

Exam Winter Semester 2022

Student Group

First Name	Surname	Matrikel Nr.

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a cross-section of 1.80 mm^2 is used. Electric power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.

Determine the current I linked to the operating voltage for heating elements.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \text{and } R = \\ 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

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Exercise E1 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A digital display shows a temperature coefficient of resistance of a thermistor. The thermistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

The power transfer resistor P is the output of the circuit and generates heat. Therefore, a solution is to use a heat sink to cool the resistor.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad \text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \\ R = 10 \text{ k}\Omega \cdot (1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2)$$

Exercise E2 Temperature-dependent Resistance**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A digital display shows a temperature coefficient of resistance of a thermistor. The thermistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

The power transfer resistor P is the output of the circuit and generates heat. Therefore, a solution is to use a heat sink to cool the resistor.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad \text{with } \Delta T = T_{\text{end}} - T_{\text{start}} \\ R = 10 \text{ k}\Omega \cdot (1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2)$$

Exercise E1 Pure Resistor Network Simplification

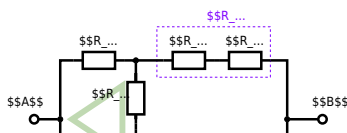
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall now be closed and integrated, $R_2 = R_3 = 100 \Omega$ and the switch between R_3 and R_4 shall be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution

$$R_{\text{eq}} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

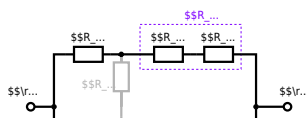
$$R_Y = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{100 \cdot 100}{100 + 100} = 50 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) = 50 + (50 + 100 + 100) \parallel (50 + 100)$$

.. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel (500 \, \Omega) \parallel (200 \, \Omega) \parallel R_{\text{eq}} = \frac{500 \, \Omega \cdot 200 \, \Omega}{500 \, \Omega + 200 \, \Omega}$$

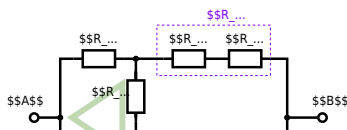
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with \$R_1 = 200 \, \Omega\$, \$R_2 = 100 \, \Omega\$, \$R_3 = 150 \, \Omega\$, \$R_4 = 100 \, \Omega\$, and the source voltage \$U_{SV} = 10 \, \text{V}\$. The result shall be given in \$\Omega\$.

Solution

$$R_{\text{eq}} = 132.8 \, \Omega$$

Now a wye-delta transformation is necessary.



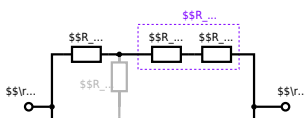
Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega) \parallel$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 || R_3) || R_4$$

$$R_{\text{eq}} = (100 \, \Omega + 200 \, \Omega || 200 \, \Omega) || (100 \, \Omega + 100 \, \Omega)$$

$$R_{\text{eq}} = (500 \, \Omega) || (200 \, \Omega)$$

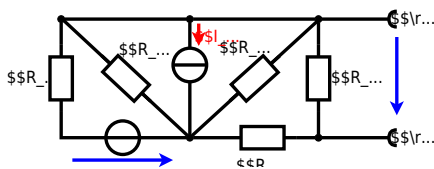
$$R_{\text{eq}} = \frac{500 \, \Omega \cdot 200 \, \Omega}{500 \, \Omega + 200 \, \Omega}$$

Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5 \, \text{V}$$

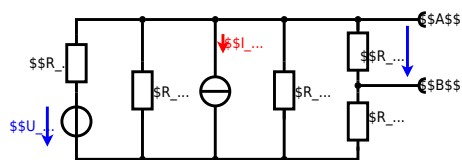
$$R_i = R_{AB} = 6 \, \Omega$$



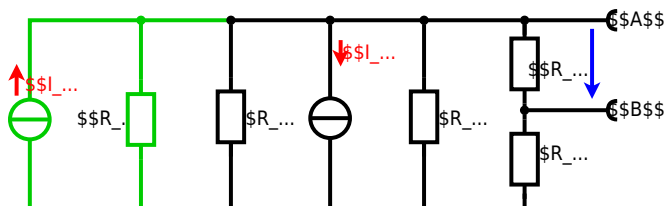
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B. $\begin{aligned} R_1 &= 5.0 \, \Omega, \quad U_2 = 6.0 \, \text{V}, \quad R_3 = 10 \, \Omega, \quad I_4 = 4.2 \, \text{A}, \\ R_5 &= 10 \, \Omega, \quad R_6 = 7.5 \, \Omega, \quad R_7 = 15 \, \Omega \end{aligned}$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_{12} and I_{45} :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_{12} - I_{45} = \frac{U_{24}}{R_{135}} - I_{45}$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{24} = U_{23} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} = (U_2 - U_4) \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} = (U_2 - U_4) \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

