# Exam Winter Semester 2022

## Student Group

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**Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)**

A heating element made of a Nichrome wire with a round cross-section is used in an electric oven.
Nichrome is a common Nickel-Chrome alloy for heating elements.
The Nichrome wire has a resistivity of $1.10\cdot 10^{-6} ~\Omega \rm{m}$.
The heating element is $3 ~\rm{m}$ long and has a diameter of $3.57 ~\rm{mm}$.

1. Calculate the resistance $R$ of the heating element.

2. The heating element is used to heat the oven to a temperature of $180 ~\rm{°C}$. For this, a power dissipation (= heat flow) of $P=40 ~\rm{W}$ is necessary.
Calculate the current $I$ needed to operate it.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 ~\rm{W}}{0.33 ~\Omega}}$$

$$\begin{align*} R &= \rho \cdot \frac{l}{A} && | \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} && \\ R &= 1.10\cdot 10^{-6} ~\Omega \rm{m} \cdot \frac{4 \cdot 3~\rm{m}}{(3.57\cdot 10^{-3}~\rm{m})^2 \cdot \pi} && \\ \end{align*}$$

Result

$$R = 0.33 ~\Omega$$

**Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)**

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Result

$$R = 0.33 ~\Omega$$

**Exercise E1 Temperature-dependent Resistance**

**(written test, approx. 6 % of a 60-minute written test, WS2022)**

A thermistor is used as a temperature sensor in a refrigeration system. The thermistor has a resistance of $10 ~\rm{k}\Omega$ at $+25 ~°\rm{C}$.
Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$
The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

2. Accordingly, explain whether the constant current or constant voltage source would be better. Give an explanation for your answer.

3. Additionally, ... the refrigeration system.

Result

\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}

Solution

The power of the resistor is: $P = U \cdot I = R \cdot I^2 = {{U^2}\over{R}}$. Therefore, a resistor transforms electrical energy into heat. This is equivalent to a heat flow. Resistive sensors might heat up the refrigeration system.
Therefore, with constant $U$ and increasing $R$ the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

Result

\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C} ) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}

**Exercise E2 Temperature-dependent Resistance**
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

A thermistor is used as a temperature sensor in a refrigeration system. The thermistor has a resistance of $10 ~\rm{k}\Omega$ at $+25 ~°\rm{C}$.
Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$
The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

2. Accordingly, explain whether the constant current or constant voltage source would be better. Give an explanation for your answer.

3. Additionally, ... the refrigeration system.

Result

\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}

Solution

The power of the resistor is: $P = U \cdot I = R \cdot I^2 = {{U^2}\over{R}}$. Therefore, a resistor transforms electrical energy into heat. This is equivalent to a heat flow. Resistive sensors might heat up the refrigeration system.
Therefore, with constant $U$ and increasing $R$ the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

Result

\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C} ) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}

**Exercise E1 Pure Resistor Network Simplification**
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following circuit with $R_1=200 ~\Omega$, $R_2=R_3=100 ~\Omega$ is given.

2. The switch shall now be closed. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

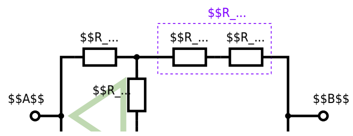
Result

Solution

\begin{align*} R_{\rm eq} &= 142.8 ~\Omega \\ \end{align*}

\begin{align*} R_{\rm eq} &= 135.3 ~\Omega \\ \end{align*}

Now a wye-delta transformation is necessary.

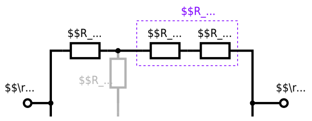



Since $R_2=R_3$ and based on the equations for the transformation, the transformed $R_Y$ is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution




The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega )&&||(100 ~\Omega + 100 ~\Omega ) &&\\ R_{\rm eq} &= (500 ~\Omega )&&||(200 ~\Omega )&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}

**Exercise E3 Pure Resistor Network Simplification**
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following circuit with $R_1=200 ~\Omega$, $R_2=R_3=100 ~\Omega$ is given.

