

Exam Winter Semester 2022

Student Group

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

2. For heating elements made of solid nichrome wire with a temperature coefficient of $1.80 \times 10^{-3} \text{ K}^{-1}$, an electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.
 Determine the current I needed for heating elements.
 The Nichrome wire has a resistivity of $1.10 \times 10^{-6} \text{ } \Omega \text{ m}$.
 The heating element is 3 m long and has a diameter of 3.57 mm .
 Solution in align^* $R = 1.10 \times 10^{-6} \text{ } \Omega \text{ m}$ align^* .
 ∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \& \quad | \text{ with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad \& \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ 1.10 \times 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \times 10^{-3} \text{ m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

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$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

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Exercise E1 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. Additionally, explain why the effect of constant resistance in a refrigeration system can be identified. The food has a resistance of $10 \text{ k}\Omega$ at 25°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \cdot \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \cdot \frac{1}{\text{K}^2}$

Result
The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

The power transfer resistance P is a function of the current I and of the voltage U . Therefore, a solution is to increase the voltage U and decrease the current I .

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \\ R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \cdot \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \cdot \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. Additionally, explain why the effect of constant resistance in a refrigeration system can be identified. The food has a resistance of $10 \text{ k}\Omega$ at 25°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \cdot \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \cdot \frac{1}{\text{K}^2}$

Result
The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermistor at -40°C .

The power transfer resistance P is a function of the current I and of the voltage U . Therefore, a solution is to increase the voltage U and decrease the current I .

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \\ R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \cdot \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \cdot \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

Exercise E1 Pure Resistor Network Simplification

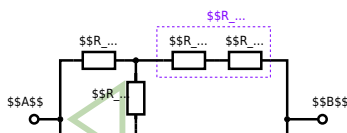
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall now be closed and integrated, $R_2 = R_3 = 100 \Omega$ and the switch between R_2 and R_3 shall be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution

$$R_{\text{eq}} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

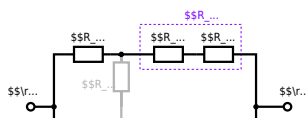
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel (500 \, \Omega) \parallel (200 \, \Omega) \parallel (500 \, \Omega \cdot 200 \, \Omega) / (500 \, \Omega + 200 \, \Omega)$$

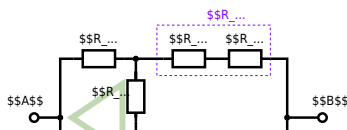
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with \$R_1=200 \, \Omega\$, \$R_2=100 \, \Omega\$, \$R_3=150 \, \Omega\$, \$R_4=100 \, \Omega\$, and \$R_5=200 \, \Omega\$.
Result given: \$R_{eq} = 132.8 \, \Omega\$.

Solution

$$R_{eq} = 132.8 \, \Omega$$

Now a wye-delta transformation is necessary.



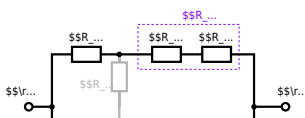
Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



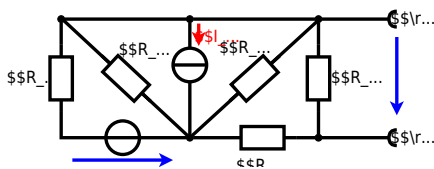
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \, \Omega + 200 \, \Omega + 200 \, \Omega) \parallel (100 \, \Omega + 100 \, \Omega) \parallel R_{\text{eq}} = \{ \{ 500 \, \Omega \cdot 200 \, \Omega \} \over {500 \, \Omega + 200 \, \Omega} \} \parallel$$

Exercise E1 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \, \text{V} \parallel R_{\text{i}} = R_{\text{AB}} = 6 \, \Omega$$

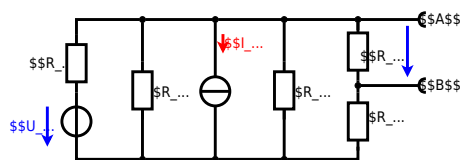


Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B.