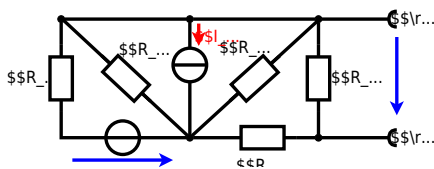
with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E4 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

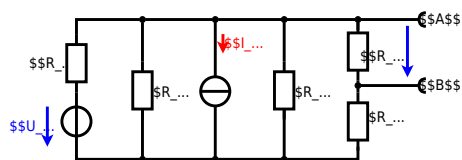
$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



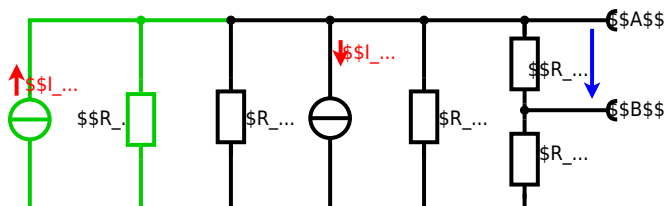
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B. $\begin{aligned} R_1 &= 5.0 \, \Omega, \quad U_2 = 6.0 \, \text{V}, \quad R_3 = 10 \, \Omega, \quad I_4 = 4.2 \, \text{A}, \\ R_5 &= 10 \, \Omega, \quad R_6 = 7.5 \, \Omega, \quad R_7 = 15 \, \Omega \end{aligned}$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



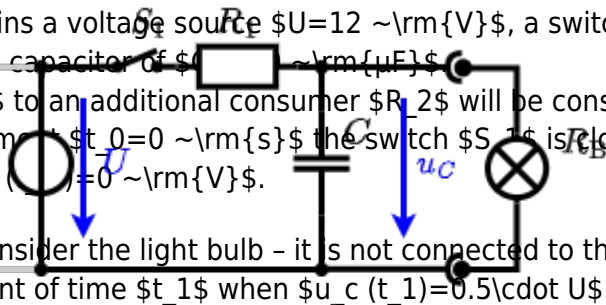
Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_{24} and I_{42} :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_{24} - I_{42} = \frac{U_{24}}{R_1} - I_{42}$$
 The resulting circuit can again be transformed:



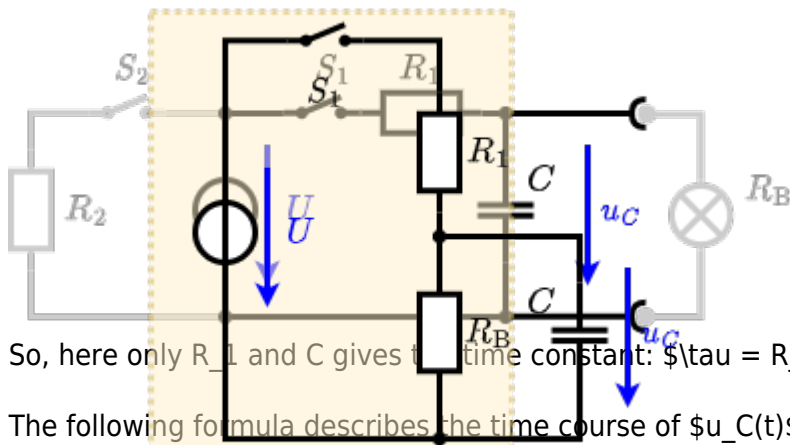
Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135}$$

The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$. An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($=0\text{ }\Omega$, short-circuit).
$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E5 Charging Capacitors

