

Result

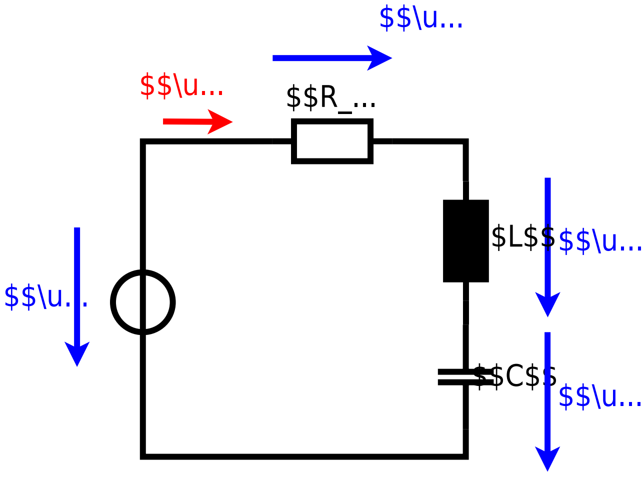
With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$: \begin{align*} I &= {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ &= {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}} \\ \end{align*}

\begin{align*} Z_L &= 2\pi \cdot f \cdot L \\ &= 2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF} \\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}}}\\ \end{align*}

$$\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}$$

$$10...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$$\u...
$$R_...
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Exercise E1 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Calculate the resistor values which have to be set in the following cases.

1. A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.
2. A RL series circuit with $L_2=4.7 ~\rm µH$ is used in AC voltage circuit ($U_2=1.0 ~\rm V$, $f_2=450 ~\rm kHz$), which generates a current $I_2=60 ~\rm mA$.
3. A RC parallel circuit with $C_3=4.7 ~\rm nF$ is used in AC voltage circuit ($U_3=1.0 ~\rm V$, $f_3=200 ~\rm kHz$), which generates a current $I_{3R}=10 ~\rm mA$ through R_3.

Solution

\begin{align*} R_1 &= \underline{X}_{C1} \\ &= {{1}\over{2\pi \cdot f_1 \cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~\rm nF}} \\ \end{align*}

Result

\begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}

Solution

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by \begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j} \cdot \omega L \end{align*} Since ${\rm j} \cdot \omega L$ is perpendicular to R_2 this can be simplified to: \begin{align*} \left| {{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left({{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}

This can be rearranged to get R_2: \begin{align*} R_2 &= \sqrt{ \left({{1 ~\rm V}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}

Result

\begin{align*} R_2 &= 10.0 ~\Omega \\ \end{align*}

Solution

Parallel circuit means that the voltage is the same on R_3 and C_3. It gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since R_3 is perpendicular to ${\rm j}\cdot {X}_{3C}$, too).

Therefore, the resulting current of the parallel circuit is given as: \begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ {I}_{3C} &= \sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2} \\ \end{align*}

Back to the first formula: \begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of a Nichrome wire with a round cross-section is used in an electric oven. A power dissipation (= heat flow) of $P=40 ~\rm{W}$ is necessary.

Nichrome is a common Nickel-Chrome alloy for heating elements.

The Nichrome wire has a resistivity of $1.10\cdot 10^{-6} ~\Omega \rm{m}$.

The heating element is $3 ~\rm{m}$ long and has a diameter of $3.57 ~\rm{mm}$.

1. Calculate the resistance R of the heating element.
2. The heating element is used to heat the oven to a temperature of $180 ~°\rm{C}$. Calculate the current I needed to operate it.

Result

\begin{align*} R=&1.10~33 ~\Omega \\ \end{align*}

Solution

\begin{align*} P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I=

\sqrt{\frac{P}{R}} = \sqrt{\frac{40 ~\rm{W}}{0.33 ~\Omega}} \end{align*}
\begin{align*} R &= \rho \cdot \frac{l}{A} && | \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} && \\ R &= 1.10\cdot 10^{-6} ~\Omega \rm{m} \cdot \frac{4 \cdot 3~\rm{m}}{(3.57\cdot 10^{-3}~\rm{m})^2 \cdot \pi} && \\ \end{align*}

Exercise E1 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

[illegible] the S_2 is open. The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

