Exam Summer Semester 2024

Student Group

First Name	Surname	Matrikel Nr.

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Exam Summer Semester 2024

Additional permitted Aids

- non-programmable calculator,
- formulary (4 one-sided DIN A4 pages)

Hits

- The duration of the exam is 120 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Electrostatics I
(written test, approx. 8 % of a 120-minute written test, SS2024)

2. Now set the charge $q_2=0$. The area charge q_4 generates a homogenous field E_4. Which value needs E_4 to have to get a resulting force of $0 ~\rm N$ on q_0?

Path

- $q_0=-1 ~\rm nC$
- $q_1=-5 ~\rm nC$

Path

In the beginning, the area charge is $q_4=0 ~\rm nC$.

The permittivity is $\varepsilon_0 = 8.854 \cdot 10^{-12} ~\rm As/Vm$

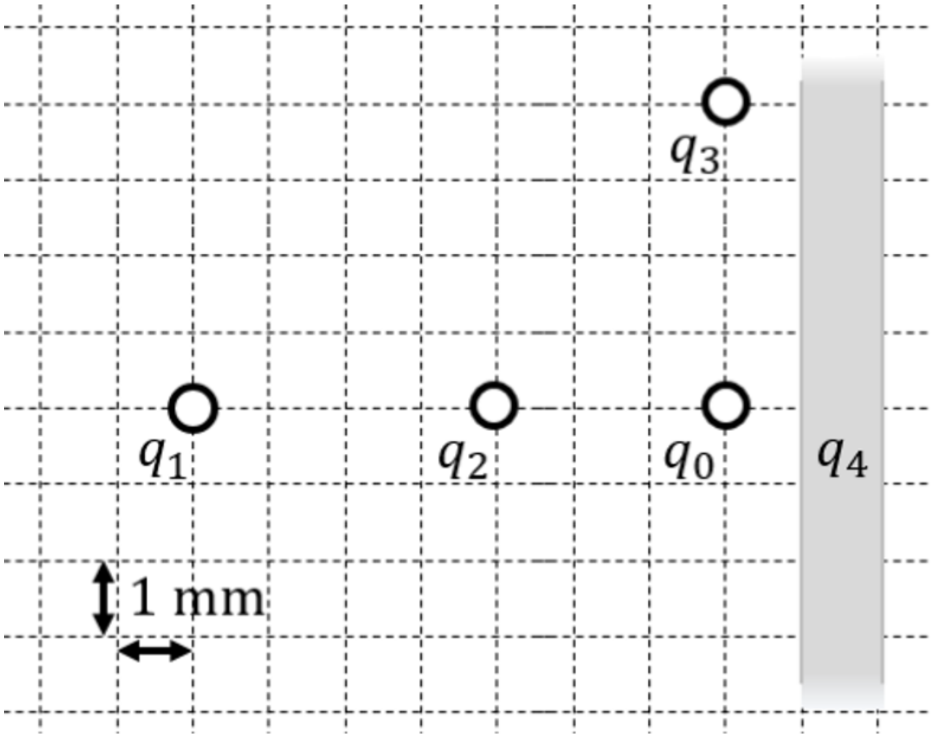
In the homogenous field the force is calculated by $F = E \cdot q$.

Here, this field has to compensate for the force $\vec{F}_{01}$ from q_1 on q_0:

$$\begin{align*} |\vec{F}_{01}| &= |E_4| \cdot |q_0| \\ \rightarrow E_4 &= {{|\vec{F}_{01}|}\over{|q_0|}} \\ &= {{917.\;.\!.\!.\! \cdot 10^{-6} {~\rm N}}\over{1 \cdot 10^{-9} ~\rm C }} \\ &= 917.\;.\!.\!.\! \cdot 10^{3}{{\rm N}\over{\rm C}} \\ &= 917.\;.\!.\!.\! \cdot 10^{3}{{\rm VAs/m}\over{\rm As}} \\ &= 917.\;.\!.\!.\! \cdot 10^{3}{{\rm V}\over{\rm m}} \\ \end{align*}$$

Result

$E_4 = 917 ~{\rm kV/m}$



1. Calculate the single forces $\vec{F}_{01}$, $\vec{F}_{02}$, $\vec{F}_{03}$, on the charge q_0!

Path

First, calculate the magnitude of the forces, like $\vec{F}_{01}$.
The force $\vec{F}_{01}$ is purely on the x-axis and therefore equal to $F_{01,x}$. \begin{align*} \vec{F}_{01} = F_{01,x} &= {{1}\over{4\pi\varepsilon}}\cdot {{q_1\cdot q_0}\over{r^2_{01}}} \\ &= {{1}\over{4\pi\cdot 8.854 \cdot 10^{-12} ~\rm As/Vm}}\cdot {{1 \cdot 10^{-9} ~\rm C \cdot 5 \cdot 10^{-9} ~\rm C}\over{(7 \cdot 10^{-3} ~\rm m)^2}} \\ &= 917.\;.\!.\!.\! \cdot 10^{-6} {\rm {{(As)^2 \cdot Vm}\over{As \cdot m^2}}} = 917.\;.\!.\!.\! \cdot 10^{-6} {\rm {{VAs}\over{m}}} = 917.\;.\!.\!.\! \cdot 10^{-6} {\rm {{Ws}\over{m}}} \\ &= 917.\;.\!.\!.\! {\rm \mu N} \quad \text{(to the right)} \end{align*}

Similarly, we get for $\vec{F}_{02}$ and $\vec{F}_{03}$ \begin{align*} \vec{F}_{02} = F_{02,x} &= -1997.\;.\!.\!.\! {\rm \mu N} \quad \text{(to the right)} \\ \vec{F}_{03} = F_{03,y} &= -1123.\;.\!.\!.\! {\rm \mu N} \quad \text{(to the top)} \\ \end{align*}

Exercise E1 Electrostatics I
(written test, approx. 8 % of a 120-minute written test, SS2024)

2. Now the charge $q_2=q_3$ are set to 0. The area charge q_4 generates a homogenous field E_4. Which value needs E_4 to have to get a resulting force of $0 ~\rm N$ on q_0?

Path

- $q_0=-1 ~\rm nC$
- $q_1=-5 ~\rm nC$

Path

$E_4 = 917{{\rm kV}\over{\rm m}}$

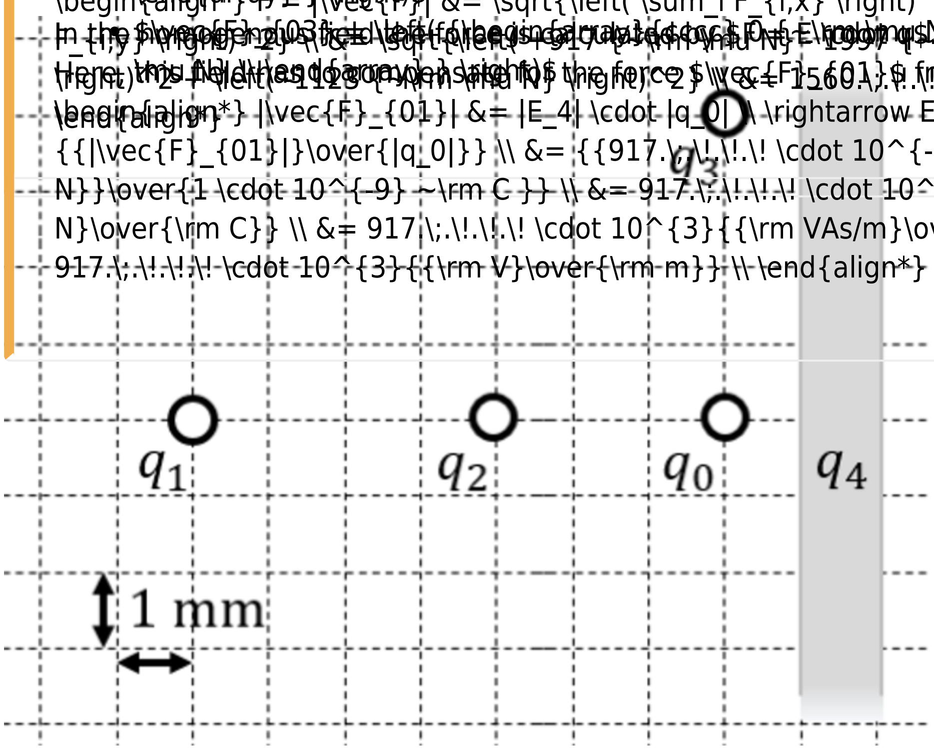
- $\vec{F}_{01} = \left({\begin{array}{cccc} +917 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{02} = \left({\begin{array}{cccc} -1997 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{03} = \left({\begin{array}{cccc} 0 {~\rm \mu N} \\ -1123 {~\rm \mu N} \\ \end{array} } \right)$

In the beginning, the area charge is $q_4=0 ~\rm nC$.

The permittivity is $\varepsilon_0 = 8.854 \cdot 10^{-12} ~\rm As/Vm$

With all the x- and y-components, we can calculate the resulting magnitude of the force with the Pythagorean Theorem: \begin{align*} F = |\vec{F}| &= \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} \\ &= \sqrt{\left(+917 {~\rm \mu N} - 1997 {~\rm \mu N} \right)^2 + \left(-1123 {~\rm \mu N} \right)^2} \\ &= 1560.\;.\!.\!.\! {~\rm \mu N} \\ \end{align*}

In the homogenous field the force is calculated by $F = E \cdot q$.
Here, this field has to compensate for the force $\vec{F}_{01}$ from q_1 on q_0: \begin{align*} |\vec{F}_{01}| &= |E_4| \cdot |q_0| \\ \rightarrow E_4 &= {{|\vec{F}_{01}|}\over{|q_0|}} \\ &= {{917.\;.\!.\!.\! \cdot 10^{-6} {~\rm N}}\over{1 \cdot 10^{-9} ~\rm C }} \\ &= 917.\;.\!.\!.\! \cdot 10^{3}{{\rm N}\over{\rm C}} \\ &= 917.\;.\!.\!.\! \cdot 10^{3}{{\rm VAs/m}\over{\rm As}} \\ &= 917.\;.\!.\!.\! \cdot 10^{3}{{\rm V}\over{\rm m}} \\ \end{align*}



1. Calculate the single forces $\vec{F}_{01}$, $\vec{F}_{02}$, $\vec{F}_{03}$, on the charge q_0!

Path

First, set up a coordinate system. Here, I choose x pointing to the right (positive values to the right) and y pointing upwards (positive values upwards).

Then, calculate the magnitude of the forces, like $\vec{F}_{01}$ (force on q_0 from q_1).
The force $\vec{F}_{01}$ is purely on the x-axis and therefore equal to $F_{01,x}$. \begin{align*} \vec{F}_{01} = F_{01,x} &= {{1}\over{4\pi\varepsilon}}\cdot {{q_1\cdot q_0}\over{r^2_{01}}} \\ &= {{1}\over{4\pi\cdot 8.854 \cdot 10^{-12} ~\rm As/Vm}}\cdot {{1 \cdot 10^{-9} ~\rm C \cdot 5 \cdot 10^{-9} ~\rm C}\over{(7 \cdot 10^{-3} ~\rm m)^2}} \\ &=

917.\;.\!.\!.\! \cdot 10^{-6} {\rm {{(As)^2 \cdot Vm}\over{As \cdot m^2}}} = 917.\;.\!.\!.\! \cdot 10^{-6} {\rm {{VAs}\over{m}}} = 917.\;.\!.\!.\! \cdot 10^{-6} {\rm {{Ws}\over{m}}} \\ &= +917.\;.\!.\!.\! {\rm \mu N} \end{align*} Since both q_0 and q_1 have the same sign for their charges, they are repelling each other. Therefore, The force $\vec{F}_{01}$ points to the right (positive value).

