

Exam Summer Semester 2022

Student Group

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Exam Summer Semester 2022

Additional permitted Aids

- non-programmable calculator,
- formulary (4 one-sided DIN A4 pages)

Hits

- The duration of the exam is 120 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Electrostatics I

(written test, approx. 10 % of a 120-minute written test, SS2022)

Given is the arrangement of the charges as in the picture below. The values of the previous results are $E_1 = 19.97 \cdot 10^{-6} \text{ N/C}$. Which value needs E_4 to have to get a resulting force of 0 N on q_0 ?

Path: $q_0 = -1 \text{ nC}$

- $q_1 = -2 \text{ nC}$

Path: $E_4 = 2310.97 \text{ V/m}$

$$\vec{F}_1 = \left(\begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \text{ N}$$

In the beginning the are 5 components, we can not calculate the resulting magnitude of the

$$|\vec{F}_1| = \sqrt{E_1^2} = 19.97 \cdot 10^{-6} \text{ N/C}$$

The perpendicularity is a position (vector) $\theta = 8.854 \cdot 10^{-12} \text{ rad}$

$$|\vec{F}_1| = |\vec{F}_2| \Rightarrow E_1 = E_2 \Rightarrow 19.97 \cdot 10^{-6} \text{ N/C} = E_2$$

In the x -direction q_1 and q_2 are in the same direction $F_1 = 19.97 \cdot 10^{-6} \text{ N}$

$$F_1 = q_0 \cdot E_1 = (-1 \text{ nC}) \cdot (19.97 \cdot 10^{-6} \text{ N/C}) = -1.997 \cdot 10^{-11} \text{ N}$$

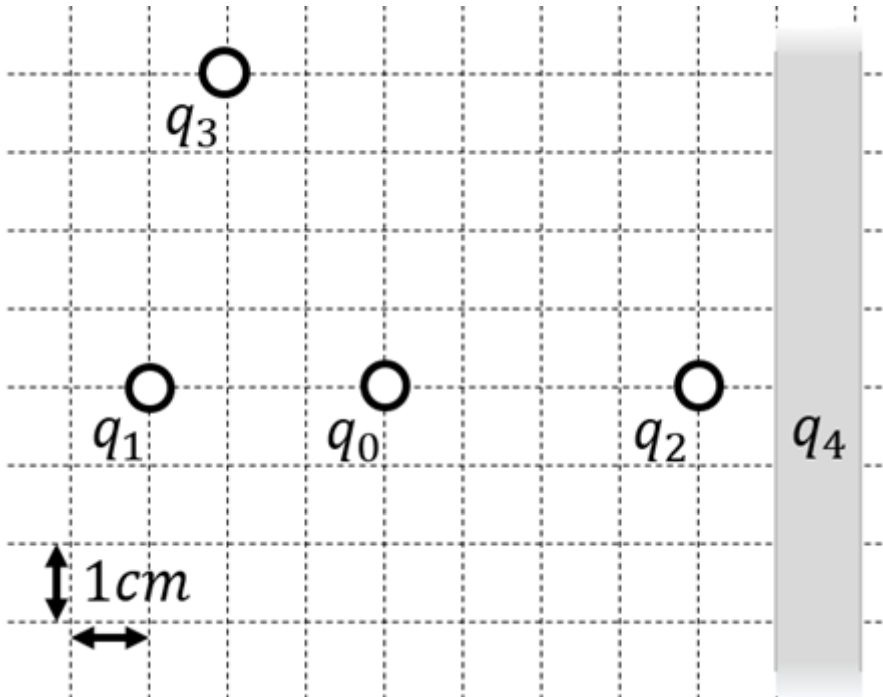
Here this finally leads to q_4 and q_0 the force $F_4 = q_0 \cdot E_4$

$$F_4 = q_0 \cdot E_4 = (-1 \text{ nC}) \cdot E_4 = -E_4 \cdot 10^{-9} \text{ C}$$

$F_1 + F_4 = 0 \Rightarrow -1.997 \cdot 10^{-11} \text{ N} - E_4 \cdot 10^{-9} \text{ C} = 0$

$$E_4 = -\frac{1.997 \cdot 10^{-11} \text{ N}}{10^{-9} \text{ C}} = 19.97 \cdot 10^{-3} \text{ N/C} = 19.97 \cdot 10^3 \text{ V/m}$$

$E_4 = 19.97 \cdot 10^3 \text{ V/m}$



1. Calculate the single forces \vec{F}_{01} , \vec{F}_{02} , \vec{F}_{03} , on the charge q_0 !

Path

First, calculate the magnitude of the forces, like \vec{F}_{01} .