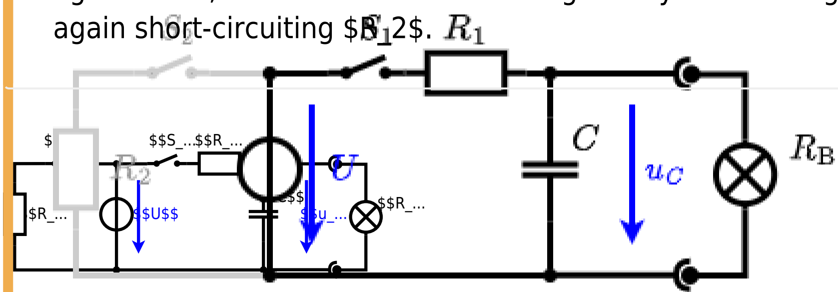
Result / Solution

Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and $R_\rm B$.

\begin{align*} [illegible] \end{align*}

Solution [illegible] \begin{align*} \Delta W_R &= \Delta W_C [illegible] \end{align*} [illegible] independent of this resistor.

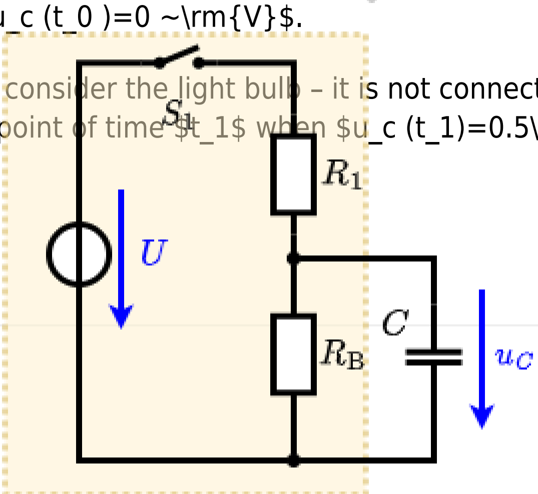
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2.

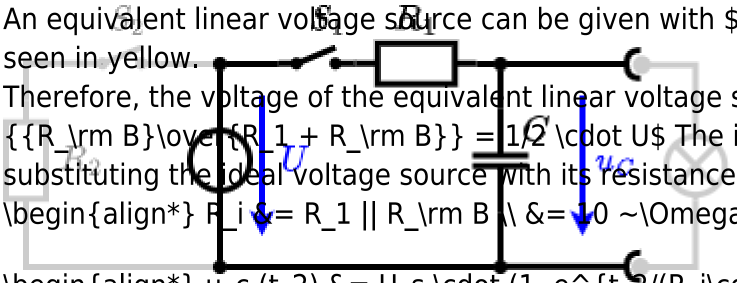


The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution





An equivalent linear voltage source can be given with U, R_1, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit). \begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

\begin{align*} u_c (t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}

Exercise E1 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following circuit with $R_1=200 ~\Omega$, $R_2=R_3=100 ~\Omega$ is given. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

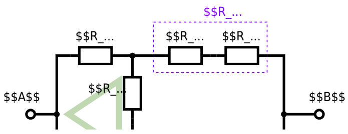
2. The switch shall now be closed.

Result

Solution

\begin{align*} R_{\rm eq} &= 142.8 ~\Omega \\ \end{align*}

Now a wye-delta transformation is necessary.

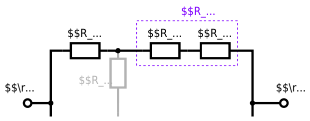


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution

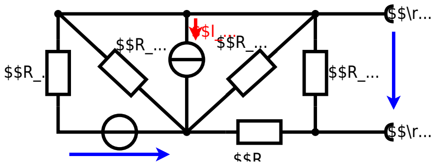


The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega)&&||(100 ~\Omega + 100 ~\Omega) &&\\ R_{\rm eq} &= (500 ~\Omega)&&||(200 ~\Omega)&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}

Exercise E4 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

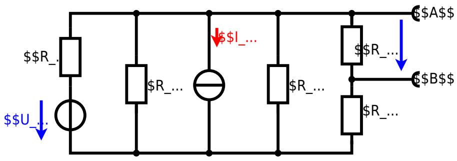
\begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}