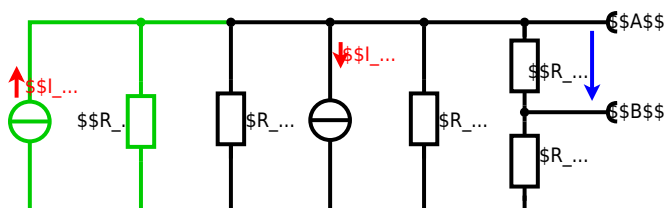
$R_1 = 5.0 \, \Omega$, $U_2 = 6.0 \, \text{V}$, $R_3 = 10 \, \Omega$, $I_4 = 4.2 \, \text{A}$, $R_5 = 10 \, \Omega$, $R_6 = 7.5 \, \Omega$, $R_7 = 15 \, \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_3 + R_4)$$

$$U_{24} = U_{23} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} = (U_2 - U_4) \cdot \frac{R_7}{R_1 \parallel R_3 \parallel R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} = (U_2 - U_4) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

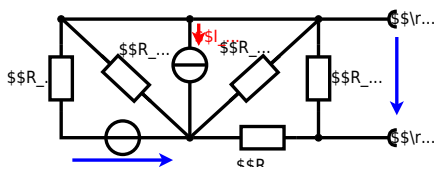
with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \quad R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E4 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

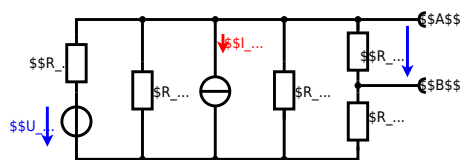
$$U_s = U_{AB} = 4.5\text{V} \quad R_i = R_{AB} = 6\Omega$$



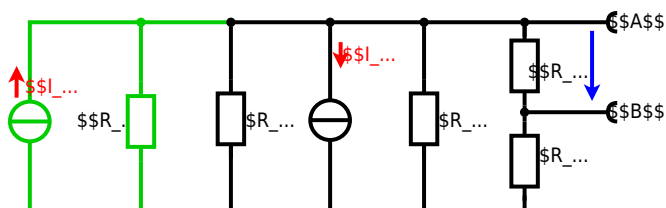
Calculate the internal resistance R_{int} and the source voltage U_{eq} of an equivalent linear voltage source on the connectors A and B. $\begin{aligned} R_1 &= 5.0 \, \Omega, \quad U_2 = 6.0 \, \text{V}, \quad R_3 = 10 \, \Omega, \quad I_4 = 4.2 \, \text{A}, \\ R_5 &= 10 \, \Omega, \quad R_6 = 7.5 \, \Omega, \quad R_7 = 15 \, \Omega \end{aligned}$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \right) - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \left(\frac{6.0\text{V}}{5.0\Omega} - 4.2\text{A} \right) \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \quad R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E1 Charging Capacitors

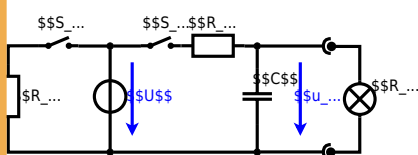
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a simplified model of a power supply unit. It consists of a DC voltage source U with an internal resistance R_1 and a switch S_1 . The voltage across the capacitor is again 0V at the moment $t_0=0\text{s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ms}$ after closing the switch.

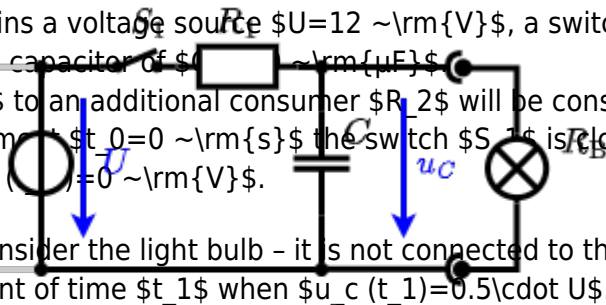
Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution: The circuit below is a simplified model of a power supply unit. It consists of a DC voltage source U with an internal resistance R_1 and a switch S_1 . The voltage across the capacitor is again 0V at the moment $t_0=0\text{s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ms}$ after closing the switch.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

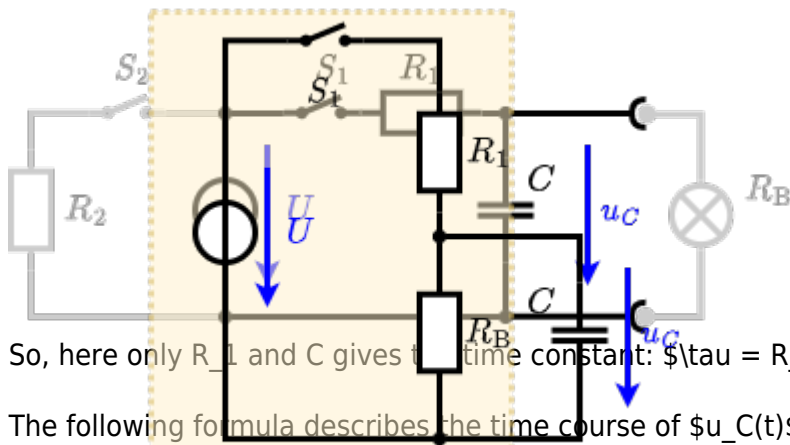


The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$

$$e^{-t/\tau} = 0.5 \quad t/\tau = \ln(0.5) \quad t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$
 An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E5 Charging Capacitors