The force \vec{F}_{01} is purely on the x -axis and therefore equal to

$$\begin{aligned} F_{01,x} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r_{01}^2} = \\ &= \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{1 \cdot 10^{-9} \text{ C} \cdot 2 \cdot 10^{-9} \text{ C}}{(3 \cdot 10^{-2} \text{ m})^2} = \\ &= 19.97... \cdot 10^{-6} \frac{\text{As}^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 19.97... \cdot 10^{-6} \frac{\text{VA}}{\text{m}} \\ &= 19.97... \mu\text{N} \quad \text{(to the right)} \end{aligned}$$

Similarly, we get for \vec{F}_{02} and \vec{F}_{03}

$$\begin{aligned} \vec{F}_{02} &= F_{02,x} = -28.09... \mu\text{N} \quad \text{(to the right)} \\ \vec{F}_{03} &= -22.47... \mu\text{N} \quad \text{(to the top left)} \end{aligned}$$

For \vec{F}_{03} , we have to calculate the x - and y -component.

This is possible, by using the angle α between the line through q_0 and q_3 and the positive x -axis (pointing to the right).

So, α has to be between 90° and 180° . It can be calculated by:

$$\begin{aligned} \alpha &= \arctan\left(\frac{-4 \text{ cm}}{+2 \text{ cm}}\right) = \pi - 1.1071... \\ &= 180^\circ - 63.4...^\circ = 116.6...^\circ \end{aligned}$$

Based on this, the x - and y -component is:

$$\begin{aligned} F_{03,x} &= |\vec{F}_{03}| \cdot \cos \alpha = 10.05... \mu\text{N} \quad \text{(to the left)} \\ F_{03,y} &= |\vec{F}_{03}| \cdot \sin \alpha = 20.10... \mu\text{N} \quad \text{(to the} \end{aligned}$$

top)} \\ \end{align*}

Exercise E2 Electrostatics I

(written test, approx. 10 % of a 120-minute written test, SS2022)

2. What is the magnitude of the force that Q_0 exerts on Q_4 ? The values of the previous results are E_4 . Which value needs E_4 to have to get a resulting force of 0 N on q_0 ?

Path

- $q_0 = -1 \text{ nC}$

- $q_1 = -2 \text{ nC}$

Path $E_4 = 2310.97 \text{ V/m}$

- $\vec{F}_{01} = \left(\begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \text{ nN}$

In the beginning, the force components, we cannot calculate the resulting magnitude of the force. The force F_{02} is purely on the y -axis and therefore equal to $F_{02,y}$.

- $F_{02,y} = \left(\begin{array}{c} 0 \\ 28.09 \\ 0 \end{array} \right) \text{ nN}$

- $F_{03} = \left(\begin{array}{c} 19.05 \\ 0 \\ 0 \end{array} \right) \text{ nN}$

- $F_{04} = \left(\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) \text{ nN}$

- $E_4 = \left(\begin{array}{c} 19.97 \\ 28.09 \\ 0 \end{array} \right) \text{ V/m}$

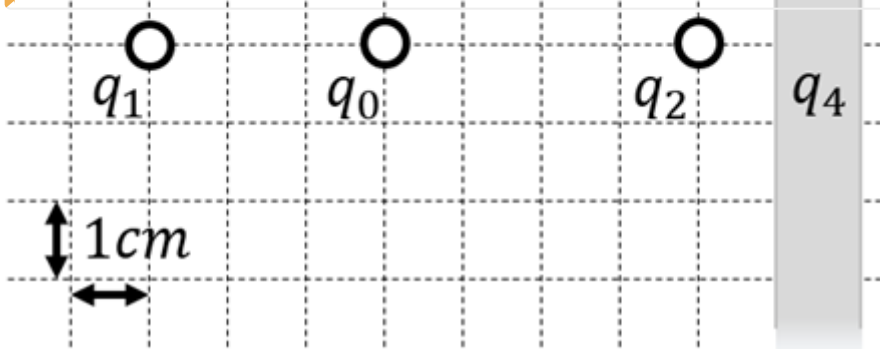
- $F_{01} = \left(\begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \text{ nN}$

- $F_{02} = \left(\begin{array}{c} 0 \\ 28.09 \\ 0 \end{array} \right) \text{ nN}$

- $F_{03} = \left(\begin{array}{c} 19.05 \\ 0 \\ 0 \end{array} \right) \text{ nN}$

- $F_{04} = \left(\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) \text{ nN}$

- $\vec{F}_{01} = \left(\begin{array}{c} 19.97 \\ 0 \\ 0 \end{array} \right) \text{ nN}$



1. Calculate the single forces \vec{F}_{01} , \vec{F}_{02} , \vec{F}_{03} , on the charge q_0 !

Path

First, calculate the magnitude of the forces, like F_{01} .