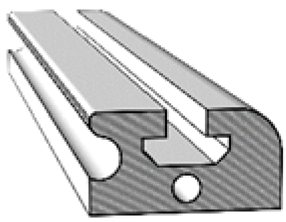
Similarly, we get for $\vec{F}_{02}$ and $\vec{F}_{03}$ \begin{align*} \vec{F}_{02} = F_{02,x} &= -1997.\;.\!.\!.\! {\rm \mu N} \\ \vec{F}_{03} = F_{03,y} &= -1123.\;.\!.\!.\! {\rm \mu N} \\ \end{align*} Since q_0 and q_2 have the different sign for their charges, they are attract each other. Therefore, The force $\vec{F}_{02}$ points to the left (negative value).
Since q_0 and q_3 have the different sign for their charges, they are attract each other. Therefore, The force $\vec{F}_{03}$ points downwards (negative value).

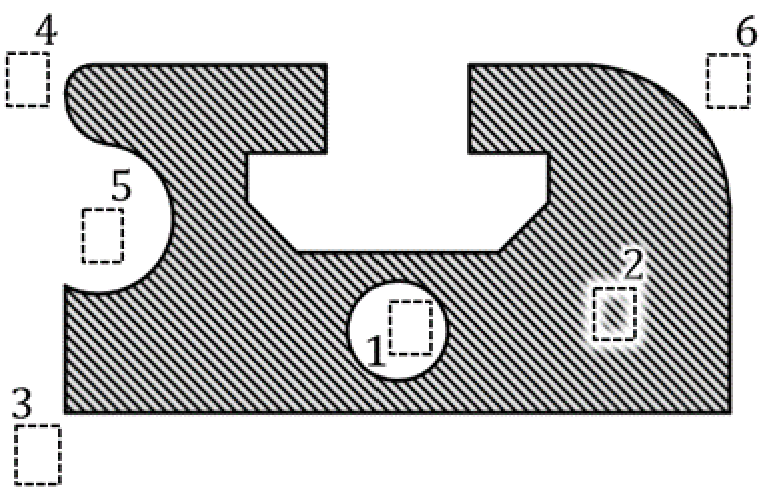
Exercise E1 Electrostatics II
(written test, approx. 10 % of a 120-minute written test, SS2024)

You must analyze an aluminum profile for usage in an environment critical for electrostatic discharge.

The figure on the right shows the cross-section of the aluminum element (hatched). During the application, it might get charged up. All areas in white consist of air (= dielectric).

Six designated areas are shown by dashed frames and numbers n, which are partly inside the object.





Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_n|$ (absolute magnitude)! Indicate also, if designated areas have quantitatively the same field strength.

Result

> $|E_1|=|E_2|=0 < |E_5| < |E_6| < |E_4| < |E_3|$

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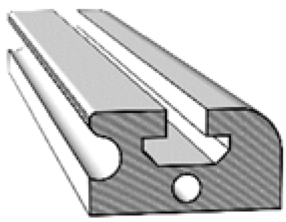

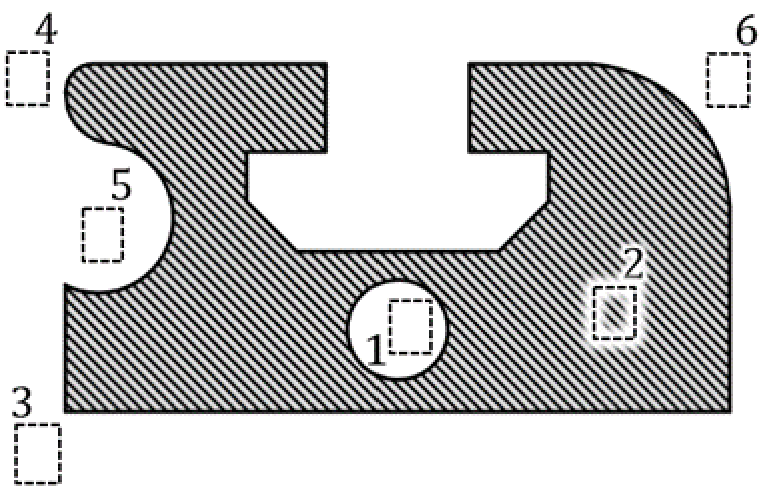

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Result

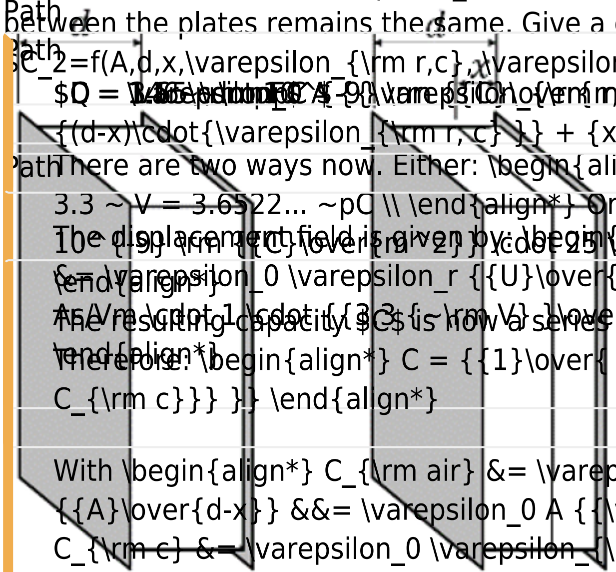
$|E_1|=|E_2|=0 < |E_5| < |E_6| < |E_4| < |E_3|$

Exercise E1 Capacitor
(written test, approx. 12 % of a 120-minute written test, SS2024)

The contaminant has $\varepsilon_{\rm r,c}>\varepsilon_{\rm r,air}$, while the distance between the plates remains the same. Give a generalized formula $C_2=f(A,d,x,\varepsilon_{\rm r,c},\varepsilon_{\rm r,air})$.

Result

Path



$C_2 = \varepsilon_0 A {{\varepsilon_{\rm r, air}}\cdot \varepsilon_{\rm r, c} }\over{ {(d-x)\cdot{\varepsilon_{\rm r, c} }} + {x\cdot{ \varepsilon_{\rm r, air}}} }}$

There are two ways now. Either: \begin{align*} Q &= C \cdot U = 1.1 ~\rm pF \cdot 3.3 ~ V = 3.6522... ~pC \\ \end{align*} Or: \begin{align*} Q &= D \cdot A = 146 \cdot 10^{-9} \rm {{C}\over{m^2}} \cdot 25 \cdot 10^{-6} ~m^2 = 3.6522... ~pC \\ \end{align*}

The displacement field is given by: \begin{align*} D &= \varepsilon_0 \varepsilon_r E \\ &= \varepsilon_0 \varepsilon_r {{U}\over{d}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 1 \cdot {{3.3 {~\rm V} }\over{200 \cdot 10^{-6} {~\rm m} }} \\ \end{align*}

The resulting capacity C is now a series circuit of $C_{\rm air}$ and $C_{\rm c}$. Therefore: \begin{align*} C = {{1}\over{ {{1}\over{C_{\rm air} }} + {{1}\over{ C_{\rm c}}} }} \end{align*}

With \begin{align*} C_{\rm air} &= \varepsilon_0 \varepsilon_{\rm r, air} {{A}\over{d-x}} &&= \varepsilon_0 A {{\varepsilon_{\rm r, air}}\over{d-x}} \\ C_{\rm c} &= \varepsilon_0 \varepsilon_{\rm r, c} {{A}\over{x}} &&= \varepsilon_0 A {{\varepsilon_{\rm r, c} }\over{x}} \\ \end{align*}

This leads to: \begin{align*} C = \varepsilon_0 A {{1}\over{ {{d-x}\over{\varepsilon_{\rm r, air} }} + {{x}\over{ \varepsilon_{\rm r, c}}} }} \end{align*}

In the following such a sensor is given with:

- Plate area : $A=25 ~\rm mm^2$
- Distance between both plates: $d=200 ~\rm \mu m$
- Air between the plates: $\varepsilon_{\rm r,air}=1$
- Supply voltage: $3.3 ~\rm V$
- Boundary effects on the end of the layers shall be ignored in the following calculations.

$\varepsilon_{0} =8.854 \cdot 10^{-12} ~\rm F/m $

1. Calculate the capacity C.

Path

\begin{align*} C &= \varepsilon_0 \varepsilon_r {{A}\over{d}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 1 \cdot {{25 \cdot 10^{-6} {~\rm m} }\over{200 \cdot

10^{-6} {~\rm m} }} \end{align*}

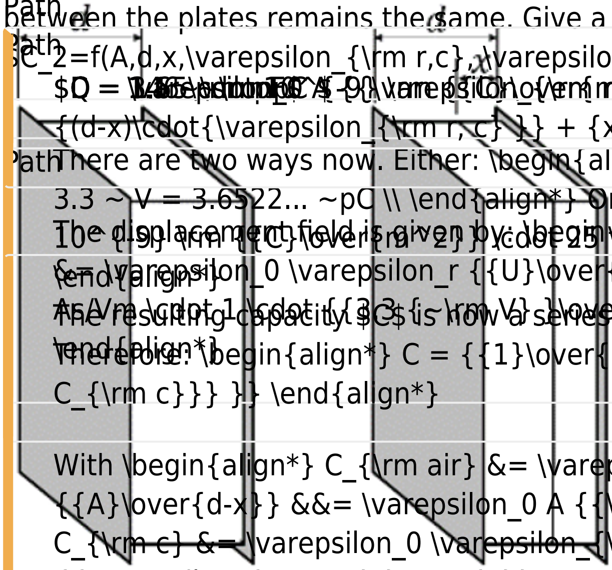
Exercise E1 Capacitor
(written test, approx. 12 % of a 120-minute written test, SS2024)

Result
voltage is applied.

The contaminant has $\varepsilon_{\rm r,c}>\varepsilon_{\rm r,air}$, while the distance between the plates remains the same. Give a generalized formula $C_2=f(A,d,x,\varepsilon_{\rm r,c},\varepsilon_{\rm r,air})$.

