

Exam Winter Semester 2022

Student Group

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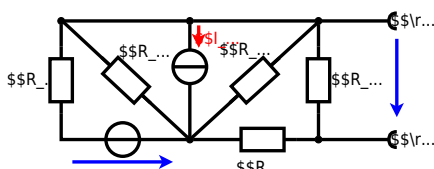
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Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

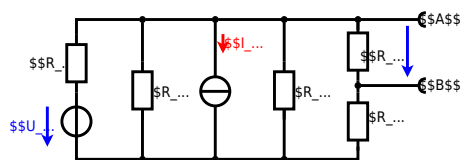
$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{ri}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



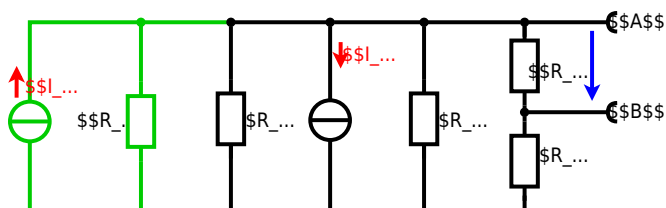
Calculated the internal resistance R_{ri} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B .
$$\begin{aligned} R_1 &= 5.0 \text{ } \Omega, \quad U_2 = 6.0 \text{ V}, \quad R_3 = 10 \text{ } \Omega, \quad I_4 = 4.2 \text{ A}, \\ R_5 &= 10 \text{ } \Omega, \quad R_6 = 7.5 \text{ } \Omega, \quad R_7 = 15 \text{ } \Omega \end{aligned}$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{24} = R_{135} \cdot I_{24} \quad \&= \left(\frac{U_2}{R_1} - I_4 \right) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between \$A\$ and \$B\$ is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \quad \&= \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \quad R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. According to question 2, the effect of temperature on the resistance of the thermistor is not linear. The thermistor has a resistance of $10\text{k}\Omega$ at 25°C and a temperature coefficient of $\alpha = 0.01\text{K}^{-1}$ and $\beta = 71 \cdot 10^{-6}\text{K}^{-2}$.

Its temperature coefficients are: $\alpha = 0.01\text{K}^{-1}$ and $\beta = 71 \cdot 10^{-6}\text{K}^{-2}$.

Result

The temperature inside the refrigeration system can reach down to -40°C .

$$R = 10\text{k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad \Delta T = T_{\text{end}} - T_{\text{start}} = -40^\circ\text{C} - 25^\circ\text{C} = -65^\circ\text{C}$$

The power of the heater is $P = \frac{U^2}{R}$. If the resistance R increases, the power P decreases. Therefore, a solution is to use a thermostat to control the heater.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = 10\text{k}\Omega \cdot (1 + 0.01\text{K}^{-1} \cdot (-65^\circ\text{C}) + 71 \cdot 10^{-6}\text{K}^{-2} \cdot (-65^\circ\text{C})^2)$$

Exercise E1 Analyzing complex Impedances

(written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance $\underline{Z}$ of the circuit shown in the figure. The components $\underline{R}$ and $\underline{X}_L$ shall be given.

After analysis, the full bidimensional complex impedance has been extracted and written in the form $\underline{Z} = R + jX_L$ where $R = 4 \Omega$ and $X_L = 5 \Omega$.

3. Calculate the physical values of the two components.

Solution: $R = 4 \Omega$ and $X_L = 5 \Omega$ (inductor) and $C = 10 \mu F$ (capacitor).

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{4 + j5} = 5.88 \angle -51.1^\circ \text{ A}$$

 The voltage across the inductor is $\underline{V}_L = j\omega L \underline{I} = j40 \cdot 5.88 \angle -51.1^\circ = 235.2 \angle 38.9^\circ \text{ V}$
 The voltage across the capacitor is $\underline{V}_C = \frac{\underline{U}}{1 + j2} = 25 \angle -63.4^\circ \text{ V}$
 The phase shift φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{5}{4}\right) = 51.1^\circ$
 With the complex part comes the physical value $L = \frac{X_L}{\omega} = \frac{5}{40} = 0.125 \text{ H}$
 The phase shift φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{5}{4}\right) = 51.1^\circ$

Exercise E1 Complex Impedance Circuit

(written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex impedance $\underline{Z}$ of the circuit shown in the figure. The components $\underline{R}$ and $\underline{X}_L$ shall be given. The voltage source is $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t) \text{ V}$ and the frequency is $f = 15 \text{ kHz}$.