(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (see the solution) consists of a 12 V voltage source, a $20\text{ }\Omega$ resistor, a $100\text{ }\mu\text{F}$ capacitor, and a $20\text{ }\Omega$ resistor. The voltage across the capacitor is again 0 V at the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

Solution

To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

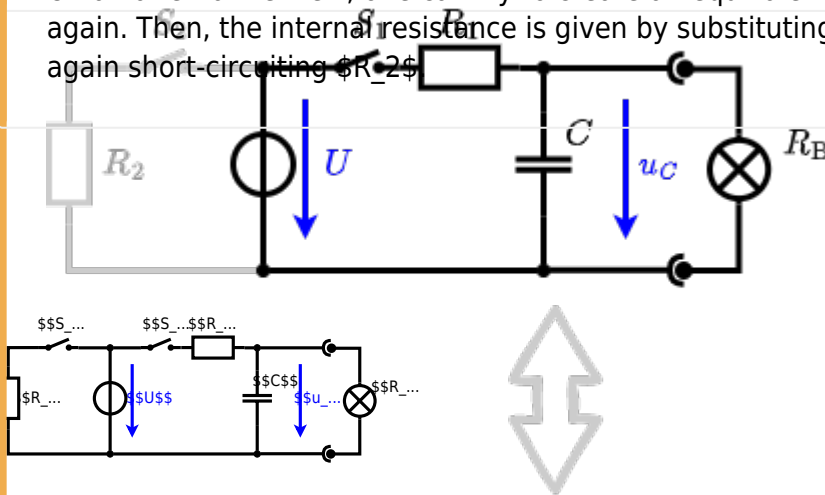
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U = 6\text{ V}$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is given by $U_s = U \cdot \frac{R_1}{R_1 + R_B}$ and the internal resistance is given by $R_i = R_1 \parallel R_B$.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

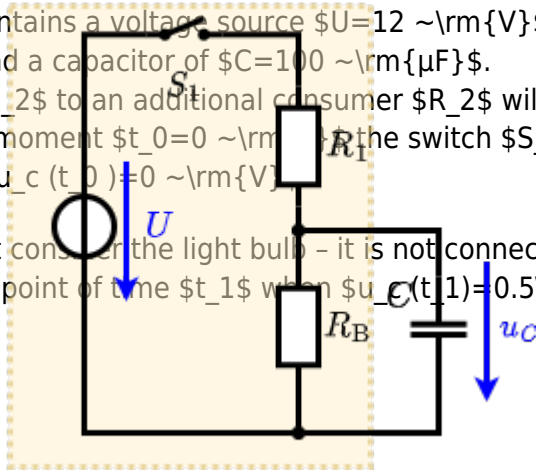


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to
$$(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = -\tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$

Exercise E1 Analyzing complex Impedances**(written test, approx. 14 % of a 60-minute written test, WS2022)**

2. Calculate the complex power $\underline{S}$ and the real power P and the reactive power Q in the circuit shown in the figure. The voltage $\underline{U}$ and the current $\underline{I}$ shall be given.

After analysis, the full bridge dimensioned impedance $\underline{Z}$ shall be extracted and the magnitude $|\underline{Z}|$ and the phase φ_Z shall be given. The voltage $\underline{U}$ and the current $\underline{I}$ shall be given.

