task_elndbo3xwi2klxuu_with_calculation

Student Group

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lorentz force, exam ee2 SS2021

Exercise E12 Lorentz Force (hard!)
(written test, approx. 10 % of a 120-minute written test, SS2021)

A $300 ~\rm km$ long, straight high-voltage direct current transmission line shall be analyzed. A current of $I = {1200} \rm A$ flows through it.

b) The picture below shows the top view again. In which of the directions shown does the resulting force act? (Independent)

Result

Horizontal component of the resulting force

A homogeneous geomagnetic field is assumed. The magnetic field strength has a vertical component of $B_{\rm v} = 40 ~\rm \mu T$ and a horizontal component of $B_{\rm h} = 20 ~\rm \mu T$.
The angle between the transmission line and the horizontal component of the field strength is $\alpha = 20°$.
The picture on the right shows the line (black), the field strength components, and the angle in front and top view for illustration purposes.

$F_{\rm h} = 14'600 ~\rm N$

Only this is perpendicular to $\vec{B}_{\rm v}$ and to $\vec{l}$ and points in the right direction by the right-hand rule.

Top View

Path

a) Calculate the force that results from the current flow on the entire conductor.
First, calculate the vertical and horizontal components and combine them accordingly.

Path

- The horizontal component $\vec{F}_{\rm h}$ of the force is based on the vertical component $\vec{B}_{\rm v}$ of the magnetic field.
- The vertical component $\vec{B}_{\rm v}$ of the magnetic field is not shown in the image but is pointing into the ground.
- It has to be perpendicular to $\vec{B}_{\rm v}$ and to $\vec{l}$. The right-hand rule has to be applied.

The force on the transmission line can be calculated via the Lorentz force $\vec{F}_\rm L$: \begin{align*} \vec{F} = I \cdot (\vec{l} \times \vec{B})

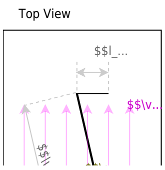
\end{align*}

Here, we have two components for the current - and therefore for the force - to evaluate.
Considering the right-hand rule (and the cross product), the vertical field $B_{\rm v}$ generates a horizontal force $F_{\rm h}$ and vice versa.

The **horizontal component** is given by

\begin{align*} F_{\rm h} &= I \cdot (l \cdot B_{\rm v}) \\ &= 1′200 {~\rm A} \cdot 300 \cdot 10^3{~\rm m} \cdot 40 \cdot 10^{-6}{~\rm {{Vs}\over{m²}}} \\ &= 14'400 ~\rm {{VAs}\over{m}} = 14'400 ~\rm {{Ws}\over{m}} = 14'400 ~\rm N \end{align*}

For the **vertical component** the angle &\alpha& has to be considered.
For the maximum $F_{\rm v}$ the angle &\alpha& has to be $90°$, therefore the $\sin$ has to be used.



\begin{align*} F_{\rm v} &= I \cdot l \cdot B_{\rm h} \cdot \sin\alpha \\ &= 1′200 {~\rm A} \cdot 300 \cdot 10^3{~\rm m} \cdot 40 \cdot 10^{-6}{~\rm {{Vs}\over{m²}}} \cdot \sin 20° \\ &= 2'462.545... ~\rm N \end{align*}

For the **overall force** F the Pythagorean theorem has to be used:

\begin{align*} F &= \sqrt{F_{\rm v}^2 +F_{\rm h}^2} \\ &= \sqrt{({14'400 ~\rm N})^2 +({2'462.545... ~\rm N})^2} \\ &= 14'609.04... ~\rm N \end{align*}