Path

Path



$Q = 3.65 \rm pC$ $D = 146 \cdot 10^{-9} \rm {C}\over{m^2}$

$C_2 = \varepsilon_0 {{A \cdot \varepsilon_{\rm r, c} }\over{ {(d-x)\cdot{\varepsilon_{\rm r, c} }} + {x\cdot{ \varepsilon_{\rm r, air}}} }}$

Path

There are two ways now. Either: \begin{align*} Q &= C \cdot U = 1.1 ~\rm pF \cdot 3.3 ~\rm V = 3.6522... ~pC \\ \end{align*} Or: \begin{align*} Q &= D \cdot A = 146 \cdot 10^{-9} \rm {C}\over{m^2}} \cdot 25 \cdot 10^{-6} ~m^2 = 3.6522... ~pC \\ \end{align*}

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$\varepsilon_{0} =8.854 \cdot 10^{-12} ~\rm F/m $

1. Calculate the capacity C.

Path

\begin{align*} C &= \varepsilon_0 \varepsilon_r {{A}\over{d}} \\ &= 8.854 \cdot 10^{-12} ~\rm As/Vm \cdot 1 \cdot {{25 \cdot 10^{-6} {~\rm m} }\over{200 \cdot 10^{-6} {~\rm m} }} \end{align*}

Exercise E1 Magnetic Field Lines
(written test, approx. 6 % of a 120-minute written test, SS2024)

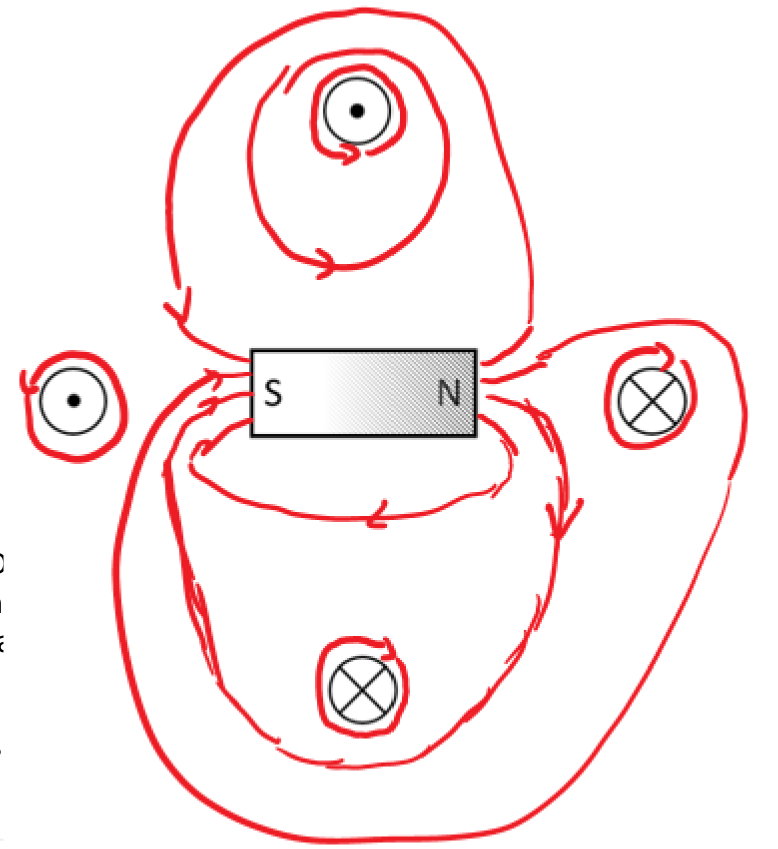
The following setup shall be given:

2. Discuss how the permanent magnet affects the H-field, based on the fundamental definition of the H-field.

- Four conductors are located perpendicular to the plane of the diagram
- All of them conduct a current with the same magnitude, but not in the same direction.
- A permanent magnet is located in between the conductors.

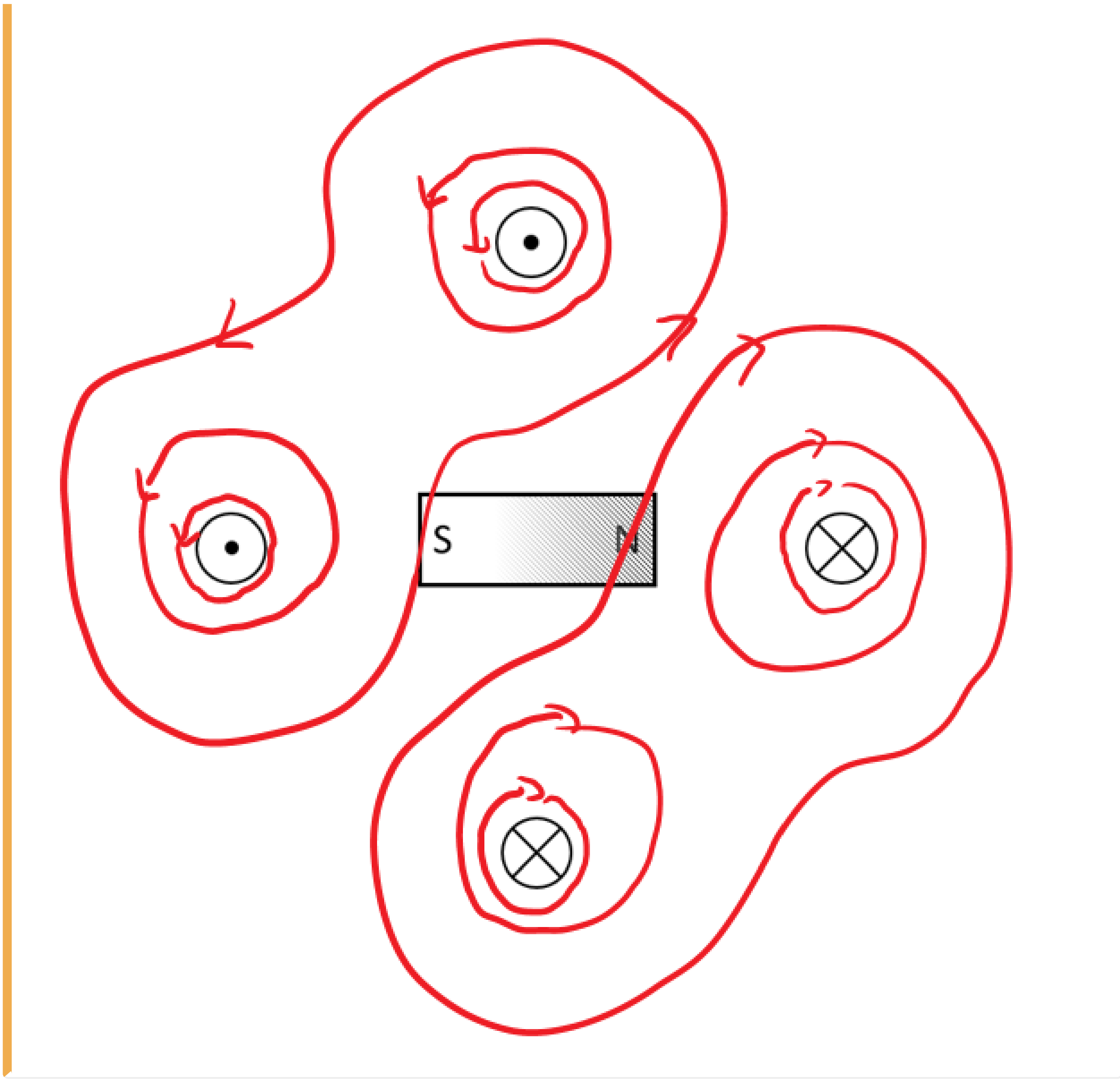
Result:

- The H-field is defined by currents $\sum I = \int H {\rm d}s$.
- In the permanent magnet, there are no free currents.
- The bound currents (of the permanent magnet) create also an H field.
- This exits on the north pole and enters the magnet on the south pole (similar to the B-field)_
- $H = B/\mu$
- The H-field from task 1 gets distracted



1. D... st 10 field lines of the H-field
qua... , and density for the shown
area

Res



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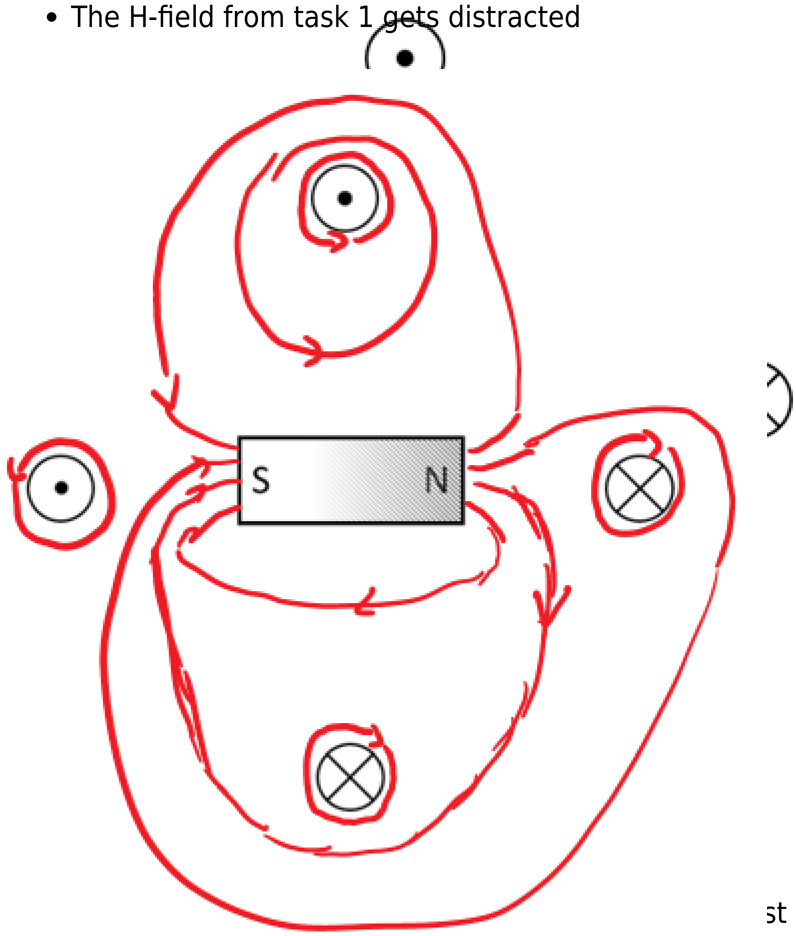

.. D … st 10 field lines of the H-field
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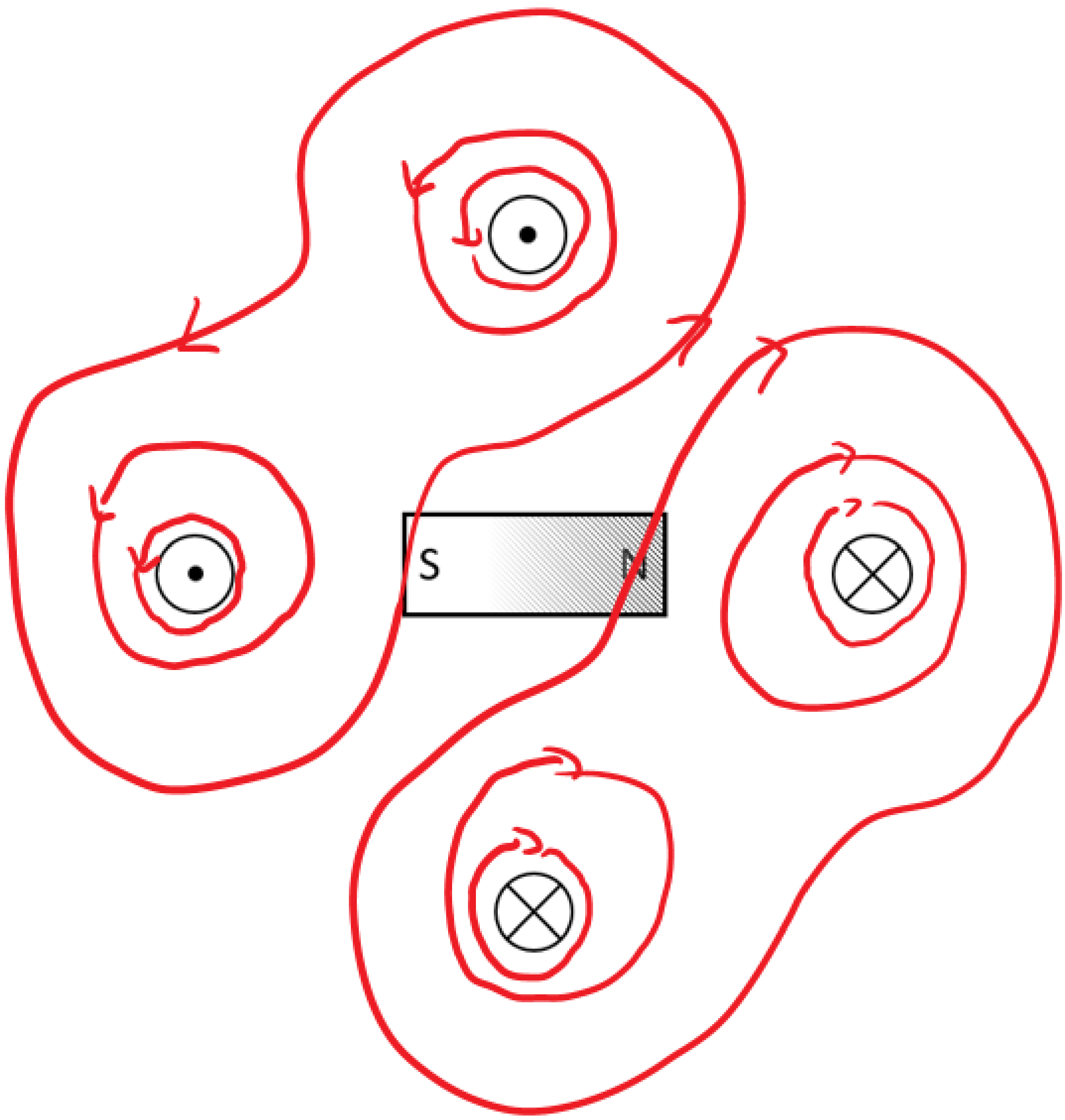
Result



Exercise E1 Fields of an coax Cable
(written test, approx. 12 % of a 120-minute written test, SS2024)

labels for the diagram.

