

6 Inductances in Circuits

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

5. Inductances in Circuits	2
5.1 Basic Circuits	2
5.1.1 Series Circuits	2
5.1.2 Parallel Circuits	2
5.2 Charging and Discharging	2
5.3 Resonance Phenomena	2
5.3.1 RLC - Series Resonant Circuit	2
5.4 Applications of Inductors	5
5.x Examples	5
Decoupling capacitor on the microcontroller	5
Crystal as Resonant Circuit	5

5. Inductances in Circuits

5.1 Basic Circuits

Focus here: uncoupled inductors!

5.1.1 Series Circuits

Based on $L = \frac{\Psi(t)}{I}$ and Kirchhoff's mesh law ($I = \text{const}$) the series circuit of inductions can be interpreted as a single current I which generates multiple linked fluxes Ψ . Since the current must stay constant in the series circuit, the following applies for multiple inductions in series connection:

$$L = \frac{\sum_i \Psi_i}{I} = \sum_i L_i$$

A similar result can be derived from the look onto the formula of the inductances of two long coils: two inductances L_1 and L_2 can be interpreted as coils with similar permeability μ_r and cross sectional area A , but differing length l_1 and l_2 . Connecting them in series is equal to adding the length of l_2 to l_1 .

5.1.2 Parallel Circuits

5.2 Charging and Discharging

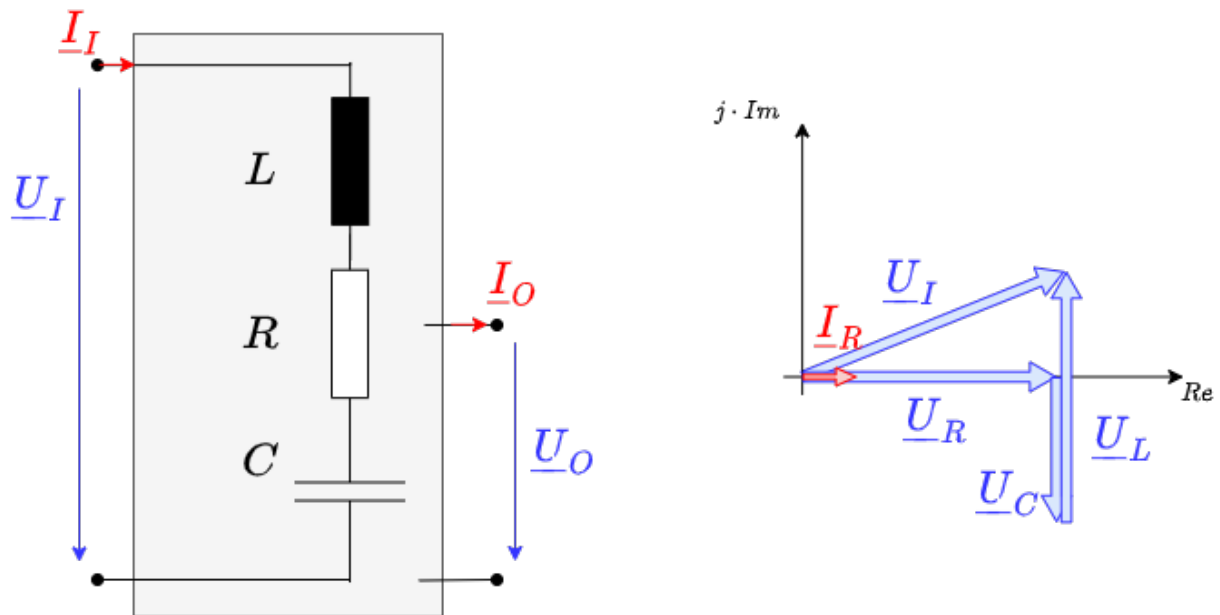
- signal time course
- comparison with RC-circuit charging / discharging

5.3 Resonance Phenomena

Link to: consideration on sinusoidal signals in EE1

5.3.1 RLC - Series Resonant Circuit

Fig. ##: circuit of the series resonant circuit



If a resistor R , a capacitor C and an inductance L are connected in series, the result is a series resonant circuit. In this case the output voltage is not clearly defined. It must be considered in the following how the voltages behave across the individual components. The total voltage (= input voltage $\underline{U}_I$) results to:

$$\underline{U} = \underline{U}_R + \underline{U}_L + \underline{U}_C$$

Since the current in the circuit must be constant, the total impedance can be determined here in a simple way:

$$\underline{U} = R \cdot \underline{I} + j \omega L \cdot \underline{I} + \frac{1}{j \omega C} \cdot \underline{I} \quad \underline{U} = \left(R + j \omega L - j \cdot \frac{1}{\omega C} \right) \cdot \underline{I} \quad \underline{Z}_{\text{ges}} = R + j \omega L - j \cdot \frac{1}{\omega C}$$