3. Calculate the physical values of the two components.

Solution $\begin{aligned} \underline{Z} &= \frac{1}{0.24} + j\omega L = \frac{1}{0.24} + j2\pi \cdot 50 \cdot 0.001 = \frac{1}{0.24} + j0.314 \end{aligned}$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50}{\frac{1}{0.24} + j0.314} = 4.68 - j1.68 \text{ A}$
 The current $\underline{I}$ and voltage $\underline{U}$ are in phase since $\varphi_Z = 0$ (pure real)
 resulting impedance $\underline{Z} = \frac{1}{0.24} + j0.314$
 Therefore, the component $0.24 \text{ }\Omega$ is a resistor with the same absolute value of $0.24 \text{ }\Omega$
 impedance $\underline{Z} = \frac{1}{0.24} + j0.314$
 $|\underline{Z}| = \sqrt{\left(\frac{1}{0.24}\right)^2 + (0.314)^2} = 4.68 \text{ }\Omega$
 $\varphi_Z = \arctan\left(\frac{0.314}{\frac{1}{0.24}}\right) = 0^\circ$
 The absolute value $|\underline{Z}|$ shall be calculated as $|\underline{Z}| = \sqrt{\left(\frac{1}{0.24}\right)^2 + (0.314)^2} = 4.68 \text{ }\Omega$
 $\varphi_Z = \arctan\left(\frac{0.314}{\frac{1}{0.24}}\right) = 0^\circ$
 With the complex part $\underline{Z}$ the physical values begin $\underline{Z} = \frac{1}{0.24} + j0.314$
 $\underline{Z} = \frac{1}{0.24} + j0.314$
 The phase φ_Z shall be calculated as $\varphi_Z = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{0.314}{\frac{1}{0.24}}\right) = 0^\circ$

Exercise E6 Analyzing complex Impedances**(written test, approx. 14 % of a 60-minute written test, WS2022)**

2. Calculate the complex power $\underline{S}$ and the real power P and the reactive power Q in the circuit shown in the figure. The voltage $\underline{U}$ and the current $\underline{I}$ shall be given.

After analysis, the full bridge dimensioned impedance $\underline{Z}$ shall be extracted and the magnitude $|\underline{Z}|$ and the phase φ_Z shall be given. The voltage $\underline{U}$ and the current $\underline{I}$ shall be given.

3. Calculate the physical values of the two components.

Solution $\begin{aligned} \underline{Z} &= \frac{1}{0.24} + j\omega L = \frac{1}{0.24} + j2\pi \cdot 50 \cdot 0.001 = \frac{1}{0.24} + j0.314 \end{aligned}$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50}{\frac{1}{0.24} + j0.314} = 4.68 - j1.68 \text{ A}$
 The current $\underline{I}$ and voltage $\underline{U}$ are in phase since $\varphi_Z = 0$ (pure real)
 resulting impedance $\underline{Z} = \frac{1}{0.24} + j0.314$
 Therefore, the component $0.24 \text{ }\Omega$ is a resistor with the same absolute value of $0.24 \text{ }\Omega$
 impedance $\underline{Z} = \frac{1}{0.24} + j0.314$
 $|\underline{Z}| = \sqrt{\left(\frac{1}{0.24}\right)^2 + (0.314)^2} = 4.68 \text{ }\Omega$
 $\varphi_Z = \arctan\left(\frac{0.314}{\frac{1}{0.24}}\right) = 0^\circ$
 The absolute value $|\underline{Z}|$ shall be calculated as $|\underline{Z}| = \sqrt{\left(\frac{1}{0.24}\right)^2 + (0.314)^2} = 4.68 \text{ }\Omega$
 $\varphi_Z = \arctan\left(\frac{0.314}{\frac{1}{0.24}}\right) = 0^\circ$
 With the complex part $\underline{Z}$ the physical values begin $\underline{Z} = \frac{1}{0.24} + j0.314$
 $\underline{Z} = \frac{1}{0.24} + j0.314$
 The phase φ_Z shall be calculated as $\varphi_Z = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{0.314}{\frac{1}{0.24}}\right) = 0^\circ$

The absolute value of the impedance is $Z = \sqrt{(0.24 \sim \Omega)^2 + (4.68 \sim \Omega)^2} = 4.7 \sim \Omega$
 $\varphi = \arctan\left(\frac{4.68 \sim \Omega}{0.24 \sim \Omega}\right) = 87.1^\circ$
 With the complex part comes the physical value: $X_L = \omega L = 2\pi \cdot 4.7 \sim \Omega$
 The phase $\varphi = 87.1^\circ$ is the angle between the voltage and the current. $\varphi = 87.1^\circ$
 $\varphi = \arctan\left(\frac{4.68 \sim \Omega}{0.24 \sim \Omega}\right) = 87.1^\circ$

Exercise E1 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$
 Parallel circuit means that the voltage is the same on R_3 and C_3
 $\frac{1}{Z} = \frac{1}{R_3} + \frac{1}{X_{C3}}$
 Back to the first formula: $R_3 \cdot I_3 = X_{C3} \cdot I_3$
 $R_3 = X_{C3} = \frac{1}{\omega C_3} = \frac{1}{2\pi \cdot 4.7 \sim \text{nF} \cdot 4 \sim \text{MHz}} = 10.0 \sim \Omega$