The following situation appears:

- Inner conductor: $+3.3 ~\rm mA$, $+10 ~\rm nC$ (current into the plane of the diagram)
- Outer conductor: $-3.3 ~\rm mA$, $ 0 ~\rm nC$ (current out of the plane of diagram)

Path

Path

- for $(0.1 ~\rm mm | 0)$: $D_{\rm i} = 3.18... ~\rm µC/m^2$
- for $(0.55 ~\rm mm| 0)$: $D_{\rm o} = 0.785... ~\rm µC/m^2$

The magnitude of the electric displacement field D can be calculated by: $\int D {\rm d}A = Q$.

Here, for any position radial to the center, the surrounding area is the surface of a cylindrical shape (here for simplicity without the round endings).

This leads to: \begin{align*} D(x) &= {{Q}\over{A}} \\ &= {{Q}\over{l \cdot 2\pi

- In general, the D-field is proportional to ${{1}\over{r}}$ for the situation between both conductors.
- Since the charges are on the surface of the conductor, there is no D-field within the conductor.

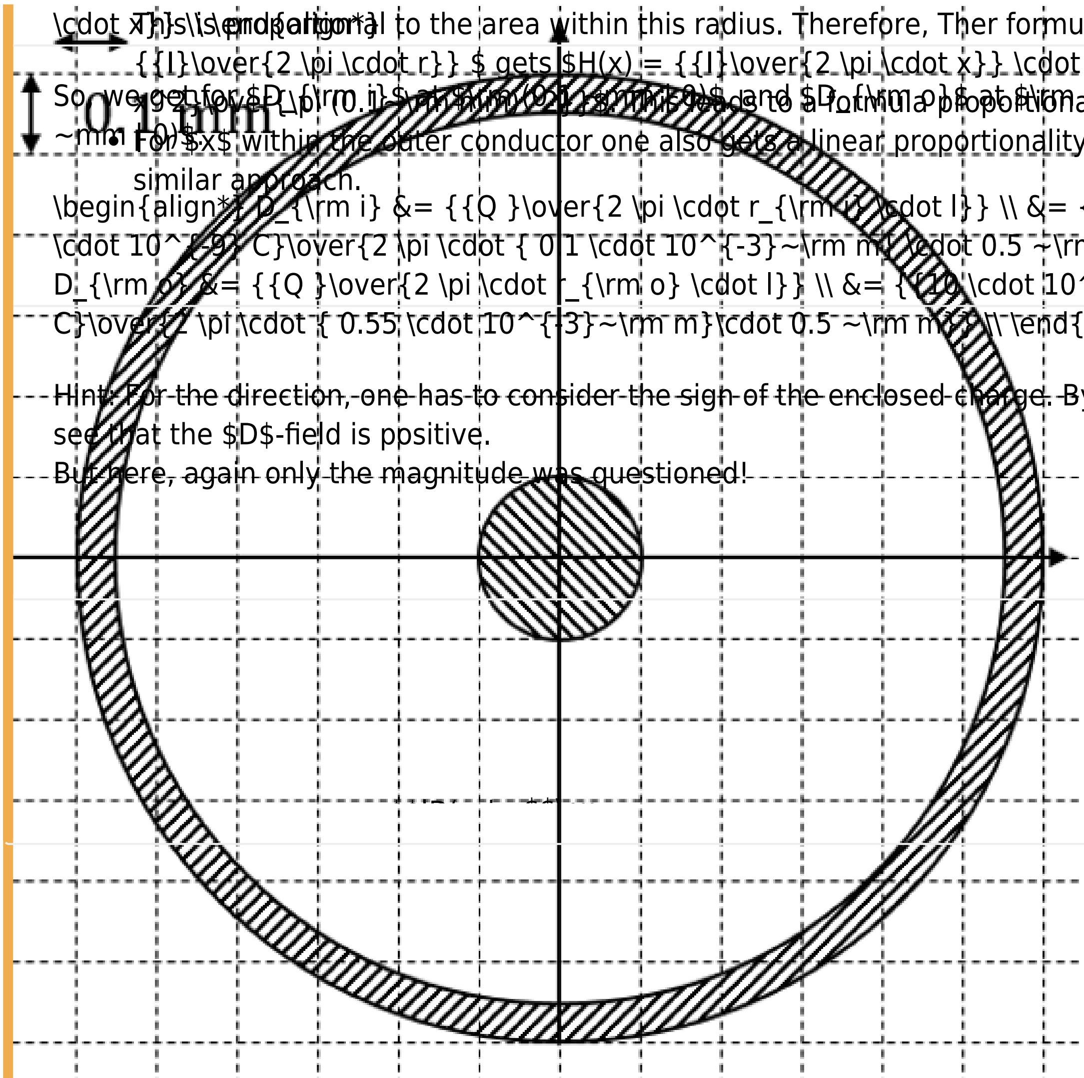
\cdot x}} \\ \end{align*}

- This is proportional to the area within this radius. Therefore, Ther formula $H = {{I}\over{2 \pi \cdot r}} $ gets $H(x) = {{I}\over{2 \pi \cdot x}} \cdot {{\pi x^2}\over{\pi (0.1 ~\rm mm)^2}}$. This leads to a formula proportional to x.
- For x within the outer conductor one also gets a linear proportionality with a similar approach.

So, we get for $D_{\rm i}$ at $\rm (0.1 ~mm | 0)$, and $D_{\rm o}$ at $\rm (0.55 ~mm | 0)$:

\begin{align*} D_{\rm i} &= {{Q }\over{2 \pi \cdot r_{\rm i} \cdot l}} \\ &= {{10 \cdot 10^{-9} C}\over{2 \pi \cdot { 0.1 \cdot 10^{-3}~\rm m} \cdot 0.5 ~\rm m}} \\ D_{\rm o} &= {{Q }\over{2 \pi \cdot r_{\rm o} \cdot l}} \\ &= {{10 \cdot 10^{-9} C}\over{2 \pi \cdot { 0.55 \cdot 10^{-3}~\rm m}\cdot 0.5 ~\rm m}} \\ \end{align*}

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the D-field is ppsitive.
But here, again only the magnitude was questioned!



. What is the magnitude of the magnetic field strength H at $\rm (0.1 ~mm | 0)$ and $\rm (0.55 ~mm | 0)$?

Path

The magnitude of the magnetic field strength H can be calculated by: $H = {{I}\over{2 \pi \cdot r}} $
So, we get for $H_{\rm i}$ at $\rm (0.1 ~mm | 0)$, and $H_{\rm o}$ at $\rm (0.55 ~mm | 0)$:

\begin{align*} H_{\rm i} &= {{I}\over{2 \pi \cdot r_{\rm i}}} \\ &= {{+3.3 A}\over{2 \pi \cdot { 0.1 \cdot 10^{-3}~\rm m}}} \\ H_{\rm o} &= {{I}\over{2 \pi \cdot r_{\rm o}}} \\ &= {{+3.3 A}\over{2 \pi \cdot { 0.55 \cdot 10^{-3}~\rm m}}} \\ \end{align*}

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the H-field on the right side points downwards.
Therefore, the sign of the H-field is negative.
But here, only the magnitude was questioned!

Exercise E1 Fields of an coax Cable
(written test, approx. 12 % of a 120-minute written test, SS2024)

A coax cable with the dimensions given in the diagram shows the cross-section with the origin in the center of the coax cable. Dimensions and labels for the diagram, and the following situation appears:

Result:

- Inner conductor: $+3.3 ~\rm mA$, $+10 ~\rm nC$ (current into the plane of the diagram)
  - for $(0.1 ~\rm mm | 0)$: $D_{\rm i} = 31.8... ~\rm µC/m^2$
- Outer conductor: $-3.3 ~\rm mA$, $ 0 ~\rm nC$ (current out of the plane of diagram)
  - for $(0.55 ~\rm mm| 0)$: $D_{\rm o} = 5.78... ~\rm µC/m^2$

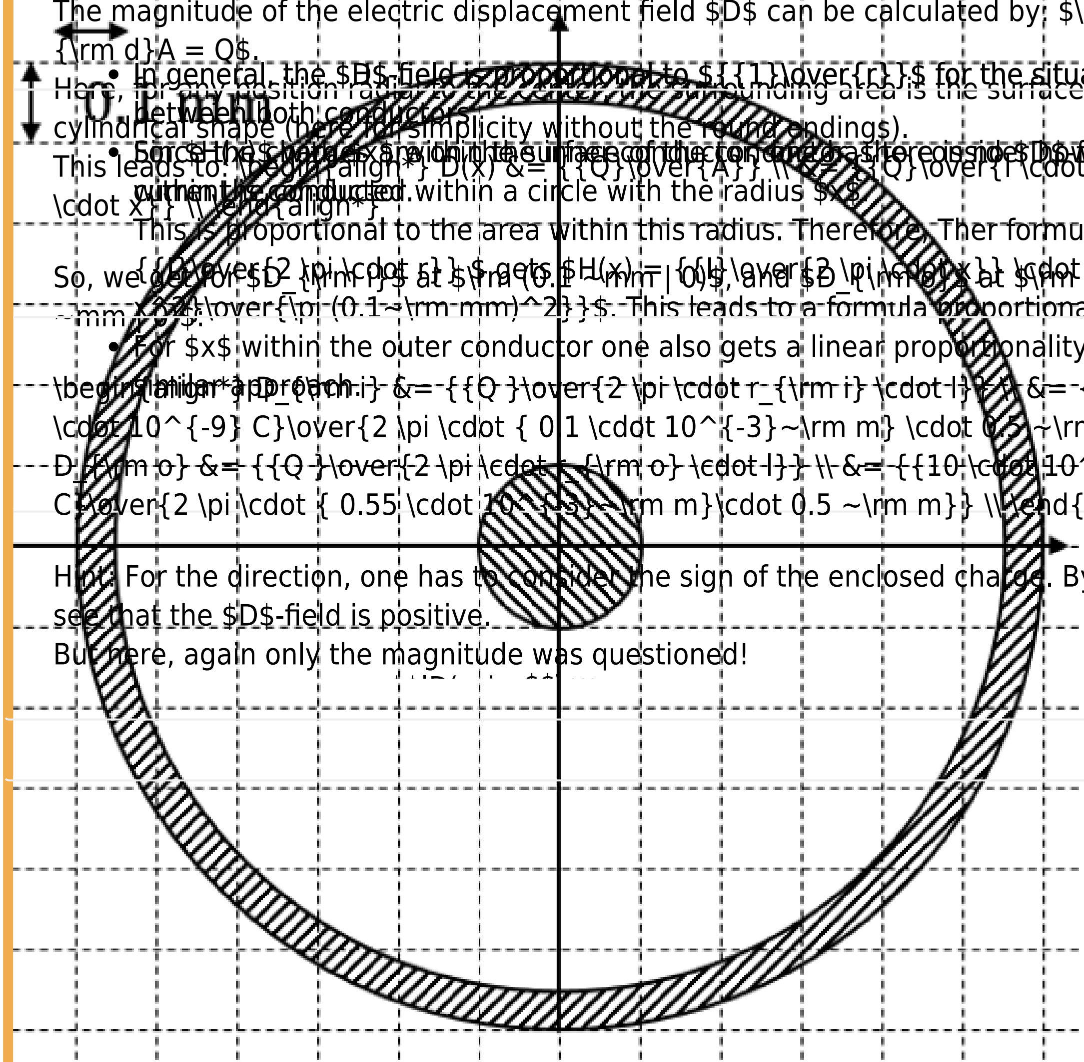
The magnitude of the electric displacement field D can be calculated by: $\int D {\rm d}A = Q$.
Here, for any position radial to the center, the surrounding area is the surface of a cylindrical shape (here for simplicity without the round endings).
This leads to: \begin{align*} D(x) &= {{Q}\over{A}} \\ &= {{Q}\over{l \cdot 2\pi \cdot x}} \\ \end{align*}

