task_tb6pi8dgh0m2e2pw_with_calculation

Student Group

First Name	Surname	Matrikel Nr.

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Charging Capacitors, dc network analysis, pure resistor network simplification, delta wye transformation, exam ee1 WS2022

Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

4. Now, an (without the light bulb) is considered, the switch S_2 is closed and $R_2=12 ~\rm{\Omega}$. The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Result

Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and $R_\rm B$.

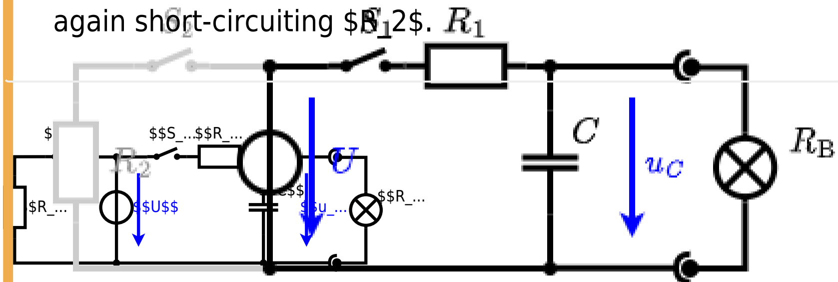
\begin{align*} \Delta t_{12} = 3.792 ~\rm{ms} \end{align*}

Solution

Under no change anything: The ideal voltage source is $U=12 ~\rm{V}$ independent of this resistor.

\begin{align*} \Delta W_C &= {{1}\over{2}} \cdot C \cdot U^2 \\ &= {{1}\over{2}}\cdot 100 ~\rm{µF} \cdot (12 ~\rm{V})^2 \end{align*}

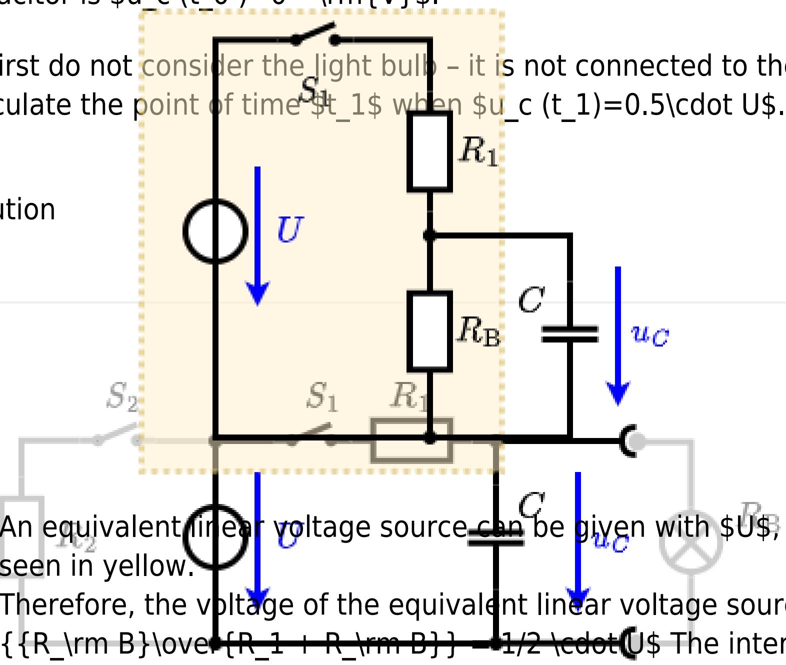
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2.



The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution



An equivalent linear voltage source can be given with U, R_1, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit).

So, here only R_1 and C gives the time constant $\tau = R_1 \cdot C$
\begin{align*} R_1 &= R_1 || R_1 \\ &= 10 ~\Omega \end{align*}

The following formula describes the time course of $U_C(t)$, which has to be set to $U_C (t_1) = 0.5 \cdot U_0$: \begin{align*} U_C (t) = U_0 \cdot (1 - e^{-t/\tau}) \end{align*}
\begin{align*} U_C(t_2) &= U_0 \cdot (1 - e^{t_2/(R_1 \cdot C)}) \\ &= {1 \over 2} \cdot U_0 \\ (1 - e^{t_2 / (10 ~\Omega \cdot 100 ~\mu F)}) &= 0.5 \end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}