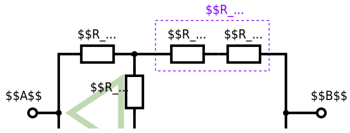
2. The switch shall now be closed. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Result

Solution

\begin{align*} R_{\rm eq} &= 142.8 ~\Omega \\ \end{align*}

Now a wye-delta transformation is necessary.

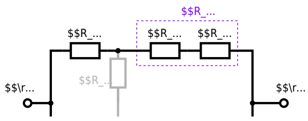



Since $R_2=R_3$ and based on the equations for the transformation, the transformed $R_Y$ is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution

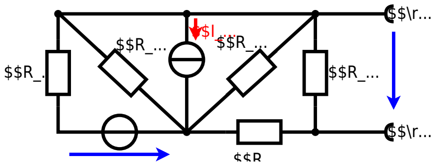



The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega )&&||(100 ~\Omega + 100 ~\Omega ) &&\\ R_{\rm eq} &= (500 ~\Omega )&&||(200 ~\Omega )&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}

**Exercise E1 Equivalent linear Source**
**(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

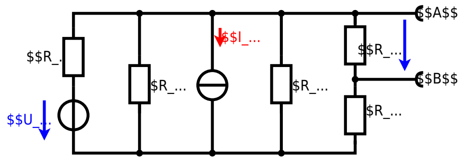
\begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}



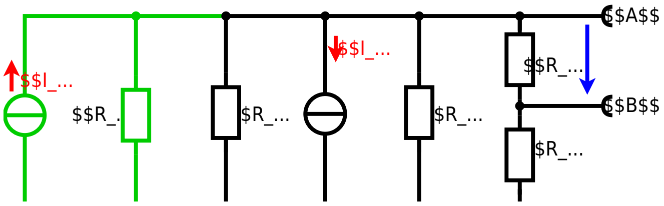

Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$. \begin{align*} R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

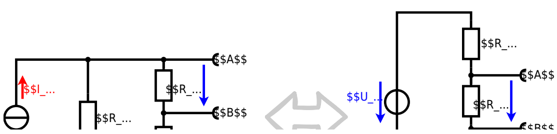
The best thing is to re-think the wiring like rubber bands and adjust them:




The linear voltage source of $U_2$ and $R_1$ can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and $R_1$:




Now a lot of them can be combined. The resistors $R_1$, $R_3$, $R_5$ are in parallel, like also $I_2$ and $I_4$: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:




Here, the $U_{24}$ is calculated by $I_{24}$ as the following: \begin{align*} U_{24}

&= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit, there is a voltage divider given by $R_{135}$, $R_6$, and $R_7$.
Therefore the voltage between $A$ and $B$ is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance $R_i$ the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || ( R_6 + R_1||R_3||R_5) \\ \end{align*}

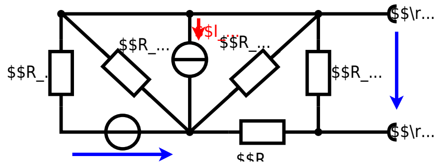
with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| ( 7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

### Exercise E4 Equivalent linear Source
**(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

\begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}




Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$. \begin{align*} R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

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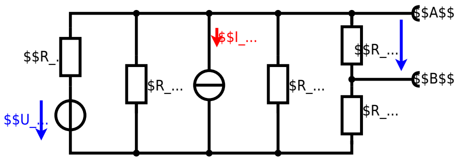
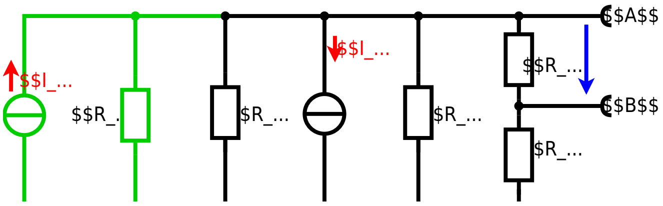