(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (see the solution) consists of a 12 V DC voltage source, a switch S_1 and a switch S_2 connected to a $20\text{ }\Omega$ resistor, a $100\text{ }\mu\text{F}$ capacitor and a light bulb R_B . At the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

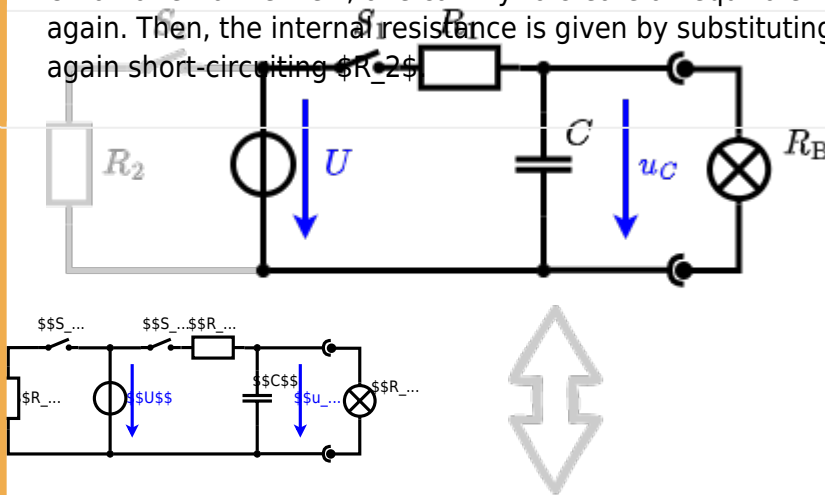
Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U = 6\text{ V}$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is given by $U_s = U \cdot \frac{R_1}{R_1 + R_B}$ and the internal resistance is given by $R_i = R_1 \parallel R_B$.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

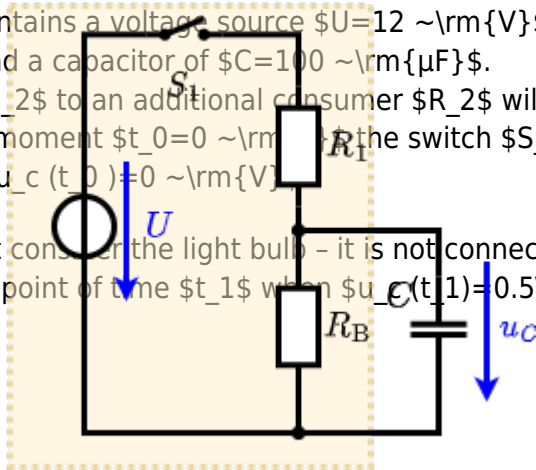


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$

It has to be rearranged to t :

$$(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) \implies t = R_1 \cdot C \cdot \ln(0.5)$$

Exercise E1 Analyzing complex Impedances**(written test, approx. 14 % of a 60-minute written test, WS2022)**

2. Calculate the phase angle of the solution $\underline{U} = 50 \angle 0^\circ$ mV, $\underline{Z} = 0.24 + j4.68 \Omega$ and $\underline{I} = 0.24 \angle -87.96^\circ$ A through the components. (R and X_L) shall be given.

After analysis, the full bridge dimensioned impedance can be extracted and the phase angle φ in phase (calculated) is $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{4.68}{0.24}\right) = 87.96^\circ$

.. Calculate the physical values of the two components.

Solution $\begin{aligned} R &= 0.24 \Omega \\ X_L &= 4.68 \Omega \end{aligned}$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{0.24 + j4.68} = 0.24 \angle -87.96^\circ$ A

The current and voltage are in phase only if the impedance is purely real. The resulting impedance is $\underline{Z} = 0.24 + j4.68 \Omega$.

Therefore, the component 4.68Ω is an inductor with the same absolute value of 4.68Ω .

Impedance $\underline{Z} = R + jX_L = 0.24 + j4.68 \Omega$

$\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{4.68}{0.24}\right) = 87.96^\circ$

The phase angle φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{4.68}{0.24}\right) = 87.96^\circ$

With the complex part (complex value) $\underline{Z} = 0.24 + j4.68 \Omega$

$\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{4.68}{0.24}\right) = 87.96^\circ$

Exercise E6 Analyzing complex Impedances**(written test, approx. 14 % of a 60-minute written test, WS2022)**

2. Calculate the phase angle of the solution $\underline{U} = 50 \angle 0^\circ$ mV, $\underline{Z} = 0.24 + j4.68 \Omega$ and $\underline{I} = 0.24 \angle -87.96^\circ$ A through the components. (R and X_L) shall be given.