So, we get for $D_{\rm i}$ at $\rm (0.1 ~mm | 0)$, and $D_{\rm o}$ at $\rm (0.55 ~mm | 0)$:

\begin{align*} D_{\rm i} &= {{Q }\over{2 \pi \cdot r_{\rm i} \cdot l}} \\ &= {{10 \cdot 10^{-9} C}\over{2 \pi \cdot { 0.1 \cdot 10^{-3}~\rm m} \cdot 0.5 ~\rm m}} \\ D_{\rm o} &= {{Q }\over{2 \pi \cdot r_{\rm o} \cdot l}} \\ &= {{10 \cdot 10^{-9} C}\over{2 \pi \cdot { 0.55 \cdot 10^{-3}~\rm m}\cdot 0.5 ~\rm m}} \\ \end{align*}

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the D-field is positive.
But here, again only the magnitude was questioned!

- for $(0.1 ~\rm mm | 0)$: $H_{\rm i} = 5.25... ~\rm A/m$
- for $(0.55 ~\rm mm| 0)$: $H_{\rm o} = 0.955... ~\rm A/m$
- In general, the H-field is proportional to ${{1}\over{r}}$ for the situation between both conductors.
- For x values within the surface of the conductor, there is not the whole current conducted within a circle with the radius x.
  This is proportional to the area within this radius. Therefore, Ther formula $H = {{I}\over{2 \pi \cdot r}} $ gets $H(x) = {{I}\over{2 \pi \cdot x}} \cdot {{\pi x^2}\over{\pi (0.1~\rm mm)^2}}$. This leads to a formula proportional to x.
- For x within the outer conductor one also gets a linear proportionality with a similar approach.



1. What is the magnitude of the magnetic field strength H at $\rm (0.1 ~mm | 0)$ and $\rm (0.55 ~mm | 0)$?

The magnitude of the magnetic field strength H can be calculated by: $H = {{I}\over{2 \pi \cdot r}} $
So, we get for $H_{\rm i}$ at $\rm (0.1 ~mm | 0)$, and $H_{\rm o}$ at $\rm (0.55

~mm | 0)$:

\begin{align*} H_{\rm i} &= {{I}\over{2 \pi \cdot r_{\rm i}}} \\ &= {{+3.3 A}\over{2 \pi \cdot { 0.1 \cdot 10^{-3}~\rm m}}} \\ H_{\rm o} &= {{I}\over{2 \pi \cdot r_{\rm o}}} \\ &= {{+3.3 A}\over{2 \pi \cdot { 0.55 \cdot 10^{-3}~\rm m}}} \\ \end{align*}

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the H-field on the right side points downwards.
Therefore, the sign of the H-field is negative.
But here, only the magnitude was questioned!

Exercise E1 Lorentz Force
(written test, approx. 8 % of a 120-minute written test, SS2024)

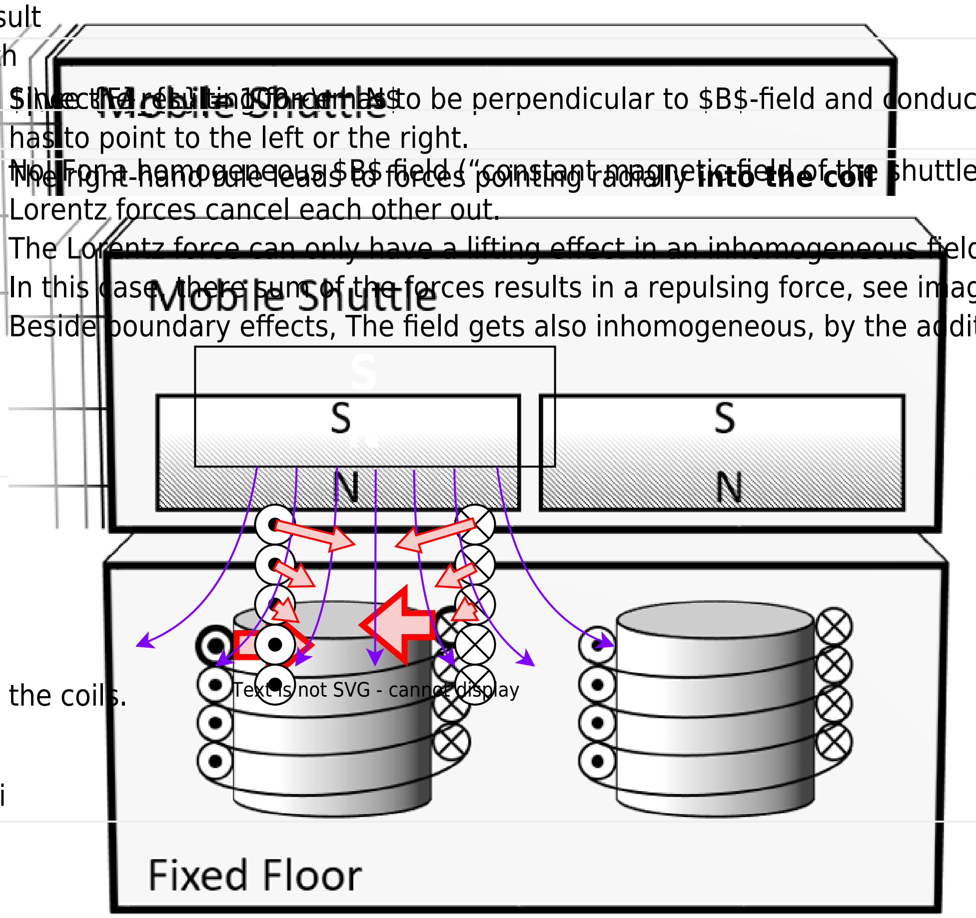
[illegible] (see image).

Result

Path

$\vec{F}_{\rm L}$ [illegible] to be perpendicular to B-field and conductor, the force has to point to the left or the right.
The right-hand rule leads to forces pointing radially **into the coil**

No! For a homogeneous B field ("constant magnetic field of the shuttle"), the Lorentz forces cancel each other out.
The Lorentz force can only have a lifting effect in an inhomogeneous field.
In this case, there sum of the forces results in a repulsing force, see image.
Beside boundary effects, The field gets also inhomogeneous, by the additional field of the coils.



A si

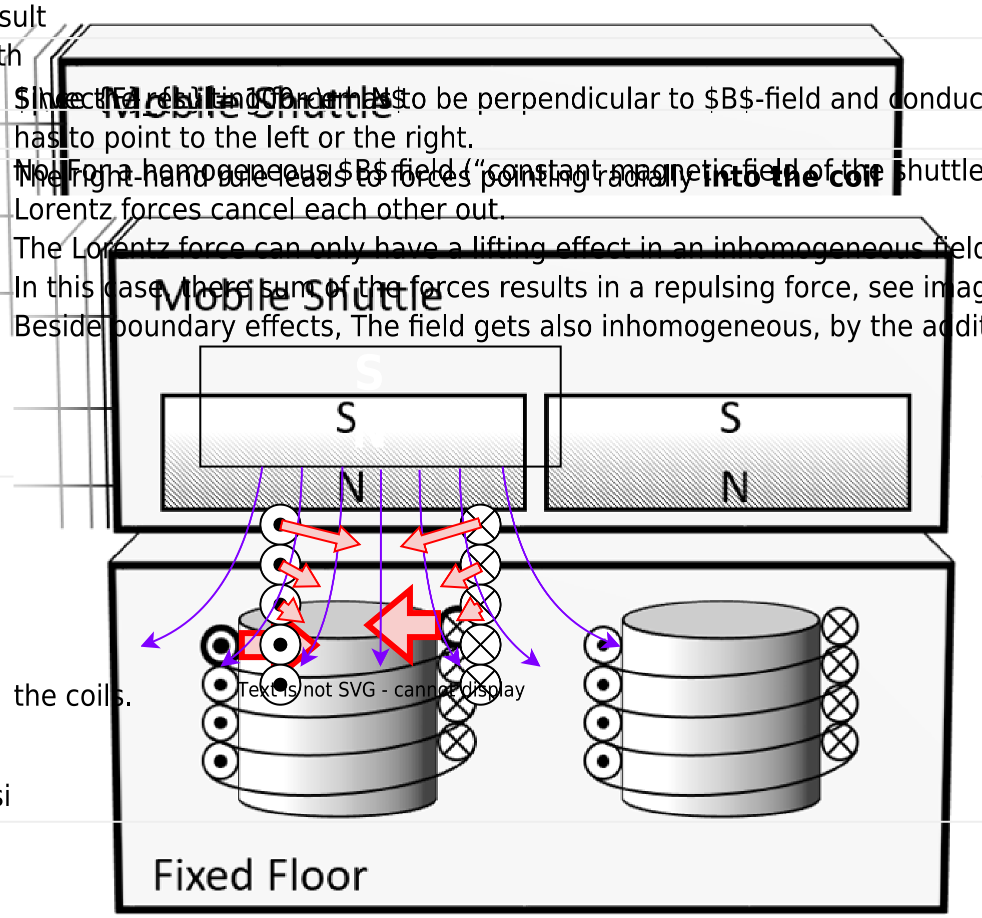
- current $I = 1.6 ~\rm A$
- magnetic field of the shuttle of is homogeneous with $B=0.5 ~\rm T$

1. Calculate the magnitude of the resulting force on one coil!

Path

The Lorentz force on a conductor the length l and the current I in a B-field is \begin{align*} |\vec{F}_{L}| &= I \cdot l \cdot B \cdot \cos(\angle \vec{B}, \vec{l}) \\ &= I \cdot (N \cdot 2\pi r) \cdot B \cdot \cos(\angle \vec{B}, \vec{l}) \\ &= \rm 1.6 ~A \cdot (500 \cdot 2\pi 40\cdot 10^{-3}~m) \cdot 0.5 ~T \cdot \cos 90° \end{align*}

Exercise E2 Lorentz Force
(written test, approx. 8 % of a 120-minute written test, SS2024)



Result

Path

Since the resulting force has to be perpendicular to B-field and conductor, the force has to point to the left or the right.
For a homogeneous B field ("constant magnetic field of the shuttle"), the Lorentz forces cancel each other out.
The Lorentz force can only have a lifting effect in an inhomogeneous field.
In this case, there sum of the forces results in a repulsing force, see image.
Beside boundary effects, The field gets also inhomogeneous, by the additional field of the coils.