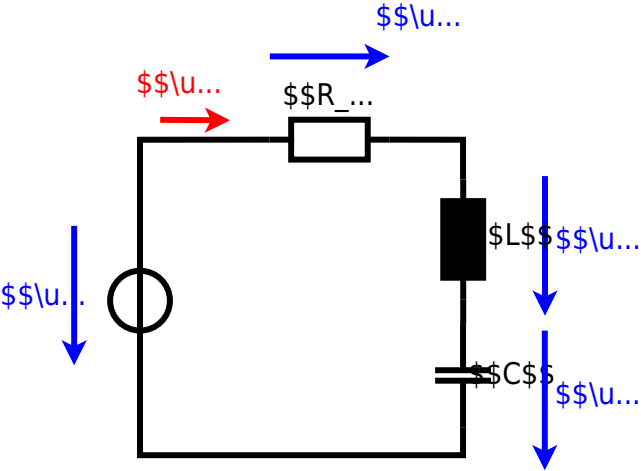
Solution: The linear source is connected with an inductor of $330 \mu H$ and a capacitor of $0.22 \mu F$, all in series.

Result

3. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

Solution: $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0 \angle 0^\circ}{0.22 \angle -51.1^\circ} = 13.6 \angle 51.1^\circ \Omega$
 $\underline{Z}_L = j\omega L = j2\pi \cdot 15 \cdot 330 \cdot 10^{-6} = j3.16 \Omega$
 $\underline{Z}_C = \frac{1}{j\omega C} = \frac{1}{j2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j15.9 \Omega$

With $\underline{Z} = 13.6 \angle 51.1^\circ \Omega$ and $\underline{U} = 3.0 \angle 0^\circ \text{ V}$, the current is $\underline{I} = \frac{\underline{U}}{\underline{Z}} = 0.22 \angle -51.1^\circ \text{ A}$
 $\underline{Z} = 13.6 \angle 51.1^\circ \Omega$
 $\underline{Z}_L = j3.16 \Omega$
 $\underline{Z}_C = -j15.9 \Omega$



Exercise E1 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit with two resistors $R_1 = 1.00 \text{ k}\Omega$ and $R_2 = 10.0 \text{ k}\Omega$ is connected to an AC voltage source $U = 10 \text{ V}$ at a frequency $f = 4 \text{ MHz}$. A capacitor $C = 40 \text{ nF}$ is connected in parallel with R_2 . Calculate the total impedance Z of the circuit.

Solution

$$Z = \sqrt{R_1^2 + \left(\frac{R_2}{\sqrt{1 + (\omega C R_2)^2}} \right)^2}$$

$$Z = \sqrt{1.00^2 + \left(\frac{10.0}{\sqrt{1 + (2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9})^2}} \right)^2}$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R_2 and C combined is given by

$$\frac{1}{Z_{R_2C}} = \frac{1}{R_2} + \frac{1}{\frac{1}{j\omega C}} \quad \Rightarrow \quad Z_{R_2C} = \frac{R_2}{1 + (\omega C R_2)^2}$$

Therefore, the resulting current of the parallel circuit is given as:

$$I = \frac{U}{Z} = \frac{U}{\sqrt{R_1^2 + \left(\frac{R_2}{\sqrt{1 + (\omega C R_2)^2}} \right)^2}}$$

Back to the first formula:

$$Z = \sqrt{R_1^2 + \left(\frac{R_2}{\sqrt{1 + (\omega C R_2)^2}} \right)^2} = \sqrt{1.00^2 + \left(\frac{10.0}{\sqrt{1 + (2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9})^2}} \right)^2} = 1.00 \text{ k}\Omega$$

Exercise E1 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A heating element made of nichrome wire with a diameter of $d = 0.5 \text{ mm}$ is used. The power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Calculate the current I needed to operate it.

The nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 0.5 \text{ mm}$.

Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \Rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \cdot W}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad \& \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ 1.10 \cdot 10^{-6} \cdot \Omega \cdot m \cdot \frac{4 \cdot 3 \cdot m}{(3.57 \cdot 10^{-3} \cdot m)^2 \cdot \pi} \quad \& \quad \end{aligned} \quad \text{align*}$$

Exercise E1 Charging Capacitors

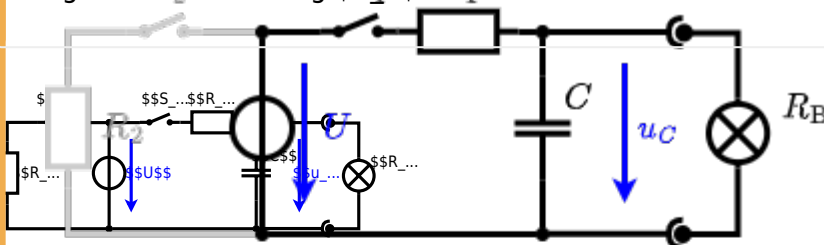
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (without the light bulb) also is the circuit of Exercise 2. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution The ideal voltage source U is in series with R_1 and R_2 . The voltage $u_c(t_2)$ is independent of R_2 .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .

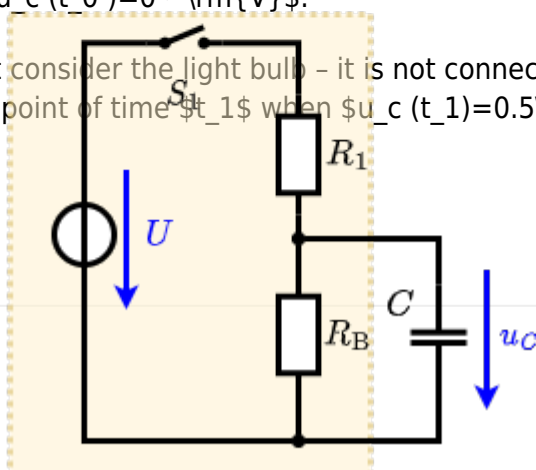


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \Omega$ and a capacitor of $C = 100 \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($= 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) \implies t = R_1 \cdot C \cdot \ln(0.5)$

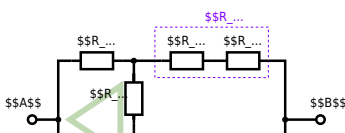
Exercise E1 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be simplified with $R_1 = 200 \Omega$, $R_2 = 100 \Omega$, $R_3 = 150 \Omega$, $R_4 = 100 \Omega$ and the voltage $U = 10 \text{ V}$. R_B is given.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

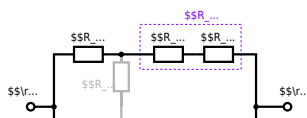
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



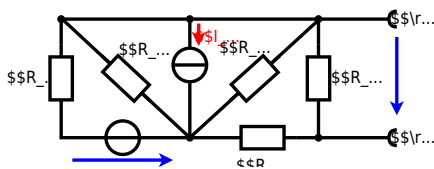
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_3) \parallel (R_4 + R_5) \parallel R_{\text{AB}} = (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel R_{\text{AB}} = (500 \, \Omega) \parallel (200 \, \Omega) \parallel R_{\text{AB}} = \frac{500 \, \Omega \cdot 200 \, \Omega}{500 \, \Omega + 200 \, \Omega} \parallel R_{\text{AB}}$$

Exercise E4 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

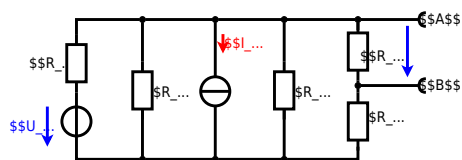
$$U_{\text{S}} = U_{\text{AB}} = 4.5 \, \text{V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \, \Omega$$