After analysis, the full bridge dimensioned impedance can be extracted and the phase angle φ in phase (calculated) is $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{4.68}{0.24}\right) = 87.96^\circ$

.. Calculate the physical values of the two components.

Solution $\begin{aligned} R &= 0.24 \Omega \\ X_L &= 4.68 \Omega \end{aligned}$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{0.24 + j4.68} = 0.24 \angle -87.96^\circ$ A

The current and voltage are in phase only if the impedance is purely real. The resulting impedance is $\underline{Z} = 0.24 + j4.68 \Omega$.

Therefore, the component 4.68Ω is an inductor with the same absolute value of 4.68Ω .

Impedance $\underline{Z} = R + jX_L = 0.24 + j4.68 \Omega$

$\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{4.68}{0.24}\right) = 87.96^\circ$

The absolute value of the impedance is $|Z| = \sqrt{(0.24)^2 + (4.68)^2} = 4.70 \text{ } \Omega$.
 The phase angle is $\phi = \arctan\left(\frac{4.68}{0.24}\right) = 87.1^\circ$.
 With the complex part comes the physical value: $X_L = \omega L = 2\pi f L = 2\pi \cdot 400 \cdot 10^{-6} = 0.251 \text{ } \Omega$.
 The phase angle is $\phi = \arctan\left(\frac{4.68}{0.24}\right) = 87.1^\circ$.
 The phase angle is $\phi = \arctan\left(\frac{4.68}{0.24}\right) = 87.1^\circ$.

Exercise E1 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$.
 The equivalent impedance for R and C combined is given by $Z = \sqrt{R^2 + X_C^2}$.
 Back to the first formula: $R_3 \cdot I_3 = X_{3C} \cdot I_3$.
 Result: $R_3 = 10.0 \text{ } \Omega$.
 Back to the first formula: $R_3 \cdot I_3 = X_{3C} \cdot I_3$.
 Result: $R_3 = 10.0 \text{ } \Omega$.

Solution

$R_1 = 1.00 \text{ } \Omega$

$R_2 = 10.0 \text{ } \Omega$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$.

Parallel circuit means that the voltage is the same on R_2 and C_2 .
 $\frac{1}{Z} = \frac{1}{R_2} + \frac{1}{X_{C2}}$.
 $\frac{1}{Z} = \frac{1}{R_2} + \frac{1}{\frac{1}{\omega C_2}}$.
 $\frac{1}{Z} = \frac{1}{R_2} + \omega C_2$.
 $Z = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2}$.

$\frac{1}{Z} = \frac{1}{R_2} + \omega C_2$.
 $Z = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2}$.
 $Z = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2}$.

Therefore, the resulting current of the parallel circuit is given as:

$I_3 = \frac{U}{Z} = \frac{U}{\frac{R_2}{1 + \omega^2 R_2^2 C_2^2}} = \frac{U(1 + \omega^2 R_2^2 C_2^2)}{R_2}$

Back to the first formula: $R_3 \cdot I_3 = X_{3C} \cdot I_3$.
 $R_3 = \frac{X_{3C}}{I_3} = \frac{X_{3C}}{\frac{U(1 + \omega^2 R_2^2 C_2^2)}{R_2}} = \frac{R_2 X_{3C}}{U(1 + \omega^2 R_2^2 C_2^2)}$

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Exercise E7 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

Exercise E1 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the equivalent impedance Z for the circuit shown in the AC network diagram of Fig. 21.3.0. The voltage source $u(t) = 3.0 \sim \text{V} \cdot \sin(2\pi \cdot 15 \sim \text{kHz} \cdot t)$ is connected in series with an inductor of $L = 330 \sim \mu\text{H}$ and a capacitor of $C = 0.22 \sim \mu\text{F}$, all in series.