Solution

$R_1 = 1.00 \sim \Omega$

$R_2 = 10.0 \sim \Omega$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by $Z = \sqrt{R^2 + X_L^2}$

Parallel circuit means that the voltage is the same on R_3 and C_3
 $\frac{1}{Z} = \frac{1}{R_3} + \frac{1}{X_{C3}}$
 Back to the first formula: $R_3 \cdot I_3 = X_{C3} \cdot I_3$
 $R_3 = X_{C3} = \frac{1}{\omega C_3} = \frac{1}{2\pi \cdot 4.7 \sim \text{nF} \cdot 4 \sim \text{MHz}} = 10.0 \sim \Omega$

$\frac{1}{Z} = \frac{1}{R_3} + \frac{1}{X_{C3}}$
 $\frac{1}{Z} = \frac{1}{10.0 \sim \Omega} + \frac{1}{10.0 \sim \Omega} = \frac{2}{10.0 \sim \Omega}$
 $Z = 5.0 \sim \Omega$

Therefore, the resulting current of the parallel circuit is given as:

$I = \frac{U}{Z} = \frac{10 \sim \text{V}}{5.0 \sim \Omega} = 2.0 \sim \text{A}$

This current is the same on R_3 and C_3
 $I = 2.0 \sim \text{A}$

Back to the first formula: $R_3 \cdot I_3 = X_{C3} \cdot I_3$
 $R_3 = X_{C3} = \frac{1}{\omega C_3} = \frac{1}{2\pi \cdot 4.7 \sim \text{nF} \cdot 4 \sim \text{MHz}} = 10.0 \sim \Omega$

Back to the first formula: $R_3 \cdot I_3 = X_{C3} \cdot I_3$
 $R_3 = X_{C3} = \frac{1}{\omega C_3} = \frac{1}{2\pi \cdot 4.7 \sim \text{nF} \cdot 4 \sim \text{MHz}} = 10.0 \sim \Omega$

$\frac{1}{Z} = \frac{1}{R_3} + \frac{1}{X_{C3}}$
 $\frac{1}{Z} = \frac{1}{10.0 \sim \Omega} + \frac{1}{10.0 \sim \Omega} = \frac{2}{10.0 \sim \Omega}$
 $Z = 5.0 \sim \Omega$

Therefore, the resulting current of the parallel circuit is given as:

$I = \frac{U}{Z} = \frac{10 \sim \text{V}}{5.0 \sim \Omega} = 2.0 \sim \text{A}$

Exercise E7 Impedances at different Frequencies

(written test, approx. 18 % of a 60-minute written test, WS2022)

Exercise E1 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the equivalent impedance Z for the circuit shown in the following diagram (of 3S) 21.B.0
 $\sim \sqrt{2} \cdot 10 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$ is the voltage source. $R_1 = 10 \Omega$, $R_2 = 20 \Omega$, $R_3 = 30 \Omega$, $L = 1 \text{ mH}$, $C = 10 \text{ nF}$.
 The resistor R_1 shall have the same absolute value of the impedance as a capacitor
 $X_C = 10 \Omega$ at $f = 15 \text{ kHz}$.

Solution:
 $R_1 = 10 \Omega$
 $R_2 = 20 \Omega$
 $R_3 = 30 \Omega$
 $L = 1 \text{ mH}$
 $C = 10 \text{ nF}$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by
 $Z_{RL} = R + j\omega L$
 Parallel circuit means that the voltage is the same on R_2 and R_3
 $\frac{1}{Z_{R2R3}} = \frac{1}{R_2} + \frac{1}{R_3}$
 $Z_{R2R3} = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{20 \cdot 30}{20 + 30} = 12 \Omega$
 $Z_{L} = j\omega L = j2\pi \cdot 15 \text{ kHz} \cdot 1 \text{ mH} = j0.942 \Omega$
 $Z_{C} = \frac{1}{j\omega C} = \frac{1}{j2\pi \cdot 15 \text{ kHz} \cdot 10 \text{ nF}} = -j1.051 \Omega$
 $Z_{R1} = 10 \Omega$
 $Z_{R2R3} = 12 \Omega$
 $Z_{L} = j0.942 \Omega$
 $Z_{C} = -j1.051 \Omega$
 $Z = Z_{R1} + Z_{R2R3} + Z_{L} + Z_{C} = 10 \Omega + 12 \Omega + j0.942 \Omega - j1.051 \Omega = 22 \Omega + j0.891 \Omega$
 $|Z| = \sqrt{22^2 + 0.891^2} = 22.018 \Omega$
 $\angle Z = \arctan\left(\frac{0.891}{22}\right) = 2.28^\circ$
 $Z = 22.018 \Omega \cdot e^{j2.28^\circ}$