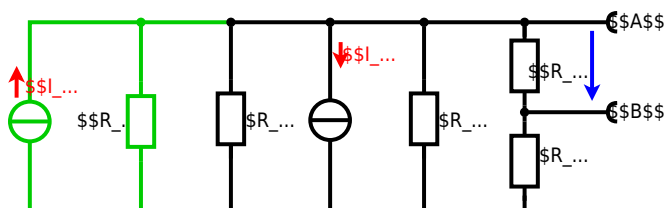
Calculate the internal resistance R_s and the source voltage U_s of an equivalent linear voltage source on the connectors A and B. $\begin{aligned} R_1 &= 5.0 \, \Omega, \quad U_2 = 6.0 \, \text{V}, \quad R_3 = 10 \, \Omega, \quad I_4 = 4.2 \, \text{A}, \\ R_5 &= 10 \, \Omega, \quad R_6 = 7.5 \, \Omega, \quad R_7 = 15 \, \Omega \end{aligned}$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$I_{24} = \frac{U_{24}}{R_6 + R_1 \parallel R_3 \parallel R_5} = \frac{(U_1 - U_4) \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_1 \parallel R_3 \parallel R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} = \frac{(U_1 - U_4) \cdot R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance (0Ω , so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5 \Omega \parallel 10 \Omega \parallel 10 \Omega = 5 \Omega \parallel 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \quad R_{AB} = 15 \Omega \parallel (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. According to question 2, the effective resistance of the refrigeration system can be identified as a resistor of 6.5Ω at 25°C and 10^{-6} K^{-2} for your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result

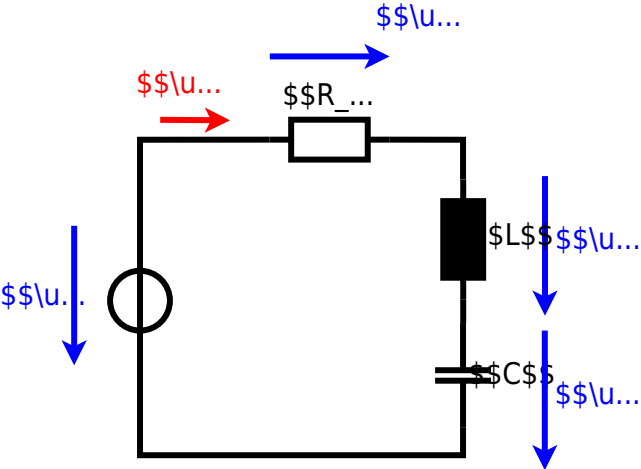
The temperature inside the refrigeration system can reach down to -40°C .

$$R = 6.5 \Omega \quad \text{at } 25^\circ\text{C}$$

The power transfer of the refrigerator is $P = \frac{U^2}{R}$ and $Q = \frac{U^2}{R} \cdot \frac{1}{2}$. Therefore, a solution is to heat the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \\ R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$



Exercise E7 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with $R_1 = 1.00 \text{ k}\Omega$ and a capacitor with $C_1 = 40 \text{ nF}$. The voltage across the resistor is $U_R = 10.0 \text{ V}$ at a frequency of $f = 4 \text{ MHz}$. Calculate the voltage across the capacitor U_C at the same frequency.

Solution

$$R_1 = 1.00 \text{ k}\Omega$$

$$C_1 = 40 \text{ nF}$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and C combined is given by

$$Z = \sqrt{R^2 + X_C^2}$$

Parallel circuit means that the voltage is the same on R and C . Since $U_R = 10.0 \text{ V}$ and $U_C = 10.0 \text{ V}$, the voltage across the combination is $U = 14.14 \text{ V}$.

Since $U_R = 10.0 \text{ V}$ and $U_C = 10.0 \text{ V}$, the voltage across the combination is $U = 14.14 \text{ V}$.