The force F_{01} is purely on the x -axis and therefore equal to $F_{01,x}$.

$\vec{F}_{01} = F_{01,x} \hat{x}$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r^2_{01}} \quad \&= \quad \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \cdot \text{As/Vm}} \cdot \frac{1 \cdot 10^{-9} \cdot \text{C} \cdot 2 \cdot 10^{-9} \cdot \text{C}}{(3 \cdot 10^{-2} \cdot \text{m})^2} \quad \&= \quad 19.97... \cdot 10^{-6} \cdot \frac{(\text{As})^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 19.97... \cdot 10^{-6} \cdot \frac{\text{VAs}}{\text{m}} = 19.97... \cdot 10^{-6} \cdot \frac{\text{Ws}}{\text{m}} \quad \&= \quad 19.97... \cdot \mu\text{N} \quad \text{(to the right)}$$

Similarly, we get for \vec{F}_{02} and \vec{F}_{03}

$$\vec{F}_{02} = F_{02,x} \quad \&= \quad -28.09... \cdot \mu\text{N} \quad \text{(to the right)}$$

$$\vec{F}_{03} \quad \&= \quad -22.47... \cdot \mu\text{N} \quad \text{(to the top left)}$$

For \vec{F}_{03} , we have to calculate the x - and y -component.

This is possible, by using the angle α between the line through q_0 and q_3 and the positive x -axis (pointing to the right).

So, α has to be between 90° and 180° . It can be calculated by:

$$\alpha = \arctan\left(\frac{-4 \text{ cm}}{+2 \text{ cm}}\right) = \pi - 1.1071... = 180^\circ - 63.4...^\circ = 116.6...^\circ$$

Based on this, the x - and y -component is:

$$|\vec{F}_{03}| \cdot \cos \alpha = 10.05... \cdot \mu\text{N} \quad \text{(to the left)}$$

$$F_{03,y} \quad \&= \quad |\vec{F}_{03}| \cdot \sin \alpha = 20.10... \cdot \mu\text{N} \quad \text{(to the top)}$$

Exercise E3 Electrostatics II

(written test, approx. 10 % of a 120-minute written test, SS2022)

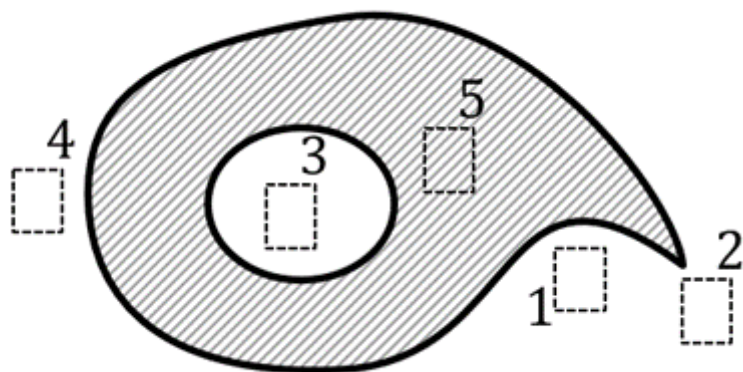
The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to $q = +1 \text{ nC}$.

In white a dielectric (e.g. vacuum) is shown.

Several designated areas are shown by dashed frames and numbers x , which are partly inside the object.

Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_x|$ (absolute magnitude)!

Indicate also, if designated areas have quantitatively the same field strength.



Result

$$|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$$

Exercise E4 Electrostatics II

(written test, approx. 10 % of a 120-minute written test, SS2022)

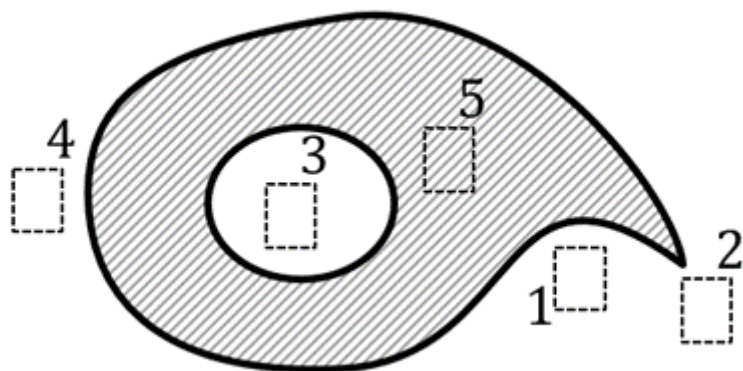
The figure below shows an arrangement of ideal metallic conductors (gray hatched) charged up to $q = +1 \text{ nC}$.