- current $I = 1.6 ~\rm A$
- magnetic field of the shuttle of is homogeneous with $B=0.5 ~\rm T$

1. Calculate the magnitude of the resulting force on one coil!

Path

> The Lorentz force on a conductor the length l and the current I in a B-field is \begin{align*} |\vec{F}_{L}| &= I \cdot l \cdot B \cdot \cos(\angle \vec{B}, \vec{l}) \\ &= I \cdot (N \cdot 2\pi r) \cdot B \cdot \cos(\angle \vec{B}, \vec{l}) \\ &= \rm 1.6 ~A \cdot (500 \cdot 2\pi 40\cdot 10^{-3}~m) \cdot 0.5 ~T \cdot \cos 90° \end{align*}

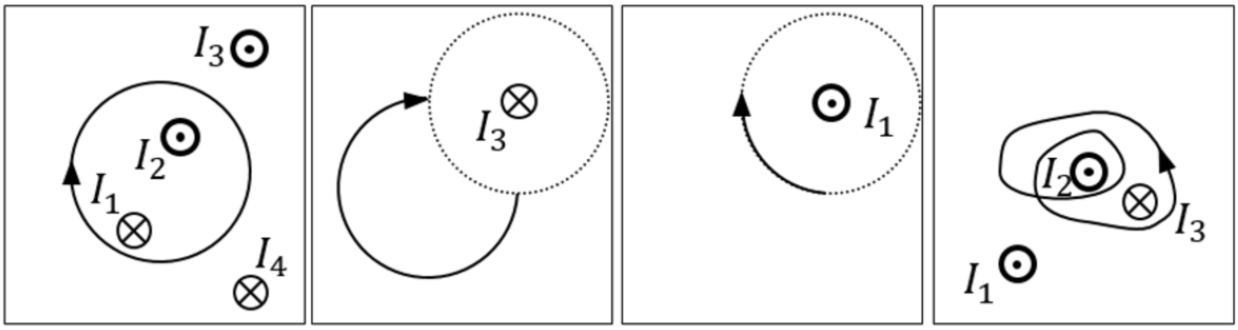
Exercise E1 Magnetic Potential
(written test, approx. 8 % of a 120-minute written test, SS2024)

Calculate the magnetic potential difference $V_\rm m$ for the following paths as shown by the solid lines.
Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1|=2 ~\rm A$
- $|I_2|=5 ~\rm A$
- $|I_3|=11 ~\rm A$
- $|I_4|=7 ~\rm A$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



1.   2.   3.   4.

Result

> Based on the right-hand rule and the part of a full revolution the following results:
>
> 1. Task: $+I_1 - I_2 = -3 ~\rm A$
>
> 2. Task: $+{{1}\over{4}} I3 = 11/4 ~\rm A$ (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)

3. Task: $-{{1}\over{4}} I1 = -0.5 ~\rm A$

4. Task: $+2 \cdot I_2 - 1 \cdot I_3 = -1 ~\rm A$

Exercise E2 Magnetic Potential
(written test, approx. 8 % of a 120-minute written test, SS2024)

Calculate the magnetic potential difference $V_\rm m$ for the following paths as shown by the solid lines.
Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1|=2 ~\rm A$
- $|I_2|=5 ~\rm A$
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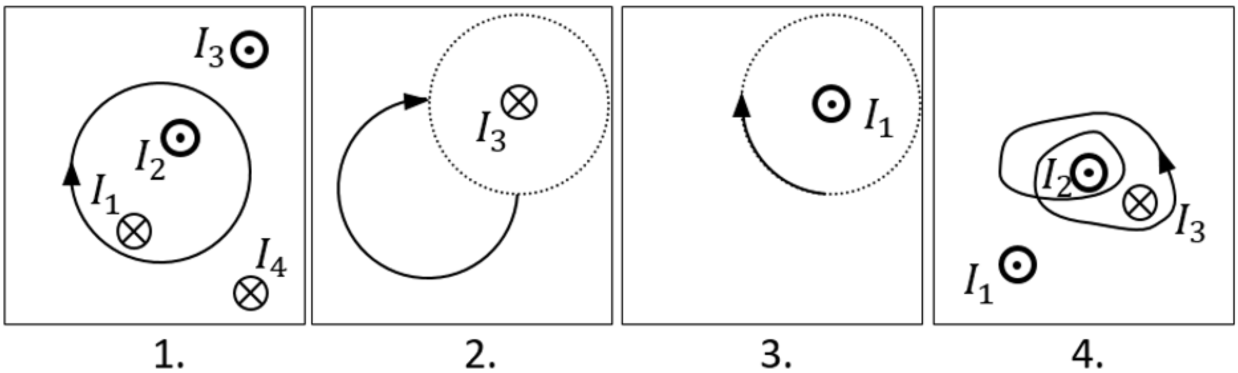
Pay attention to the signs of the currents (given by the diagrams) and of the results!



Result

Based on the right-hand rule and the part of a full revolution the following results:

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3. Task: $-{{1}\over{4}} I1 = -0.5 ~\rm A$

4. Task: $+2 \cdot I_2 - 1 \cdot I_3 = -1 ~\rm A$

Exercise E1 Self-Induction
(written test, approx. 8 % of a 120-minute written test, SS2024)

A coil with a length of $30 ~\rm cm$ and a radius of $2 ~\rm cm$ has 500 turns.
The current through the coil changes linearly from 0 to $3 ~\rm A$ in $0.02 ~\rm ms$.
The arrangement is located in air ($\mu_{\rm r}=1$).

2. Determine the induced voltage in the coil during the change in current.

Result

Path

$\mu_0= 4\pi \cdot 10^{-7} ~\rm Vs/Am$

$ L = 1.32 \rm mH$ $ u_{\rm ind} = -197 ~\rm V$

1. Calculate the (self-)inductance of the coil.

Path

For the linear change of the current the formula of the induced voltage can also be linearized: \begin{align*} u_{\rm ind} &= - L \cdot {{ {\rm d} i }\over{ {\rm d} t }} \\ &\rightarrow - L \cdot {{ {\Delta} i }\over{ {\Delta} t }} \\ &= - 1.32 \cdot 10^{-3} \cdot {{3 A}\over{0.02 \cdot 10^{-3} s}} \end{align*}

The formula for the induction of a long coil is: \begin{align*} L &= \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{A}\over{l}} \\ &= 4\pi \cdot 10^{-7} {~\rm Vs/Am} \cdot (500)^2 \cdot {{\pi \cdot (2\cdot 10^{-2} ~\rm m)^2}\over{ 2 \cdot 10^{-2} ~\rm m}} \\ \end{align*}

Exercise E1 Self-Induction
(written test, approx. 8 % of a 120-minute written test, SS2024)

A coil with a length of $30 ~\rm cm$ and a radius of $2 ~\rm cm$ has 500 turns.
The current through the coil changes linearly from 0 to $3 ~\rm A$ in $0.02 ~\rm ms$.
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Result

Path

$\mu_0= 4\pi \cdot 10^{-7} ~\rm Vs/Am$

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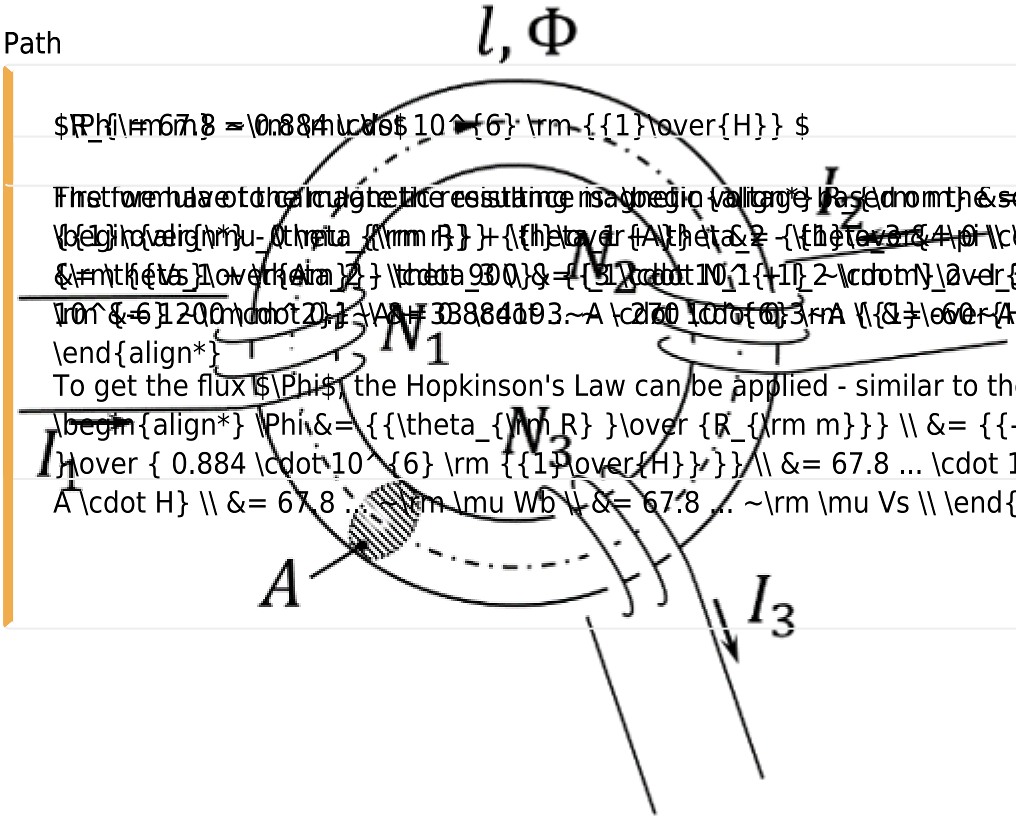
Exercise E3 Magnetic Circuit
(written test, approx. 9 % of a 120-minute written test, SS2024)

A toroidal core (ferrite, $\mu_{\rm r}=900$) has a cross-sectional area of $A=300 ~\rm mm^2$ and an average circumference of $l=3 ~\rm dm$.

2. Calculate the resulting magnetic flux in the core.

Result

Path



$R_{\rm m} = 0.884 \cdot 10^{6} \rm {{1}\over{H}} $

$\Phi \rm = 67.8 ~\rm \mu Vs$

The formula for the magnetic resistance is \begin{align*} R_{\rm m} &= {{1}\over{\mu_0 \mu_{\rm r}}} \cdot {{l}\over{A}} \\ &= {{1}\over{4\pi \cdot 10^{-7} ~\rm Vs/Am \cdot 900}} \cdot {{3 \cdot 10^{-1} ~\rm m}\over{300 \cdot 10^{-6} ~\rm m^2}} \\ &= 0.88419... \cdot 10^{6} ~\rm {{1}\over{H}} \end{align*}

First we have to calculate the resulting magnetic voltage based on the sources: \begin{align*} \theta_{\rm R} &= \theta_1 - \theta_2 - \theta_3 \\ &= N_1 \cdot I_1 - N_2 \cdot I_2 - N_3 \cdot I_3 \\ &= 1200 \cdot 0.1 ~\rm A - 33 \cdot 3 ~\rm A - 270 \cdot 0.3 ~\rm A \\ &= -60 ~\rm A \end{align*}
To get the flux Φ, the Hopkinson's Law can be applied - similar to the Ohm's Law: \begin{align*} \Phi &= {{\theta_{\rm R} }\over {R_{\rm m}}} \\ &= {{-60~\rm A }\over { 0.884 \cdot 10^{6} \rm {{1}\over{H}} }} \\ &= 67.8 ... \cdot 10^{-6} { \rm A \cdot H} \\ &= 67.8 ... ~\rm \mu Wb \\ &= 67.8 ... ~\rm \mu Vs \\ \end{align*}

On the core, there are three coils with:

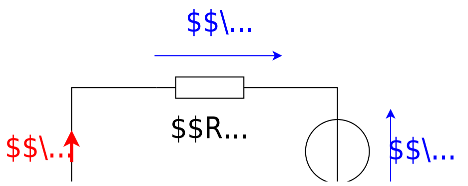
- Coil 1: $N_1 = 1200$, $I_1=100 ~\rm mA$
- Coil 2: $N_2 = 33 $, $I_2= 3 ~\rm A$
- Coil 3: $N_3 = 270 $, $I_3=0.3 ~\rm A$

Refer to the drawing for the direction of the windings, current, and flux!