Therefore, the resulting current of the parallel circuit is given as:

$$I = \frac{U}{Z} = \frac{14.14 \text{ V}}{\sqrt{1.00^2 + 4.7^2}} = 3.0 \text{ mA}$$

Back to the first formula: $U_C = I \cdot X_C = 3.0 \text{ mA} \cdot 4.7 \text{ k}\Omega = 14.1 \text{ V}$

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Exercise E1 Resistance of a Wire by Resistivity

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A heating element made of nichrome wire with a diameter of $d = 0.5 \text{ mm}$ and a length of $L = 3 \text{ m}$ is used for heating. The power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Calculate the current I and the voltage U for heating elements.

The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 0.5 mm .

Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \cdot W}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \& \quad l \text{ with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad \& \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ 1.10 \cdot 10^{-6} \cdot \Omega \cdot \frac{4 \cdot 3 \cdot m}{(3.57 \cdot 10^{-3} \cdot m)^2 \cdot \pi} \quad \& \quad \end{aligned} \quad \text{align*}$$

Exercise E5 Charging Capacitors

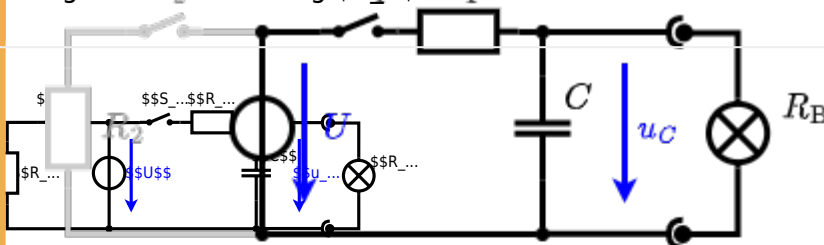
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (without the light bulb) also consists of a DC voltage source $U = 12 \text{ V}$, a resistor $R_1 = 20 \text{ }\Omega$, a capacitor $C = 100 \text{ }\mu\text{F}$, and a switch S_1 . The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_C(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and S_1 .

Solution The ideal voltage source U and the resistor R_1 in series can be replaced by an equivalent voltage source $U_{\text{eq}} = \frac{U}{2} = 6 \text{ V}$ and an internal resistance $R_{\text{eq}} = R_1 = 20 \text{ }\Omega$.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_1 .

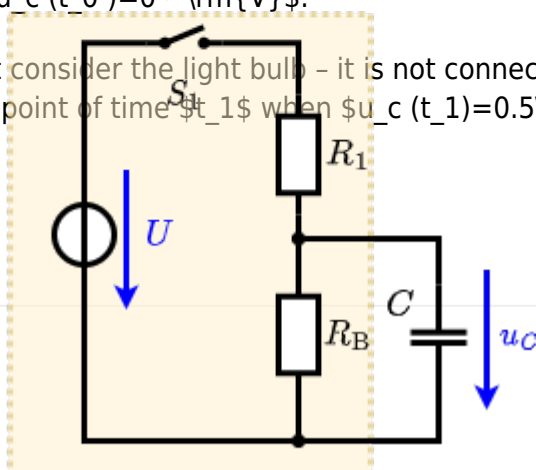


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($= 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to
$$(1 - e^{-t/\tau}) = 0.5$$

$$e^{-t/\tau} = 0.5 \quad \ln(0.5) = -t/\tau \quad t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$

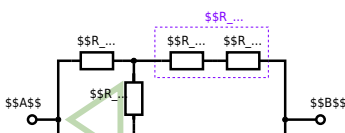
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be simplified with $R_1 = 200 \Omega$, $R_2 = 100 \Omega$, $R_3 = 150 \Omega$, $R_4 = 100 \Omega$ and the voltage source $U = 10 \text{ V}$. Result: R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

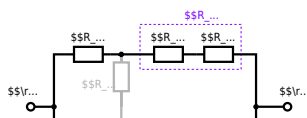
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \{ \{ 500 \sim \Omega \cdot 200 \sim \Omega \} \over {500 \sim \Omega + 200 \sim \Omega} \} \parallel$$

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