In white a dielectric (e.g. vacuum) is shown.

Several designated areas are shown by dashed frames and numbers x , which are partly inside the object.

Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_x|$ (absolute magnitude)!

Indicate also, if designated areas have quantitatively the same field strength.



Result

$$|E_3|=|E_5|=0 < |E_1| < |E_4| < |E_2|$$

Exercise E5 Electron Velocity in Semiconductors (written test, approx. 6 % of a 120-minute written test, SS2022)

A current of $I=1\text{ mA}$ flows through a cross-sectional area $A=10\text{ }\mu\text{m}^2$ in a semiconductor.

The electron density in the semiconductor is given by the number of dopant atoms per volume.

The doping shall provide 1 donor atom (= one electron) per 10^{10} silicon atoms. The elementary charge of silicon is $e_0 = 1.602 \cdot 10^{-19}\text{ As}$ (about 41 % of the speed of light). The molar volume of silicon is $V_{\text{mol,Si}} = 12 \cdot 10^{-6}\text{ m}^3/\text{mol}$, with $N_{\text{A}} = 6.022 \cdot 10^{23}$ silicon atoms per 1 mol .

The elementary charge is given as: $e_0 = 1.602 \cdot 10^{-19}\text{ As}$

What is the average electron velocity v_e in this semiconductor?

Path

The following formula gives the speed, where n_e is the number of electrons per volume.
$$v_e = \frac{I}{n_e \cdot e_0 \cdot A}$$

n_e can be derived from the overall number of Si-atoms per volume ($\frac{N_{\text{A}}}{V_{\text{mol,Si}}}$) and the fraction k_{Donators} of these atoms, which got substituted by donators.
$$n_e = \frac{N_{\text{A}}}{V_{\text{mol,Si}}} \cdot k_{\text{Donators}} \cdot e_0 \cdot A$$

Putting in the numbers:
$$v_e = \frac{1 \cdot 10^{-3}\text{ A}}{6.022 \cdot 10^{23} / \text{mol} \cdot 12 \cdot 10^{-6}\text{ m}^3/\text{mol} \cdot 10^{-10} \cdot 1.602 \cdot 10^{-19}\text{ As} \cdot 10 \cdot (10^{-6}\text{ m})^2}$$

Exercise E6 Electron Velocity in Semiconductors (written test, approx. 6 % of a 120-minute written test, SS2022)

Result Current of $I=1\text{ mA}$ flows through a cross-sectional area $A=10\text{ }\mu\text{m}^2$ in a semiconductor.

The electron density in the semiconductor is given by the number of dopant atoms per volume.

The doping shall provide 10^{16} dopant atoms (about 1% of the speed of light) silicon atoms. The molar volume of silicon is $V_{\text{mol,Si}} = 12 \cdot 10^{-6} \text{ m}^3/\text{mol}$, with $N_{\text{A}} = 6.022 \cdot 10^{23}$ silicon atoms per 1 mol .

The elementary charge is given as: $e_0 = 1.602 \cdot 10^{-19} \text{ As}$

What is the average electron velocity v_e in this semiconductor?

Path

The following formula gives the speed, where n_e is the number of electrons per volume.
$$v_e = \frac{I}{n_e \cdot e_0 \cdot A}$$

n_e can be derived from the overall number of Si-atoms per volume ($\frac{N_{\text{A}}}{V_{\text{mol,Si}}}$) and the fraction k_{Donators} of these atoms, which got substituted by donators.
$$n_e = \frac{N_{\text{A}}}{V_{\text{mol,Si}}} \cdot k_{\text{Donators}} \cdot e_0 \cdot A$$

Putting in the numbers:
$$v_e = \frac{1 \cdot 10^{-3} \text{ A}}{6.022 \cdot 10^{23} \text{ 1/mol} \cdot 12 \cdot 10^{-6} \text{ m}^3/\text{mol} \cdot 10^{-10} \cdot 1.602 \cdot 10^{-19} \text{ As} \cdot 10 \cdot (10^{-6} \text{ m})^2}$$

Exercise E7 Capacitor
(written test, approx. 7 % of a 120-minute written test, SS2022)

Given: Dielectric material capacitor with the following dimensions: $c=0.1 \text{ }\mu\text{m}$

Result: ϵ_r of air ($\epsilon_r=1$), while the thickness of the dielectric material remains the same

Path: Length of layer overlap: $l=1.5 \text{ mm}$
Distance between single layers: $d=1.0 \text{ }\mu\text{m}$

- Depth of component: $w=0.7 \text{ mm}$
- ϵ_r of dielectric material (from the picture): 3 left-side and 3 right-side layers.