1. Draw the equivalent magnetic circuit that fully represents the setup.
Name all the necessary magnetic resistances, fluxes, and voltages.

Result

- Since the material, and diameter of the core is constant, one can directly simplify the magnetic resistor into a single $R_\rm m$.
- For the orientation of the magnetic voltages θ_1, θ_2, and θ_3, the orientation of the coils and the direction of the current has to be taken into account by the right-hand rule.
- There is only one flux Φ
- The magnetic voltages are antiparallel to the flux for sources and parallel for the load.



Exercise E4 Magnetic Circuit
(written test, approx. 9 % of a 120-minute written test, SS2024)

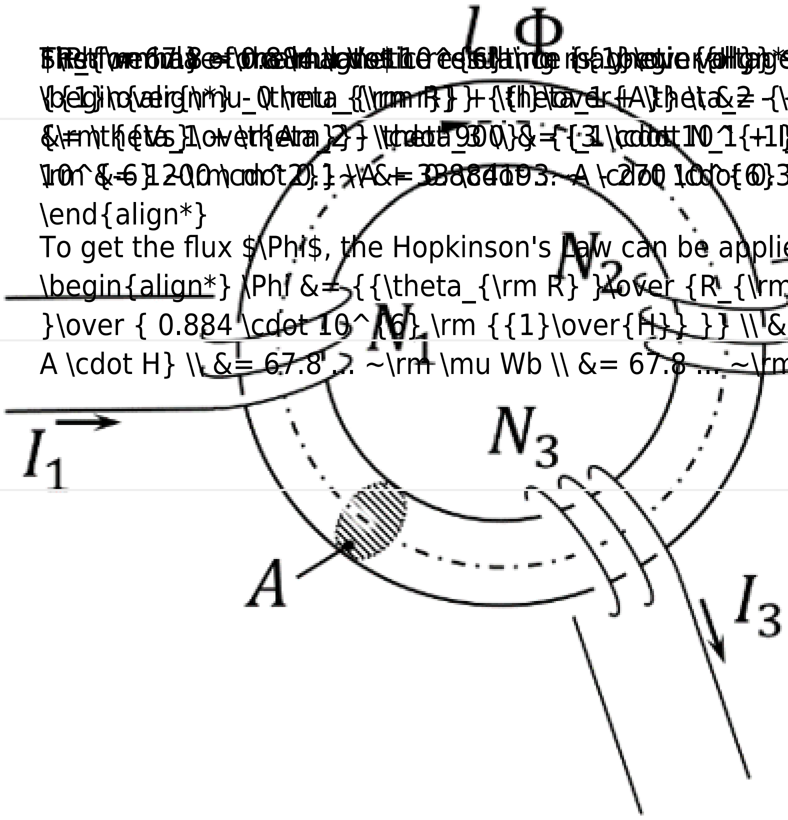
A toroidal core (ferrite, $\mu_{\rm r}=900$) has a cross-sectional area of $A=300 ~\rm mm^2$ and an average circumference of $l=3 ~\rm dm$.

1. Calculate the magnetic resistance of the core.

Result

Path

[illegible]
\end{align*}
To get the flux Φ, the Hopkinson's Law can be applied - similar to the Ohm's Law: \begin{align*} \Phi &= {{\theta_{\rm R} }\over {R_{\rm m}}} \\ &= {{-60~\rm A }\over { 0.884 \cdot 10^{6} \rm {{1}\over{H}} }} \\ &= 67.8 ... \cdot 10^{-6} { \rm A \cdot H} \\ &= 67.8 ... ~\rm \mu Wb \\ &= 67.8 ... ~\rm \mu Vs \\ \end{align*}



On the core, there are three coils with:

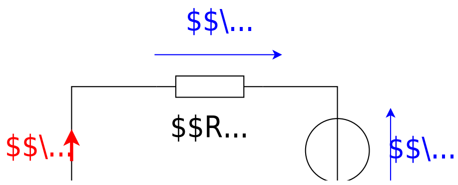
- Coil 1: $N_1 = 1200$, $I_1=100 ~\rm mA$
- Coil 2: $N_2 = 33 $, $I_2= 3 ~\rm A$
- Coil 3: $N_3 = 270 $, $I_3=0.3 ~\rm A$

Refer to the drawing for the direction of the windings, current, and flux!

1. Draw the equivalent magnetic circuit that fully represents the setup.
Name all the necessary magnetic resistances, fluxes, and voltages.

Result

- Since the material, and diameter of the core is constant, one can directly simplify the magnetic resistor into a single $R_\rm m$.
- For the orientation of the magnetic voltages θ_1, θ_2, and θ_3, the orientation of the coils and the direction of the current has to be taken into account by the right-hand rule.
- There is only one flux Φ
- The magnetic voltages are antiparallel to the flux for sources and parallel for the load.



Exercise E1 Magnetic Circuit
(written test, approx. 10 % of a 120-minute written test, SS2024)

... in the shown circuit?

Result

Path

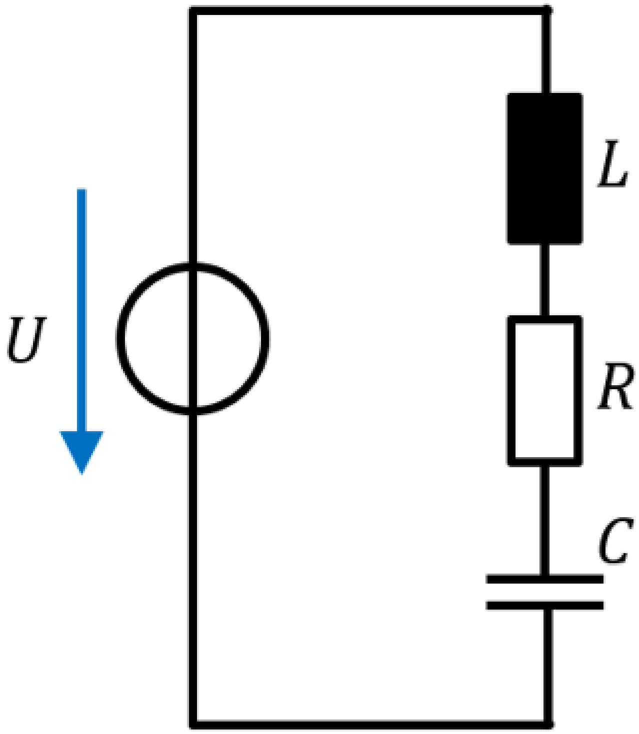
$U_C = 100 ~\rm V$

- $f_r = 39.79 ~\rm MHz$ / $|\underline{Z}_{RLC}| = 83.0 ~\rm m\Omega$
- $\varphi = 0°$ / $|\underline{Z}_{RLC}(f_r)| = 20.0 ~\rm m\Omega$

The formula for the resonance frequency f_r is: $$f_r = {{1}\over{2\pi \sqrt{LC}}} = {{1}\over{2\pi \sqrt{1.6 \cdot 10^{-9} {~\rm H} \cdot 10 \cdot 10^{-9} {~\rm F}}}} = 39.788... ~\rm MHz$$

The voltage on the ideal capacity is the input voltage by the Q-factor increased. $$U_C = U_{\rm s} \cdot Q = U_{\rm s} \cdot \sqrt{{L}\over{C}} \cdot {{1}\over{R}} = 5 {~\rm V} \cdot \sqrt{{1.6 \cdot 10^{-9} {~\rm H}}\over{10 \cdot 10^{-9} {~\rm F}}} \cdot {{1}\over{0.020~\Omega}} = 100 ~\rm V$$

The impedance at resonance is purely the resistance.



A given capacitor shall have the following values:

- $C=10 ~\rm nF$
- $R=20 ~\rm m\Omega$
- $L=1.6 ~\rm nH$

1. What is the impedance Z_{RLC} of this real capacitor for $f_0=44 ~\rm MHz$? (Phase and magnitude)

Path

The impedance is based on the resistance R and the reactance $X_{LC}= {\rm j}\cdot (X_L - X_C)$: \begin{align*} \underline{Z}_{RLC} &= R + {\rm j}\cdot (X_L - X_C) \\ &= R + {\rm j}\cdot (\omega L - {{1}\over{\omega C}}) \\ &= R + {\rm j}\cdot (2\pi f \cdot L - {{1}\over{2\pi f \cdot C}}) \\ \end{align*}

The reactive part is \begin{align*} X_{LC} &= 2\pi f \cdot L - {{1}\over{2\pi f \cdot C}} \\ &= 2\pi 44 \cdot 10^{6} {~\rm MHz} \cdot 1.6 \cdot 10^{-9} {~\rm H} - {{1}\over{2\pi \cdot 10^{6} {~\rm MHz} \cdot 10 \cdot 10^{-9} {~\rm F}}} \\ &= +0.08062... ~\Omega \\ \end{align*}

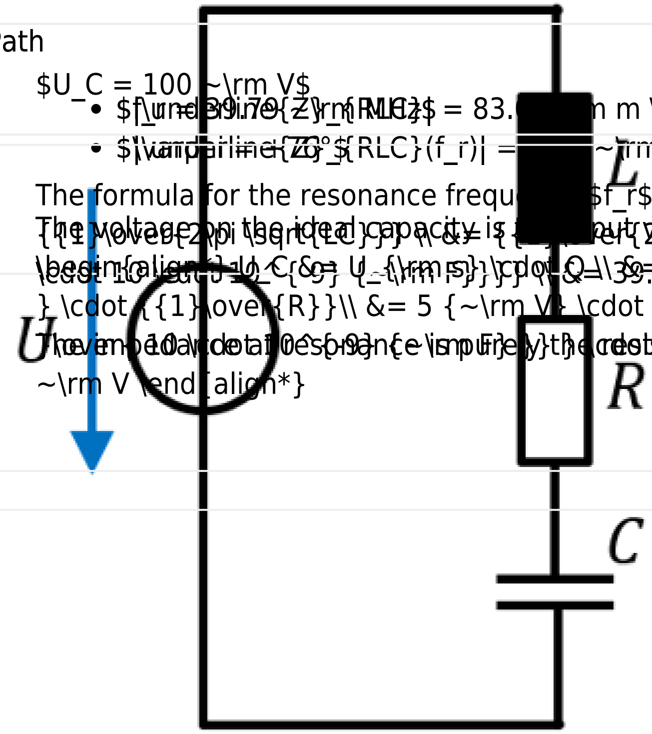
To get the magnitude of the impedance $|\underline{Z}_{RLC}|$ one can use the Pythagorean Theorem: \begin{align*} |\underline{Z}_{RLC}| &= \sqrt{R^2 + X_{LC}^2} \\ &= \sqrt{(0.020~\Omega)^2 + (0.08062... ~\Omega)^2} \\ &= 0.0830 ... ~\Omega \\ \end{align*}

For the phase φ the $\arctan$ can be applied: \begin{align*} \varphi &= \arctan \left({{X_{LC}}\over{R}} \right) \\ &= \arctan \left({{0.08062...

~\Omega}\over{0.020 ~\Omega}} \right) \\ &= 1.3276 ... \hat{=} +76° \\
\end{align*}

Exercise E5 Magnetic Circuit
(written test, approx. 10 % of a 120-minute written test, SS2024)

... in the shown circuit?