Exercise E1 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

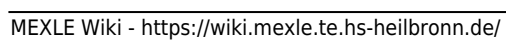
1. Calculate the equivalent impedance Z for the circuit shown in the following diagram (of 3S) 21.B.0
 $\sim \sqrt{2} \cdot 10 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$ is the voltage source. $R_1 = 10 \Omega$, $R_2 = 20 \Omega$, $R_3 = 30 \Omega$, $L = 1 \text{ mH}$, $C = 10 \text{ nF}$.
 The resistor R_1 shall have the same absolute value of the impedance as a capacitor
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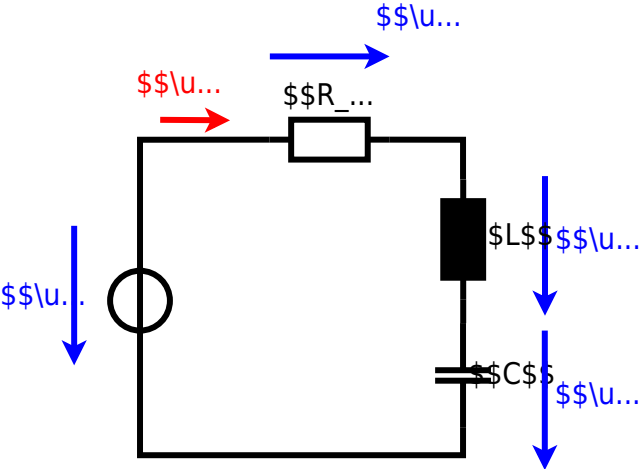
Solution:
 $R_1 = 10 \Omega$
 $R_2 = 20 \Omega$
 $R_3 = 30 \Omega$
 $L = 1 \text{ mH}$
 $C = 10 \text{ nF}$

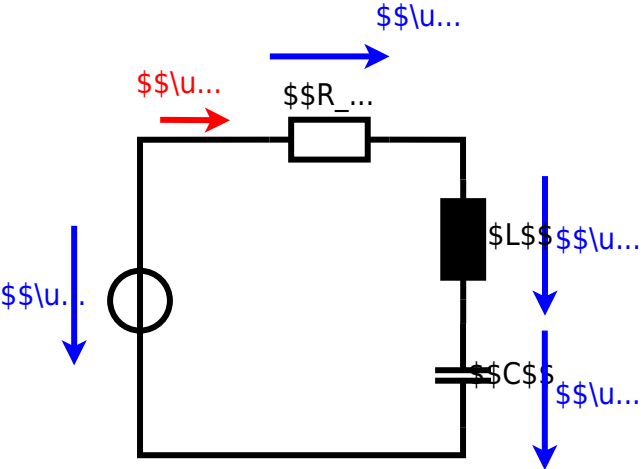
Result:
 $Z = 22.018 \Omega \cdot e^{j2.28^\circ}$
 Draw the circuit diagram of the given circuit.
 Label all components, voltages, and currents.

$Z = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + j\omega L + \frac{1}{j\omega C}}}$
 $Z = \frac{1}{\frac{1}{10} + \frac{1}{20 + j0.942 - j1.051}}$
 $Z = \frac{1}{\frac{1}{10} + \frac{1}{19.858}}$
 $Z = 4.82 \Omega$

Result:
 $Z = 4.82 \Omega$
 $\angle Z = \arctan\left(\frac{0.891}{22}\right) = 2.28^\circ$
 $Z = 4.82 \Omega \cdot e^{j2.28^\circ}$
 $\underline{Z} = R + jX_L - jX_C = R + j\omega L - \frac{1}{j\omega C}$
 $\underline{Z} = 10 + j0.942 - j1.051 = 10 - j0.109$
 $|\underline{Z}| = \sqrt{10^2 + 0.109^2} = 10.006 \Omega$
 $\angle \underline{Z} = \arctan\left(\frac{-0.109}{10}\right) = -0.62^\circ$
 $\underline{Z} = 10.006 \Omega \cdot e^{-j0.62^\circ}$







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