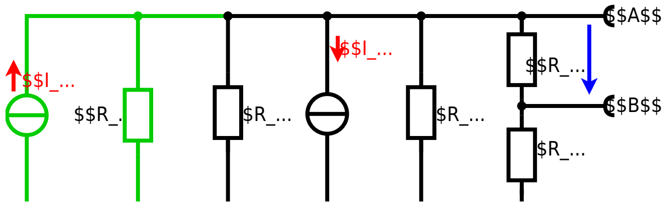
Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$. \begin{align*} R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

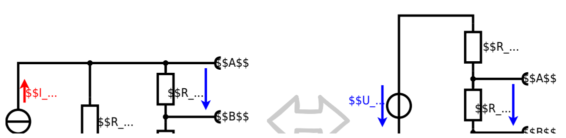
The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and R_1:



Now a lot of them can be combined. The resistors R_1, R_3, R_5 are in parallel, like also I_2 and I_4: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following: \begin{align*} U_{24}

&= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit, there is a voltage divider given by R_{135}, R_6, and R_7.
Therefore the voltage between A and B is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || (R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| (7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

A thermistor is used as a temperature sensor in a refrigeration system. The thermistor has a resistance of $10 ~\rm{k}\Omega$ at $+25 ~°\rm{C}$.
Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$
The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

2. Additionally, explain [illegible]

3. Regarding question 2: [illegible] refrigeration system [illegible]? Give an explanation for your answer.

Result

\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}

Result

The power of the resistor is $P = U \cdot I = R \cdot I^2 = {{U^2}\over{R}}$ [illegible] equivalent to the heat flow.
Resistors transfer electrical energy out of the circuit and generate heat. Therefore, a [illegible] should [illegible] heat up the refrigeration system.
Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

Solution

\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C}) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}

Exercise E6 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

A complex impedance shall be analyzed. The complex impedance $\underline{Z}$ consists of two components. (R and $\underline{X}_1$) shall be given.

This component shall be dimensioned in such a way that the current and voltage are in phase.

$$\underline{Z} = \left({{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega$$

Solution

1. Calculate the physical values of the two components.

Solution

$$\underline{Z} = \left({{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ = \left({{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ = \left({{2}\over{9+16}} \cdot (3-4{\rm j}) + 5{\rm j} \right) ~\Omega \\ = 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega$$

With the complex part comes the physical value:

$$X_L = \omega L \\ L = {{X_L}\over{2\pi \cdot f}} \\ = {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}}$$

Result

$$R = 0.24 ~\Omega \\ L = 2.26 ~\rm{mH}$$

2. Now the ideal voltage source $U = 50 ~\rm{V}$, $f = 300 ~\rm{Hz}$ is connected to the two components. Calculate the current in phase of the current.

Solution

$$\underline{I} = {{\underline{U}}\over{\underline{Z}}} \\ = {{50 ~\rm{V}}\over{0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega}} \\ = {{50 ~\rm{V}}\over{0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega}} \cdot {{0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega}\over{0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega}}$$

The absolute value $|\underline{I}|$ can be calculated as:

$$|\underline{I}| = {{|\underline{U}|}\over{|\underline{Z}|}} \\ = {{50 ~\rm{V}}\over{| 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega |}} \\ = {{50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}}$$

The phase φ_i can be calculated as

$$\varphi_i = \arctan \left({{\Im()}\over{\Re()}} \right) \\ = \arctan \left({{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right)$$

Result

$$|\underline{I}| = 10.7 ~\rm{A} \\ \varphi_i = -87.06°$$

3. A third component $\underline{X}_2$ shall be added in series to the two components. After connecting, the current and voltage are in phase. Calculate the component and its value.

Solution

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.

Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$

$$X_C = {{1}\over{\omega \cdot C}} = X_L \\ C = {{1}\over{\omega \cdot X_L}} \\ = {{1}\over{2\pi \cdot 300 ~\rm{Hz} \cdot 4.68 ~\Omega}}$$

Result

$$C = 103 ~\rm{µF}$$

Exercise E8 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

A circuit is supplied by a voltage source $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ with an internal resistance of $10 ~\Omega$.