Path

The capacity can be derived from the geometry by:
$$C = \epsilon_0 \epsilon_r \frac{A}{d}$$

The air builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as
$$C_{\text{Air}} = \frac{C \cdot C_{\text{dielectric}}}{C + C_{\text{Air}}}$$

The capacity of air is
$$C_{\text{Air}} = \frac{\epsilon_0 \epsilon_r N A}{d} = 8.854 \cdot 10^{-12} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}} = 0.465 \dots \text{ nF}$$

By this the overall capacity is
$$C_{\text{C}} = \frac{0.139 \dots \text{ nF} \cdot 0.465 \dots \text{ nF}}{0.139 \dots \text{ nF} + 0.465 \dots \text{ nF}}$$

How many "multiple plates" N do we have to consider?
 For this, we have to count facing areas A_0 . There are $N=5$.

The material shall have a dielectric permittivity of $\epsilon_r=3$.
 The following calculations shall ignore boundary effects on the end of the layers.

$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ As/Vm}$

.. What is the field strength in the dielectric material between the layer, when a voltage of $U=6.3 \text{ V}$ is applied?

Path

The electric field strength E is given by:
$$E = \frac{U}{d} = \frac{6.3 \text{ V}}{1 \cdot 10^{-6} \text{ m}} = 6.3 \cdot 10^6 \text{ V/m}$$

Therefore, the formula is
$$C = \frac{\epsilon_0 \epsilon_r N A}{d} = 8.854 \cdot 10^{-12} \cdot 3 \cdot \frac{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}}$$

Exercise E8 Capacitor
(written test, approx. 7 % of a 120-minute written test, SS2022)

Given the multiple capacitor shown in the left side below, with the following dimensions: $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ As/Vm}$

Results of third layer capacitor, $\epsilon_r = 1.5$, with the thickness of the dielectric material remains the same between single layers: $d = 1.0 \text{ ~}\mu\text{m}$
 What is the capacitance C ?

- Number of layers (as shown in the picture): 3 left-side and 3 right-side layers.

$$E = \frac{Q}{\epsilon_0 \epsilon_r A} = \frac{U}{d}$$

The capacity can be derived from the geometry by: $C = \epsilon_0 \epsilon_r \frac{N \cdot l \cdot w}{d}$

For the area A we have multiple plates with the area $A_0 = l \cdot w$ facing each other builds another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as $C_{\text{Air}} = \frac{\epsilon_0 \epsilon_r N \cdot l \cdot w}{d}$

$$C_{\text{Air}} = \frac{\epsilon_0 \epsilon_r N \cdot l \cdot w}{d} = 8.854 \cdot 10^{-12} \cdot 5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3} \cdot 1 \cdot 10^{-6}$$

The material shall have a dielectric permittivity of $\epsilon_r = 1.5$.
 The following calculations shall ignore boundary effects on the end of the layers.

$$C_{\text{Air}} = \frac{\epsilon_0 \epsilon_r N \cdot l \cdot w}{d} = \frac{8.854 \cdot 10^{-12} \cdot 5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}} = 0.465 \text{ nF}$$

What is the field strength in the dielectric material between the layer, when a voltage of $U = 6.3 \text{ V}$ is applied?

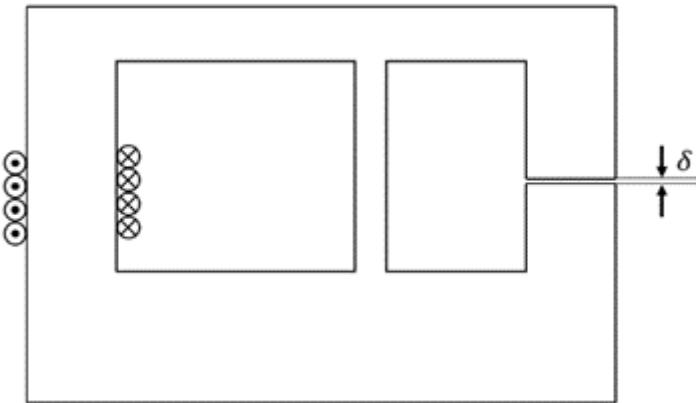
$$E = \frac{U}{d} = \frac{6.3 \text{ V}}{1 \cdot 10^{-6} \text{ m}} = 6.3 \cdot 10^6 \text{ V/m}$$

$$C = \frac{\epsilon_0 \epsilon_r N \cdot l \cdot w}{d} = \frac{8.854 \cdot 10^{-12} \cdot 3 \cdot 5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{1 \cdot 10^{-6}} = 0.165 \text{ nF}$$

Exercise E9 Magnetic Circuit
(written test, approx. 7 % of a 120-minute written test, SS2022)

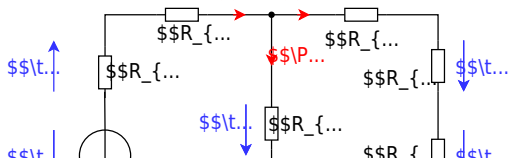
The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages. The components shall be designed in such a way, that the magnetic resistance is constant in it.