Solution
Result

$$Z = 19.8 \sim \Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 10^3 \cdot 0.22 \cdot 10^{-6}} = -j15.9 \sim \Omega$$

$$Z_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 10^3 \cdot 330 \cdot 10^{-6} = j31.8 \sim \Omega$$

$$Z = R + jX_L - jX_C = 19.8 \sim \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by $Z_L = j\omega L$.
 Parallel circuit means that the voltage is the same on R_1 and R_2 .

$$\frac{1}{Z} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow Z = \frac{R_1 R_2}{R_1 + R_2}$$

 Since R_1 and R_2 are in parallel, the voltage across them is the same.

$$U = I \cdot Z \Rightarrow I = \frac{U}{Z}$$

 The resulting current of the parallel circuit is given as:

$$I = \frac{U}{Z} = \frac{U}{\frac{R_1 R_2}{R_1 + R_2}} = U \cdot \frac{R_1 + R_2}{R_1 R_2}$$

 This can be simplified to:

$$I = U \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

 Back to the first formula:

$$Z = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow R_1 R_2 = Z (R_1 + R_2)$$

$$R_1 R_2 = Z R_1 + Z R_2$$

$$R_1 R_2 - Z R_1 - Z R_2 = 0$$

$$R_1 (R_2 - Z) - Z R_2 = 0$$

$$R_1 (R_2 - Z) = Z R_2$$

$$R_1 = \frac{Z R_2}{R_2 - Z}$$

Exercise E1 Complex Impedance Circuit

(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the equivalent impedance Z for the circuit shown in the AC network diagram of Fig. 21.3.0. The voltage source $u(t) = 3.0 \sim \text{V} \cdot \sin(2\pi \cdot 15 \sim \text{kHz} \cdot t)$ is connected in series with an inductor of $L = 330 \sim \mu\text{H}$ and a capacitor of $C = 0.22 \sim \mu\text{F}$, all in series.

Solution
Result

$$Z = 19.8 \sim \Omega$$

Draw the circuit diagram of the given circuit.
 Label all components, voltages, and currents.

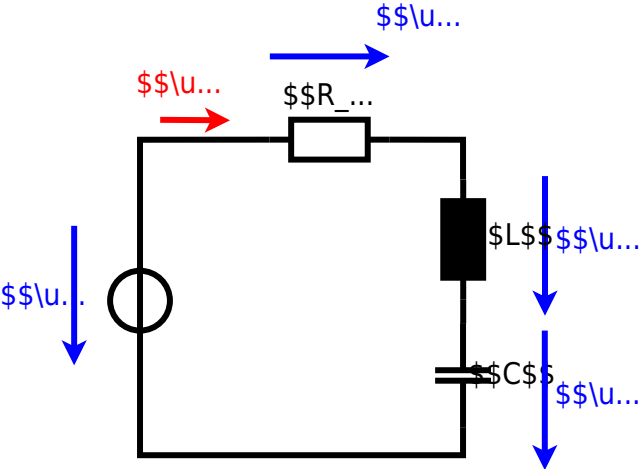
$$Z = \frac{U}{I} = \frac{3.0 \sim \text{V}}{0.15 \sim \text{A}} = 19.8 \sim \Omega$$

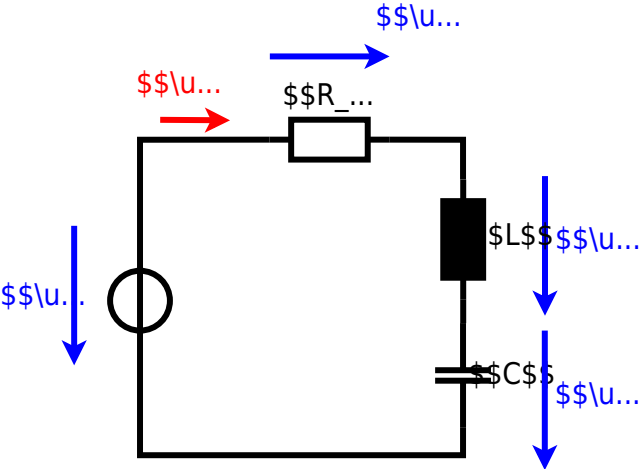
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$$Z_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 10^3 \cdot 330 \cdot 10^{-6} = j31.8 \sim \Omega$$

$$Z = R + jX_L - jX_C = 19.8 \sim \Omega$$







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