This linear source is connected with an inductor of $330 ~{\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

1. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

2. Calculate the complex impedances $\underline{Z}_C$, $\underline{Z}_L$ and $\underline{Z}$. Choose an appropriate scaling factor and write it down.

Solution

$$Z_C = {{1}\over{2\pi \cdot f \cdot C}} \\ = {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}$$

$$Z_L = 2\pi \cdot f \cdot L \\ = 2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}$$

Result

$$Z_L = 31.1 ~\Omega \\ Z_C = 48.2 ~\Omega \\ Z = 19.8 ~\Omega$$

3. Calculate the current I through the circuit as well as the phase shift φ.

Solution

$$Z = {{\hat{U}}\over{\hat{I}}} \\ \hat{I} = {{\hat{U}}\over{Z}}$$

With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$:

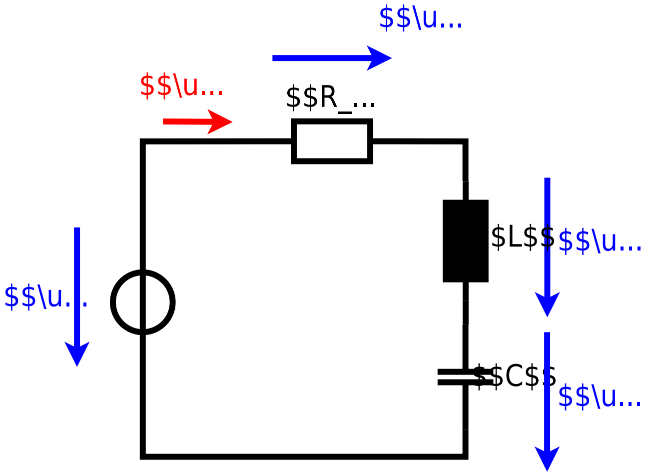
$$I = {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ = {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}}$$

Result

$$I = 107.3 ~{\rm mA}$$

$$\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}$$

`$$10...`


$$\u...
$$\u...
$$R_...
$L$$
$$\u...
$$\u...
$$C$$
$$\u...

Exercise E7 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Calculate the resistor values which have to be used in the following circuits.

2. A series circuit with ... $L_2=4.7 ~\rm µH$ ... $f_2=450 ~\rm kHz$, which generates a current ...

3. A RC parallel circuit with $C_3=4.7 ~\rm µF$ is used in an AC voltage circuit ($U_3=1.0 ~\rm V$, $f_3=2.0 ~\rm kHz$) which generates a current $I_{3R}=1.0 ~\rm A$ through R_3.

Result

1. A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

Solution

Solution

\begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}

Solution

\begin{align*} R_2 &= 7.0 ~\Omega \\ \end{align*}

\begin{align*} R_3 &= 10.0 ~\Omega \\ \end{align*}

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by \begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j} \cdot \omega L \end{align*} Since ${\rm j} \cdot \omega L$ is perpendicular to R_2 this can be simplified to: \begin{align*} \left| {{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left({{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}

This can be rearranged to get R_2: \begin{align*} R_2 &= \sqrt{ \left({{U}\over{I}} \right)^2 - X_{L2}^2 } \\ &= \sqrt{ \left({{1~{\rm V}}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}

Parallel circuit means that the voltage is the same on R_3 and C_3.

\begin{align*} R_1 &= |\underline{X}_{C1}| \\ &= {{1}\over{2\pi \cdot f \cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~\rm nF}} \\ \end{align*}

... gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since R_3 is perpendicular to ${\rm j}\cdot {X}_{3C}$, too).

Therefore, the resulting current of the parallel circuit is given as: \begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ {I}_{3C} &= \sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2} \end{align*}

Back to the first formula: \begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of a Nichrome wire with a round cross-section is used in an electric oven. For this a power dissipation (= heat flow) of $P=40 ~\rm{W}$ is necessary.

Nichrome is a common Nickel-Chrome alloy for heating elements.

The Nichrome wire has a resistivity of $1.10\cdot 10^{-6} ~\Omega \rm{m}$.

The heating element is $3 ~\rm{m}$ long and has a diameter of $3.57 ~\rm{mm}$.

1. Calculate the resistance R of the heating element.

2. The heating element needs to be run at a temperature of $180 ~°\rm{C}$.

Calculate the current I needed to operate it.