The linear voltage source of $U_2$ and $R_1$ can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and $R_1$:

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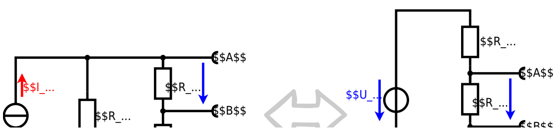

Here, the $U_{24}$ is calculated by $I_{24}$ as the following: \begin{align*} U_{24}

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On the right side of the last circuit, there is a voltage divider given by $R_{135}$, $R_6$, and $R_7$.
Therefore the voltage between $A$ and $B$ is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance $R_i$ the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || ( R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| ( 7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

**Exercise E1 Charging Capacitors**
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

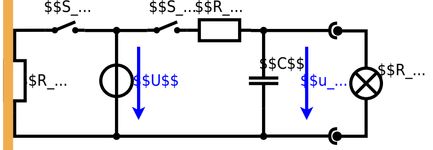
[illegible] the voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch $S_1$ is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Result
Solution

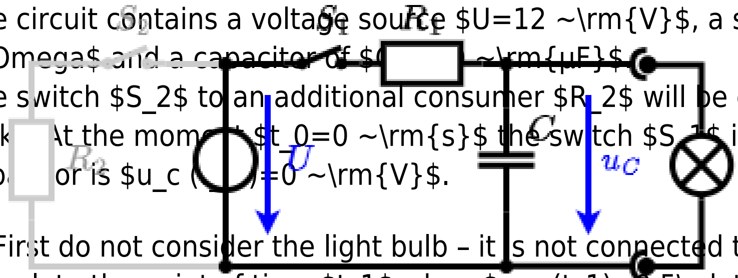
Hint: To solve this, first create an equivalent linear voltage source from $U$, $R_1$, and $R_\rm B$.

[illegible]

[illegible]
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting $R_2$.



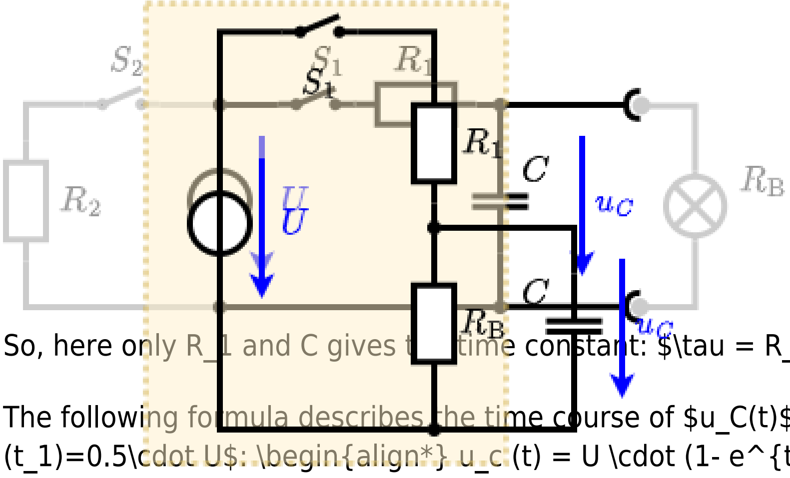
The circuit contains a voltage source $U=12 ~\rm{V}$, a switch $S_1$, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C= [illegible] ~\rm{µF}$.
The switch $S_2$ to an additional consumer $R_2$ will be considered to be open for the first task. At the moment $t_0=0 ~\rm{s}$ the switch $S_1$ is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.



1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time $t_1$ when $u_c (t_1)=0.5\cdot U$.



Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to $t$ \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}

An equivalent linear voltage source can be given with $U$, $R_1$, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit). \begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}

\begin{align*} u_c (t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}

**Exercise E5 Charging Capacitors**
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

[illegible] The switch $S_2$ is still open. The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch $S_1$ is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Result

Solution

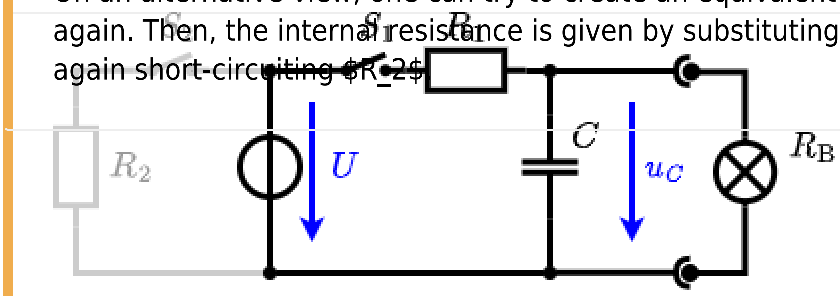
Hint: To solve this, first create an equivalent linear voltage source from $U$, $R_1$, and $R_\rm B$.

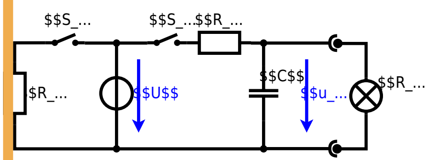
\begin{align*} [illegible] 3.792 ~\rm{V} [illegible] \end{align*}

Solution

It does not change anything. The ideal voltage source provides the voltage of $U=12 ~\rm{V}$ independent of this resistor. \begin{align*} \Delta W_R &= \Delta W_C &= {{1}\over{2}} C \cdot U^2 \\ &= {{1}\over{2}} \cdot 100 ~\rm{µF} \cdot (12 ~\rm{V})^2 \end{align*}

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting $R_2$

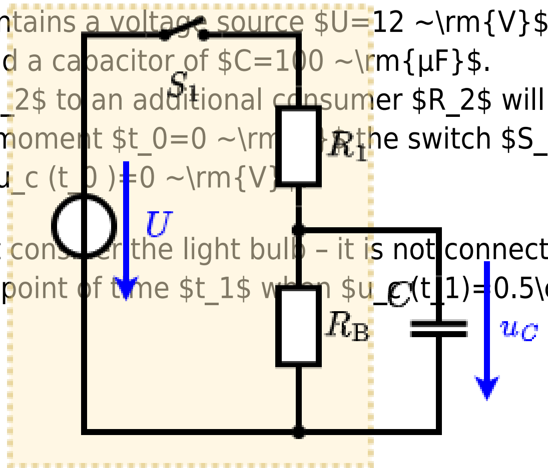






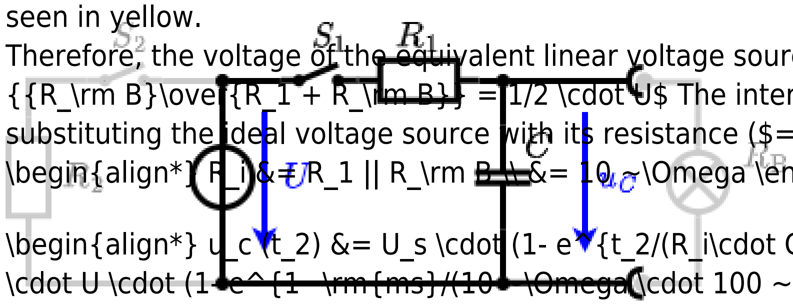
The circuit contains a voltage source $U=12 ~\rm{V}$, a switch $S_1$, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch $S_2$ to an additional consumer $R_2$ will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch $S_1$ is closed, the voltage across the capacitor is $u_c (t_0 )=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit. Calculate the point of time $t_1$ when $u_C(t_1)=0.5\cdot U$.