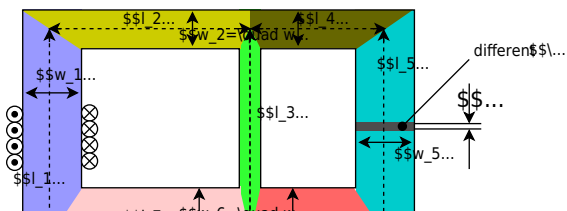
Formulas are not necessary.



Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each "diversion".

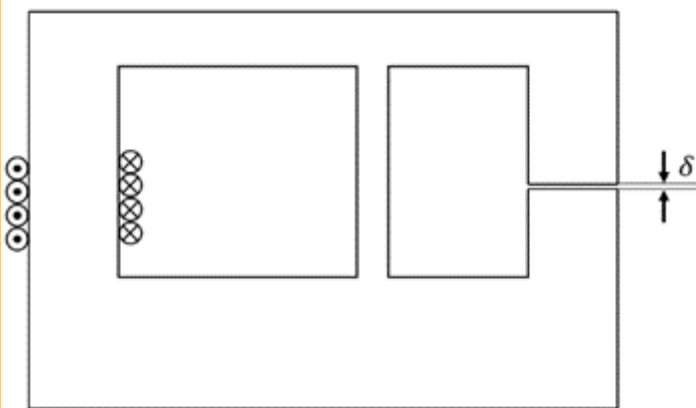
$$R_{\text{m}} = \frac{1}{\mu_0 \mu_r} \frac{l}{w \cdot h}$$




Exercise E10 Magnetic Circuit
(written test, approx. 7 % of a 120-minute written test, SS2022)

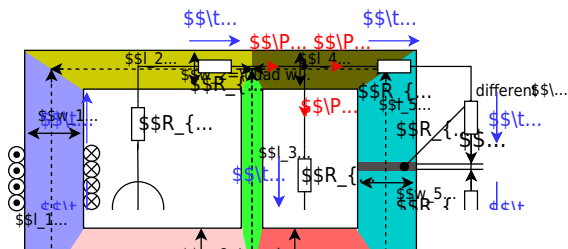
The magnetic setup below shall be given. Draw the equivalent magnetic circuit to represent the setup fully. Name all the necessary magnetic resistances, fluxes, and voltages. The components shall be designed in such a way, that the magnetic resistance is constant in it.

Formulas are not necessary.



Path

Watch for parts of the magnetic circuit, where the width and material are constant. These parts represent the magnetic resistors which have to be calculated individually. Be aware, that every junction creates a branch with a new resistor, like for an electrical circuit - there must be a node on each "diversion".

$$R_m = \frac{l}{\mu_0 \mu_r w h}$$


Exercise E1 Self Induction

(written test, approx. 8 % of a 120-minute written test, SS2022)

2. A circuit breaker is used to protect a DC voltage source, which is fused with a circuit breaker. The circuit breaker has a DC voltage source, which is fused with a circuit breaker.

Sketch the diagram of the circuit with the current $I(t) = I_0 \sin(\omega t)$ and the induced current $i_{ind}(t)$ in the circuit breaker. The current $I(t)$ is induced linearly down to 0 A within $1 \mu s$.

(The inner resistance of the motor shall be neglected.)

$$u_{ind}(t) = 3150 \text{ V}$$

Path

... Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

Result

- external voltage of the voltage source U_{ext}
- voltage $u_{\text{ind}}(t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

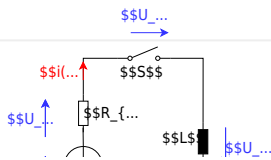
The induced voltage can be calculated by linearizing the following:

$$u_{\text{ind}}(t) = -L \frac{di}{dt} \Rightarrow u_{\text{ind}}(t) = -L \frac{\Delta i}{\Delta t}$$

With the given details:

$$u_{\text{ind}}(t) = -L \frac{0 - I}{t_1 - t_0} = 50 \cdot 10^{-6} \text{ H} \cdot \frac{63 \text{ A}}{1 \cdot 10^{-6} \text{ s}} = 3150 \frac{\text{Vs}}{\text{A}} \cdot \frac{\text{A}}{\text{s}}$$