Path

$U_C = 100 ~\rm V$

- $f_r = 39.79 ~\rm MHz$ = 83.0 ~\rm m \Omega$
- $\varphi = 76°$ ... |Z_{RLC}(f_r)| = ~\rm m \Omega$

The formula for the resonance frequency f_r is: \begin{align*} f_r &= {{1}\over{2\pi \sqrt{LC}}} \\ &= {{1}\over{2\pi \sqrt{1.6 \cdot 10^{-9} {~\rm H} \cdot 10 \cdot 10^{-9} {~\rm F}}}} \\ &= 39.788... ~\rm MHz \\ \end{align*}

The voltage on the ideal capacity is the input voltage by the Q-factor increased. \begin{align*} U_C &= U_{\rm s} \cdot Q \\ &= U_{\rm s} \cdot \sqrt{ {{L}\over{C}} } \cdot {{1}\over{R}}\\ &= 5 {~\rm V} \cdot \sqrt{ {{ 1.6 \cdot 10^{-9} {~\rm H} \over 10 \cdot 10^{-9} {~\rm F}} } \cdot {{1}\over{0.020~\Omega}}\\ &= 100 ~\rm V \end{align*}

The impedance at resonance is purely the resistance.

A given capacitor shall have the following values:

- $C=10 ~\rm nF$
- $R=20 ~\rm m\Omega$
- $L=1.6 ~\rm nH$

1. What is the impedance Z_{RLC} of this real capacitor for $f_0=44 ~\rm MHz$? (Phase and magnitude)

Path

The impedance is based on the resistance R and the reactance $X_{LC}= {\rm j}\cdot (X_L - X_C)$: \begin{align*} \underline{Z}_{RLC} &= R + {\rm j}\cdot (X_L - X_C) \\ &= R + {\rm j}\cdot (\omega L - {{1}\over{\omega C}}) \\ &= R + {\rm j}\cdot (2\pi f \cdot L - {{1}\over{2\pi f \cdot C}}) \\ \end{align*}

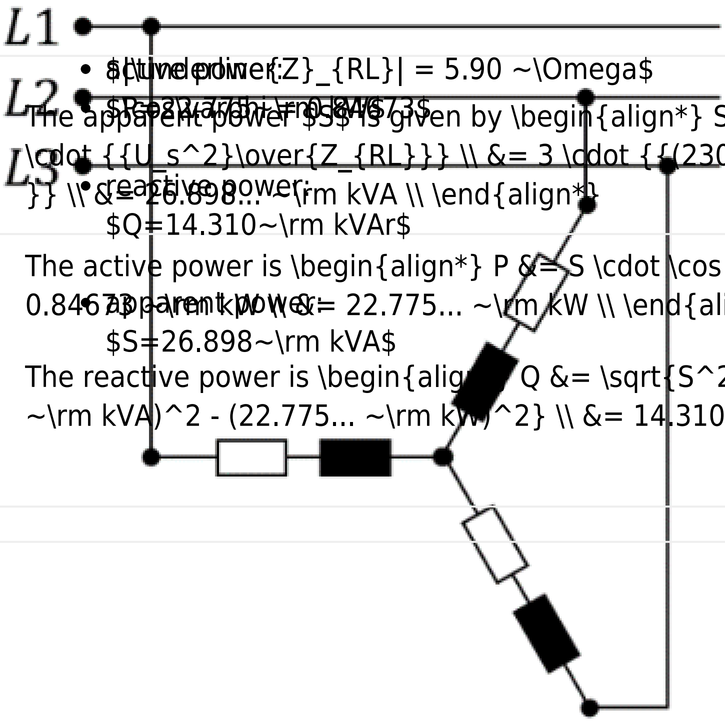
The reactive part is \begin{align*} X_{LC} &= 2\pi f \cdot L - {{1}\over{2\pi f \cdot C}} \\ &= 2\pi 44 \cdot 10^{6} {~\rm MHz} \cdot 1.6 \cdot 10^{-9} {~\rm H} - {{1}\over{2\pi \cdot 10^{6} {~\rm MHz} \cdot 10 \cdot 10^{-9} {~\rm F}}} \\ &= +0.08062... ~\Omega \\ \end{align*}

To get the magnitude of the impedance $|\underline{Z}_{RLC}|$ one can use the Pythagorean Theorem: \begin{align*} |\underline{Z}_{RLC}| &= \sqrt{R^2 + X_{LC}^2} \\ &= \sqrt{(0.020~\Omega)^2 + (0.08062... ~\Omega)^2} \\ &= 0.0830 ... ~\Omega \\ \end{align*}

For the phase φ the $\arctan$ can be applied: \begin{align*} \varphi &= \arctan \left({{X_{LC}}\over{R}} \right) \\ &= \arctan \left({{0.08062... ~\Omega}\over{0.020 ~\Omega}} \right) \\ &= 1.3276 ... \hat{=} +76° \\ \end{align*}

Exercise E1 Magnetic Circuit
(written test, approx. 10 % of a 120-minute written test, SS2024)

A symmetric and balanced three-phase motor is connected to a $230 ~\rm V$ / $400 ~\rm V$ / $50 ~\rm Hz$ three-phase power net. Each single string has a resistor $R=5 ~\Omega$ and an inductance of $L=10 ~\rm mH$.



2. Calculate the true power, the apparent power, and the reactive power of the motor.

Result

- active power: $P=22.775~\rm kW$
- reactive power: $Q=14.310~\rm kVAr$
- apparent power: $S=26.898~\rm kVA$

Path

- $|\underline{Z}_{RL}| = 5.90 ~\Omega$
- $\cos\varphi = 0.84673$

The apparent power S is given by \begin{align*} S &= 3 \cdot U_s \cdot I_s \\ &= 3 \cdot {{U_s^2}\over{Z_{RL}}} \\ &= 3 \cdot {{(230~\rm V)^2}\over{ 5.90 ~\Omega }} \\ &= 26.898... ~\rm kVA \\ \end{align*}

The active power is \begin{align*} P &= S \cdot \cos \varphi \\ &= 26.898... \cdot 0.84673 ~\rm kW \\ &= 22.775... ~\rm kW \\ \end{align*}

The reactive power is \begin{align*} Q &= \sqrt{S^2 - P^2} \\ &= \sqrt{ (26.898... ~\rm kVA)^2 - (22.775... ~\rm kW)^2} \\ &= 14.310...~\rm kVAr \\ \end{align*}

1. Calculate the $\cos \varphi$, and the magnitude of the impedance $|Z|$ for a single string.

Path

The phase φ is given by: \begin{align*} \varphi &= \arctan \left({{X_{L}}\over{R}} \right) \\ &= \arctan \left({{2\pi \cdot f \cdot L}\over{R}} \right) \\ &= \arctan \left({{2\pi \cdot 50 {~\rm Hz} \cdot 10 \cdot 10^{-3} {~\rm H}}\over{5 ~\Omega}} \right) \\ &= 0.5609 ... \hat{=} +32° \\ \end{align*}

With this, the $\cos \varphi$ becomes \begin{align*} \cos \varphi &= \cos(0.5609 ...) \\ &= 0.84673...\\ \end{align*}

The impedance is given by: \begin{align*} |\underline{Z}_{RL}| &= \sqrt{X_{L}^2 + R^2} \\ &= \sqrt{(2\pi \cdot f \cdot L)^2 + R^2} \\ &= \sqrt{(2\pi \cdot 50 {~\rm Hz} \cdot 10 \cdot 10^{-3} {~\rm H})^2 +(5 ~\Omega)^2} \\ &= 5.905... ~\Omega\\ \end{align*}

Exercise E6 Magnetic Circuit
(written test, approx. 10 % of a 120-minute written test, SS2024)

A symmetric and balanced three-phase motor is driven with a $230 ~\rm V$ / $400 ~\rm V$ / $50 ~\rm Hz$ three-phase power net. Each single string has a resistor $R=5 ~\Omega$ and an inductance of $L=10 ~\rm mH$.

2. Calculate the true power, the apparent power, and the reactive power of the motor.

Result

- $\cos \varphi = 0.84673$
- $|\underline{Z}_{RL}| = 5.90 ~\Omega$

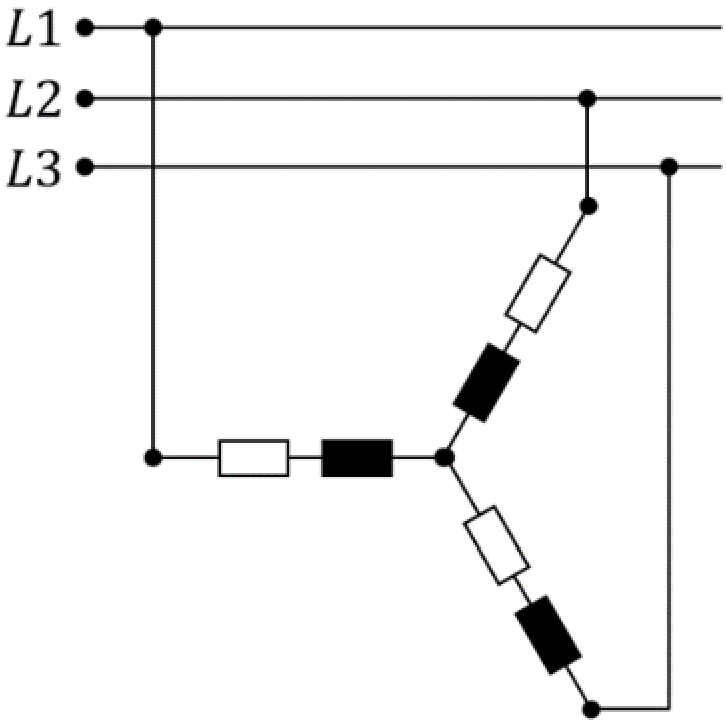
Path

The apparent power S is given by \begin{align*} S &= 3 \cdot U_s \cdot I_s \\ &= 3 \cdot {{U_s^2}\over{Z_{RL}}} \\ &= 3 \cdot {{(230~\rm V)^2}\over{ 5.90 ~\Omega }} \\ &= 26.898... ~\rm kVA \\ \end{align*}

The active power is \begin{align*} P &= S \cdot \cos \varphi \\ &= 26.898... \cdot 0.84673 ~\rm kW \\ &= 22.775... ~\rm kW \\ \end{align*}

The reactive power is \begin{align*} Q &= \sqrt{S^2 - P^2} \\ &= \sqrt{ (26.898... ~\rm kVA)^2 - (22.775... ~\rm kW)^2} \\ &= 14.310...~\rm kVAr \\ \end{align*}

- active power: $P=22.775~\rm kW$
- reactive power: $Q=14.310~\rm kVAr$
- apparent power: $S=26.898~\rm kVA$



1. Calculate the $\cos \varphi$, and the magnitude of the impedance $|Z|$ for a single string.

Path

The phase φ is given by: \begin{align*} \varphi &= \arctan \left({{X_{L}}\over{R}} \right) \\ &= \arctan \left({{2\pi \cdot f \cdot L}\over{R}} \right) \\ &= \arctan \left({{2\pi \cdot 50 {~\rm Hz} \cdot 10 \cdot 10^{-3} {~\rm H}}\over{5 ~\Omega}} \right) \\ &= 0.5609 ... \hat{=} +32° \\ \end{align*}

With this, the $\cos \varphi$ becomes \begin{align*} \cos \varphi &= \cos(0.5609 ...) \\ &= 0.84673...\\ \end{align*}

The impedance is given by: \begin{align*} |\underline{Z}_{RL}| &= \sqrt{X_{L}^2 + R^2} \\ &= \sqrt{(2\pi \cdot f \cdot L)^2 + R^2} \\ &= \sqrt{(2\pi \cdot 50 {~\rm Hz} \cdot 10 \cdot 10^{-3} {~\rm H})^2 +(5 ~\Omega)^2} \\ &= 5.905... ~\Omega\\ \end{align*}

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