Solution

An equivalent linear voltage source can be given with $U$, $R_1$, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit). \begin{align*} R_i &= R_1 || R_\rm B &= 10 ~\Omega \end{align*}



\begin{align*} u_c(t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1 ~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}
So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to $t$ \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}

**Exercise E1 Analyzing complex Impedances**
**(written test, approx. 14 % of a 60-minute written test, WS2022)**

components. ($R$ and $\underline{X}_1$) shall be given.

After analysis, the following formula for the impedance was extracted \begin{align*} \underline{Z} = \left( {{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega \end{align*}

Solution

1. Calculate the physical values of the two components.

\begin{align*} C = 103 ~\rm{µF} \end{align*}

Solution

\begin{align*} R &= 0.24 ~\Omega \\ L &= 2.26 ~\rm{mH} \end{align*}

Solution

\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }}

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.

Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$ \begin{align*} X_C &= {{1}\over{\omega \cdot C}} = X_L \\ C &= {{1}\over{\omega \cdot X_L}} \\ &= {{1}\over{2\pi \cdot 300 ~\rm{Hz} \cdot 4.68 ~\Omega}} \\ \end{align*}

\begin{align*} \underline{Z} &= \left( {{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left( {{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4j}} + 5{\rm j} \right) ~\Omega \\ &= \left( {{2}\over{9+16 }} \cdot (3-4{\rm j} ) + 5{\rm j} \right) ~\Omega \\ &= \left( 0.24 + 4.68 {\rm j} \right) ~\Omega \\ &= 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega \\ &= R + {\rm j} X_L \\ \end{align*}

The absolute value $|\underline{I}|$ can be calculated as \begin{align*} |\underline{I}| &= {{50 ~\rm{V}}\over{| 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega |}} \\ &= {{50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}} \end{align*}

With the complex part comes the physical value: \begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \\ \end{align*}

The phase $\varphi_i$ can be calculated as \begin{align*} \varphi_i &= \arctan \left( {{\Im()}\over{\Re()}} \right) \\ &= \arctan \left( {{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

\begin{align*} |\underline{I}| &= 10.7 ~\rm{A} \\ \varphi_i &= -87.06° \end{align*}

**Exercise E6 Analyzing complex Impedances**
**(written test, approx. 14 % of a 60-minute written test, WS2022)**

components. ($R$ and $\underline{X}_1$) shall be given.

After analysis, the following formula for the impedance was extracted \begin{align*} \underline{Z} = \left( {{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega \end{align*}

Solution

1. Calculate the physical values of the two components.

\begin{align*} C = 103 ~\rm{µF} \end{align*}

Solution

\begin{align*} R &= 0.24 ~\Omega \\ L &= 2.26 ~\rm{mH} \end{align*}

Solution

\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }}

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.

Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$ \begin{align*} X_C &=

\begin{align*} \underline{Z} &= \left( {{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left( {{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4j}} +

5{1}\over{\omega C} = X_L - {1}\over{\omega C} \\ &= {\rm j}
\right) \\ &= 0.24
~\Omega} \over {\rm j} 0.24 ~\Omega - 4.68 ~\Omega \\ &= 50
~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}} \end{align*}

With the complex part comes the physical value: \begin{align*} X_L &= \omega L \\ L &= {X_L}\over{2\pi \cdot f} \\ &= {4.68 ~\Omega}\over{2\pi \cdot 300 ~{\rm Hz}} \\ \end{align*}

The phase $\varphi$ can be calculated as: \begin{align*} \varphi &= \arctan \left( {\rm Im}\over{\rm Re} \right) \\ &= \arctan \left( {{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

**Exercise E1 Impedances at different Frequencies**
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

Calculate the resistor values $R_1$, $R_2$ and $R_3$ in the following circuits.