\$\$u_{\text{ind}}(t) = \dots



Exercise E11 Self Induction
(written test, approx. 8 % of a 120-minute written test, SS2022)

A motor with a maximum current of $I = 63 \text{ A}$, which is fused with a circuit breaker that has a DC voltage source and which is fused with a circuit breaker. Sketch the breaker response $i(t)$ and $u_{\text{ind}}(t)$ with a current of $I = 63 \text{ A}$ at $t = 0$ and the induced voltage is reduced linearly down to 0 A within $1 \mu\text{s}$. (The inner resistance of the motor shall be neglected.)

$$u_{\text{ind}}(t) = 3150 \text{ V}$$

Path

.. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

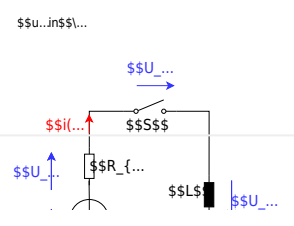
- external voltage of the voltage source U_{ext}
- voltage $u_{\text{ind}}(t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

The induced voltage can be calculated by linearizing the following:

$$u_{\text{ind}}(t) = -L \frac{di}{dt} \Rightarrow u_{\text{ind}}(t) = -L \frac{\Delta i}{\Delta t}$$

With the given details:
$$u_{\text{ind}}(t) = -L \frac{dI}{dt} = 50 \cdot 10^{-6} \cdot \frac{63 \cdot A}{1 \cdot 10^{-6} \cdot s} = 3150 \frac{Vs}{A} \cdot \frac{A}{s}$$



Exercise E12 Series Resonant Circuit
 (written test, approx. 10 % of a 120-minute written test, SS2022)

2. The input voltage magnitude of $U_{\text{eff}} = 10 \text{ V}$ is applied on the inductor. The circuit of an RLC network consists of a resistor $R = 10 \text{ }\Omega$, an inductor $L = 10 \text{ mH}$, and a capacitor $C = 10 \text{ nF}$.
 In this case, the magnitude of the voltage U_C would be $U_{C0} = Z_{\text{RLC}}$.
 Which value would C_0 have for the given f_0 ?

- Path 1: $C = 10 \text{ nF}$
- Path 2: $C = 100 \text{ nF}$
- Path 3: $C = 1 \text{ }\mu\text{F}$
- Path 4: $C = 10 \text{ }\mu\text{F}$

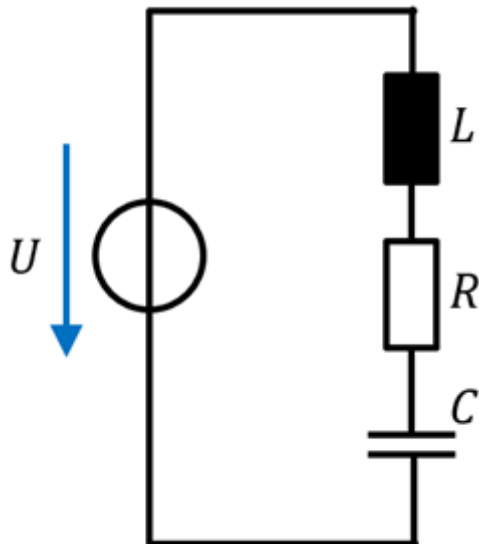
The resonance frequency is given as
$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10 \cdot 10^{-3} \cdot 10 \cdot 10^{-9}}} = 205 \text{ kHz}$$

 The calculated impedance of $Z_{\text{RLC}} = 20.5 \text{ }\Omega$ is set equal to $X_C(f_0)$.

$$Z_{\text{RLC}} = X_{C0} \Rightarrow \frac{1}{2\pi f \cdot C} \Rightarrow C = \frac{1}{2\pi f \cdot Z_{\text{RLC}}}$$

 At resonance, the impedance is given purely by the resistor.

With values:
$$C = \frac{1}{2\pi \cdot 100 \cdot 10^6 \cdot 20.5} = 10.6 \text{ nF}$$



1. What is the impedance \underline{Z}_{RLC} of this real capacitor for $f_0=100 \text{ ~\rm MHz}$? (Phase and magnitude)

Path

The impedance \underline{Z}_{RLC} is given by:
$$\underline{Z}_{RLC} = R + \underline{X}_L + \underline{X}_C \quad \&= R + \{\rm j\}\omega L - \{\{\rm j\}\over{\omega C}\} \quad \&= R + \{\rm j\}\cdot \left(\omega L - \frac{1}{\omega C}\right) \quad \&= R + \{\rm j\}\cdot X_{LC}$$