As the magnitude of the (input) voltage U or the (input or total) impedance Z and the phase result to:

$$U = \sqrt{U_R^2 + (U_L - U_C)^2} = \sqrt{U_R^2 + (U_L - U_C)^2}$$

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$\varphi_u = \varphi_Z = \arctan \frac{\omega L - \frac{1}{\omega C}}{R}$$

There are now 3 different situations to distinguish:

- If $U_L > U_C$ the whole setup behaves like an ohmic-inductive load. This is the case at high frequencies.
- If U_L equals U_C , the total input voltage U is applied to the resistor. In this case, the total resistance Z is minimal and only ohmic.
Thus, the current I is then maximal. If the current is maximum, then the responses of the capacitance and inductance - their voltages - are also maximum. This situation is the

resonance case.

- If $U_L < U_C$ then the whole setup behaves like a resistive-capacitive load. This is the case at low frequencies.

Again, there seems to be an excellent frequency, namely when $U_L = U_C$ or $Z_C = Z_L$ holds:

$$\begin{aligned} \frac{1}{\omega_0 C} &= \omega_0 L \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad 2\pi f_0 &= \frac{1}{\sqrt{LC}} \quad \rightarrow \boxed{f_0 = \frac{1}{2\pi \sqrt{LC}}} \end{aligned}$$

The frequency f_0 is called **resonance frequency**.

	$\quad$	$f \rightarrow 0$	$\quad$	$f = f_0$	$\quad$	$f \rightarrow \infty$
voltage U_R at the resistor		$\mathbf{0}$		U since the impedances just cancel		$\mathbf{0}$
voltage U_L at the inductor		$\mathbf{0}$ because ωL becomes very small		$\omega_0 L \cdot I = \omega_0 L \cdot \frac{U}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} \cdot U$		U since ωL becomes very large
$\mathbf{U}$ voltage U_C at the capacitor		$\mathbf{U}$ because $\frac{1}{\omega C}$ becomes very large		$\frac{1}{\omega_0 C} \cdot I = \frac{1}{\omega_0 C} \cdot \frac{U}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} \cdot U$		$\mathbf{0}$ because $\frac{1}{\omega C}$ becomes very small

The calculation in the table shows that in the resonance case, the voltage across the capacitor or inductor deviates from the input voltage by a factor $\frac{1}{R} \sqrt{\frac{L}{C}}$. This quantity is called **quality or Q-factor** Q_S :

$$\boxed{Q_S = \frac{U_C}{U} \Big|_{\omega = \omega_0} = \frac{U_L}{U} \Big|_{\omega = \omega_0} = \frac{1}{R} \sqrt{\frac{L}{C}}}$$

The quality can be greater than, less than or equal to 1.

- If the quality is very high, the overshoot of the voltages at the impedances becomes very large in the resonance case. This is useful and necessary in various applications, e.g. in an RLC element as an antenna.
- If the Q is very small, overshoot is no longer seen. Depending on the impedance at which the output voltage is measured, a high-pass or low-pass is formed similar to the RC or RL element. However, this has a steeper slope in the blocking range. This means that the filter effect is better.

The reciprocal of the Q is called **attenuation** d_S . This is specified when using the circuit as a non-overshooting filter.

$$\boxed{d_S = \frac{1}{Q_S} = R \sqrt{\frac{C}{L}}}$$

5.4 Applications of Inductors

- ferrite bead
- Decoupling
- Filter
- unwanted coupling and circuit design

5.x Examples

Decoupling capacitor on the microcontroller

[Simulation in Falstad](#). Note: The simulation gives a highly simplified picture. The response of the microcontroller is shown reduced to a triangular signal, since the slope of the voltages cannot be represented. A real simulation requires a powerful SPICE program in which the [conduction theory](#) can be represented.

Further details can be found [here \(practice\)](#), [here \(layout\)](#), also [Layout](#) or [Layout](#).

Crystal as Resonant Circuit

[Falstad Simulation](#)

[frequency response](#)

[setup for the design](#)

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