1. A resistor $R_1$ shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

2. A series circuit with $L_2=4.7 ~\rm µH$ and an AC voltage source ($U_2=1.0 ~\rm V$, $f_2=450 ~\rm kHz$) generates a current of ...

3. A parallel circuit with $C_3=...$ used in an AC voltage source ($U_3=...$, $f_3=...$) ... generates a current $I_3=60 ~\rm mA$ and $I_{3R}=1.0 ~\rm A$ through $R_3$.

Result

Solution

Solution \begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}

Solution \begin{align*} R_2 &= 10.0 ~\Omega \\ \end{align*}

Solution \begin{align*} R_3 &= 7... ~\Omega \\ \end{align*}

\begin{align*} R_1 &= |\underline{X}_{C1}| \\ &= {{1}\over{2\pi \cdot f \cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~\rm nF}} \\ \end{align*}

A series circuit means that the current is constant on every component.

The equivalent impedance for $R$ and $L$ combined is given by \begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j} \cdot \omega L \end{align*} Since ${\rm j} \cdot \omega L$ is perpendicular to $R_2$ this can be simplified to: \begin{align*} \left| {{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left( {{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}

This can be rearranged to get $R_2$: \begin{align*} R_2 &= \sqrt{ \left( {{U}\over{I}} \right)^2 - X_{L2}^2 } \\ &= \sqrt{ \left( {{1 ~\rm V}}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}

Parallel circuit means that the voltage is the same on $R_3$ and $C_3$.

... gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since $R_3$ is perpendicular to ${\rm j}\cdot{X}_{3C}$, too).

Therefore, the resulting current of the parallel circuit is given as: \begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ {I}_{3C} &= \sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2} \end{align*}

Back to the first formula: \begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}

**Exercise E7 Impedances at different Frequencies**
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

1. A resistor $R_1$ shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

Solution

Solution

Solution

$$R_1 = 1.00 ~\Omega$$

$$R_3 = 10.0 ~\Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for $R$ and $L$ combined is given by

$${{\underline{U}}\over{\underline{I}}} = R_2 + \underline{X}_{L2} = R_2 + {\rm j}\cdot\omega L$$

Since ${\rm j}\cdot\omega L$ is perpendicular to $R_2$ this can be simplified to:

$$\left|{{\underline{U}}\over{\underline{I}}}\right|^2 = |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left({{U}\over{I}}\right)^2 = {R_2}^2 + {X_{L2}}^2$$

Parallel circuit means that the voltage is the same on $R_3$ and $C_3$.

$$R_1 = |\underline{X}_{C1}| = {{1}\over{2\pi \cdot f \cdot C_1}}$$

$\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since $R_3$ is perpendicular to ${\rm j}\cdot{X}_{3C}$, too).

Therefore, the resulting current of the parallel circuit is given as:

$$\underline{I}_{3} = \underline{I}_{3R} + \underline{I}_{3C}$$

This can be rearranged to get:

$$R_2 = \sqrt{ \left({{U}\over{I}}\right)^2 - X_{L2}^2 }$$

$$R_2 = \sqrt{ \left( {{1~\rm V}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 }$$

Back to the first formula:

$$R_3 \cdot {I}_{3R} = {X}_{3C} \cdot {I}_{3C} \\ R_3 = {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ = {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}}$$

**Exercise E1 Complex Impedance Circuit**
**(written test, approx. 15 % of a 60-minute written test, WS2022)**

A linear voltage source ... $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ ... with an internal resistance of $10 ~\Omega$.