Putting in the numbers, only for the reactive part X_{LC} :
$$X_{LC} = 2\pi \cdot f_0 \cdot L - \frac{1}{2\pi \cdot f_0 \cdot C} \quad \&= 2\pi \cdot 100 \cdot 10^6 \text{ ~\rm Hz} \cdot 60 \cdot 10^{-12} \text{ ~\rm H} - \frac{1}{2\pi \cdot 100 \cdot 10^6 \text{ ~\rm Hz} \cdot 10 \cdot 10^{-9} \text{ ~\rm F}}$$

$$\&= -121.45... \text{ ~\rm m}\Omega$$

With the real and imaginary parts, we can derive the magnitude and phase:
$$Z_{RLC} = \sqrt{R^2 + X_{LC}^2} \quad \&= \sqrt{(88 \text{ ~\rm m}\Omega)^2 + (-121.45 \text{ ~\rm m}\Omega)^2} \quad \&= 150.0... \text{ ~\rm m}\Omega$$

$$\varphi = \arctan\left(\frac{X_{LC}}{R}\right) = \arctan\left(\frac{-121.45 \text{ ~\rm m}\Omega}{88 \text{ ~\rm m}\Omega}\right) = -54.07...^\circ$$

Exercise E13 Series Resonant Circuit
(written test, approx. 10 % of a 120-minute written test, SS2022)

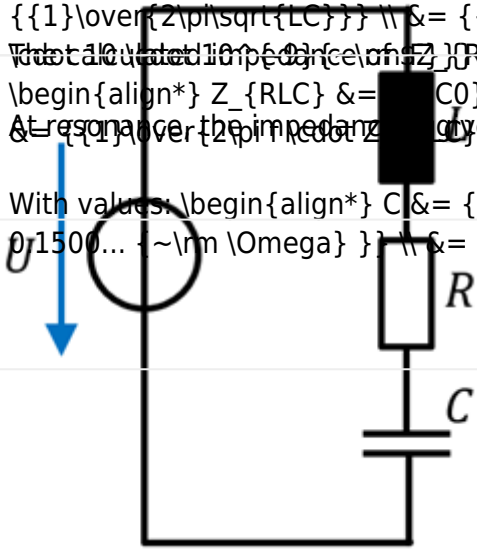
2. What is the resonance magnitude of R (in Ω) for a real capacitor in series with an inductor of inductance L .

At resonance, the magnitude of the total impedance Z_{RLC} would be $X_{C0} = Z_{RLC}$. Which value would C_0 have for the given f_0 ?

- Path
- $C = 10 \text{ nF}$
 - $C = 10.6 \text{ nF}$
 - $C = 10.6 \text{ pF}$
 - $C = 10.6 \text{ mF}$

The resonance frequency is given as $f_r = \frac{1}{2\pi\sqrt{LC}}$ $\Leftrightarrow C = \frac{1}{4\pi^2 f_r^2 L}$
 The calculated impedance of Z_{RLC} has to be set equal to X_C
 $Z_{RLC} = R + j(\omega L - \frac{1}{\omega C}) \Leftrightarrow R = \frac{1}{\omega C}$
 At resonance the impedance is only given by the resistor.

With values: $C = \frac{1}{2\pi \cdot 100 \cdot 10^6 \cdot 10.6 \cdot 10^{-9}} = 1.5 \cdot 10^{-6} \text{ F} = 1.5 \text{ }\mu\text{F}$



1. What is the impedance \underline{Z}_{RLC} of this real capacitor for $f_0 = 100 \text{ MHz}$? (Phase and magnitude)

Path

The impedance \underline{Z}_{RLC} is given by: $\underline{Z}_{RLC} = R + \underline{X}_L + \underline{X}_C = R + j\omega L - \frac{j}{\omega C}$

Putting in the numbers, only for the reactive part \underline{X}_{LC} : $\underline{X}_{LC} = 2\pi \cdot 100 \cdot 10^6 \cdot 60 \cdot 10^{-12} - \frac{j}{2\pi \cdot 100 \cdot 10^6 \cdot 10 \cdot 10^{-9}} = -121.45 \text{ m}\Omega$

With the real and imaginary parts, we can derive the magnitude and phase: $Z_{RLC} = \sqrt{R^2 + X_{LC}^2} = \sqrt{(88 \text{ m}\Omega)^2 + (-121.45 \text{ m}\Omega)^2} = 150.0 \text{ m}\Omega$

```
\begin{align*} \varphi &= \arctan \left( \frac{X_{LC}}{R} \right) &= \arctan \\ \left( \frac{-121.45 \text{ m}\Omega}{88 \text{ m}\Omega} \right) &= -0.9437... \\ &= -54.07...^\circ \end{align*}
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