Result

Solution

\begin{align*} R=&110.33 ~\rm{m}\Omega \end{align*}

Solution

\begin{align*} P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I=

\sqrt{\frac{P}{R}} = \sqrt{\frac{40 ~\rm{W}}{0.33 ~\Omega}} \end{align*} \begin{align*} R &= \rho \cdot \frac{l}{A} && | \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} && \\ R &= 1.10\cdot 10^{-6} ~\Omega \rm{m} \cdot \frac{4 \cdot 3~\rm{m}}{(3.57\cdot 10^{-3}~\rm{m})^2 \cdot \pi} && \\ \end{align*}

Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

[illegible] The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Result

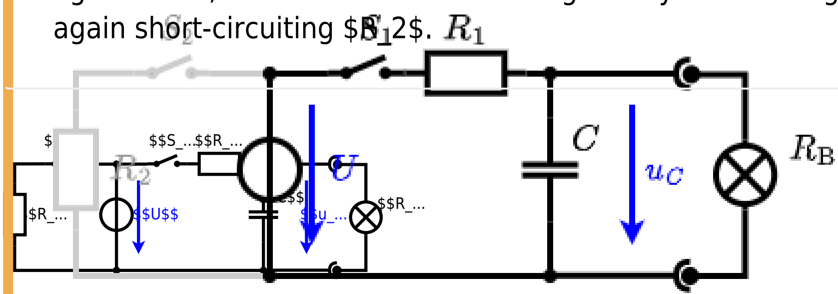
Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and $R_\rm B$.

[illegible]

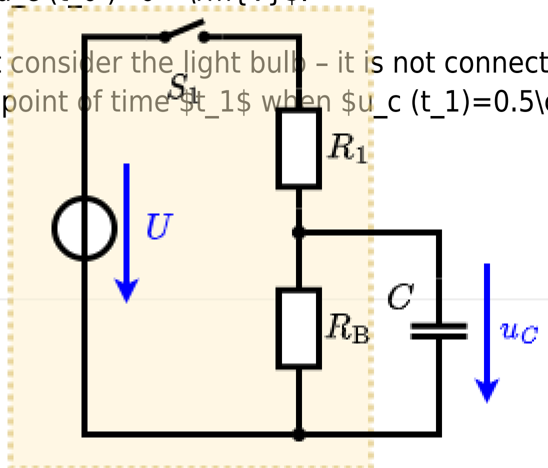
Solution

[illegible] $U=12~\rm{V}$ [illegible]

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2.

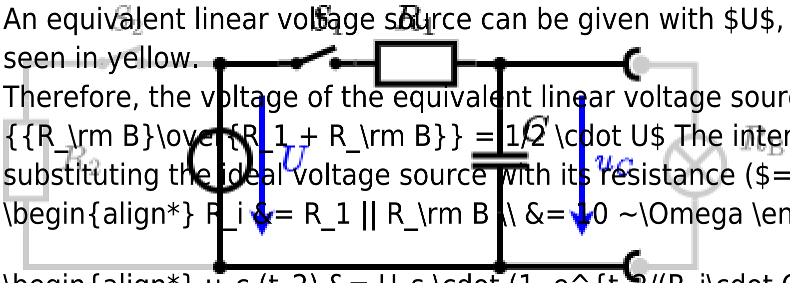


The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.



1. First do not consider the light bulb – it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution



An equivalent linear voltage source can be given with U, R_1, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit). \begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}

\begin{align*} u_c (t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}

Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following circuit with $R_1=200~\Omega$, $R_2=R_3=100~\Omega$ and ... is given.

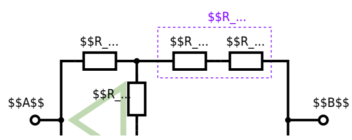
2. The switch shall now be closed. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Result

Solution

\begin{align*} R_{\rm eq} &= 132.8 ~\Omega \\ \end{align*}

Now a wye-delta transformation is necessary.

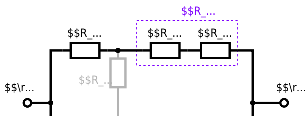


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega)&&||(100 ~\Omega + 100 ~\Omega) &&\\ R_{\rm eq} &= (500 ~\Omega)&&||(200 ~\Omega)&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}