This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

1. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

Result

Solution

Solution

Result

$$Z_L = 31.1 ~\Omega \\ Z_C = 48.2 ~\Omega \\ Z = 19.8 ~\Omega$$

$$I = 107.3 ~{\rm mA}$$

$$Z = {{\hat{U}}\over{\hat{I}}} \\ \hat{I} = {{\hat{U}}\over{Z}}$$

$$Z_C = {{1}\over{2\pi \cdot f \cdot C}} = {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}$$

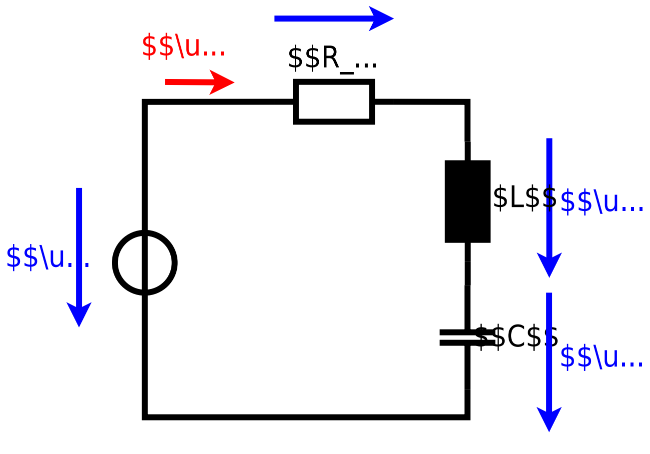
With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$:

$$I = {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} = {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}}$$

$$Z_L = 2\pi \cdot f \cdot L = 2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}$$

$$\underline{Z} = R + \underline{Z}_L + \underline{Z}_C \\ = R + j \cdot {Z}_L - j \cdot {Z}_C \\ = R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| = \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }$$

$$10...


$$\u...
$$\u...
$$R_...
$L$$
$$\u...
$$\u...
$$C$$
$$\u...

### Exercise E8 Complex Impedance Circuit
### (written test, approx. 15 % of a 60-minute written test, WS2022)

A linear voltage source generates a sinusoidal signal $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ with an internal resistance of $10 ~\Omega$.
This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

1. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

2. Calculate the complex impedances $\underline{Z}_C$, $\underline{Z}_L$ and $|\underline{Z}|$.

Solution

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}\\ \end{align*}

\begin{align*} Z_L &= 2\pi \cdot f \cdot L\\ &= 2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}\\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}}}\\ \end{align*}

\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}

Result

\begin{align*} Z_L &= 31.1 ~\Omega \\ Z_C &= 48.2 ~\Omega \\ Z &= 19.8 ~\Omega \end{align*}

3. Calculate the current $I$ provided by the voltage source.

Solution

\begin{align*} Z &= {{\hat{U}}\over{\hat{I}}} \\ \hat{I} &= {{\hat{U}}\over{Z}} \\ \end{align*}

With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$: \begin{align*} I &= {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ &= {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}} \\ \end{align*}

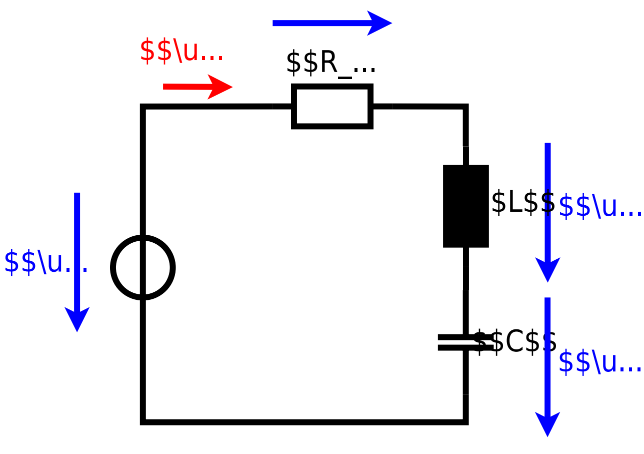
Result

\begin{align*} I = 107.3 {\rm mA} \end{align*}

4. Draw the complete phasor diagram of the voltages $\underline{U}_C$, $\underline{U}_L$, $\underline{U}_R$ and $\underline{U}$ by the voltage source.
Choose an appropriate scaling factor and write it down.

Solution

Result


$$\u...
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$$R_...
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