Block 16 - Ampère's Law and Magnetomotive Force (MMF)

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Block 16 - Ampère's Law and Magnetomotive Force (MMF)

Learning objectives

> After this 90-minute block, you can
>
> - …

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

> 1. …

Core content

Generalization of the Magnetic Field Strength

So far, only the rotational symmetric problem on a single wire was considered in formula, when the current I ant the length s of a magnetic field line around it is given:

\begin{align*} \quad H_\varphi ={I\over{s}} = {{I}\over{2 \cdot \pi \cdot r}} \quad \Leftrightarrow

\quad I = H_\varphi \cdot {s} \quad \quad \quad | \quad \text{applies only to the long, straight conductor} \end{align*}

Now, this shall be generalized. For this purpose, we will look back at the electric field.
For the electric field strength E of a capacitor with two plates at a distance of s and the potential difference U holds:

\begin{align*} U = E \cdot s \quad \quad | \quad \text{applies to capacitor only} \end{align*}

In words: The potential difference is given by adding up the field strength along the path of a probe from one plate to the other.
This was extended to the vltage between to points 1 and 2. Additionally, we know by Kirchhoff's voltage law that the voltage on a closed path is “0”.

\begin{align*} U_{12} &= \int_1^2 \vec{E} \; {\rm d}\vec{s} \\ U &= \oint \vec{E} \; {\rm d}\vec{s} =0 \\ \end{align*}

We can now try to look for similarities. Also for the magnetic field, the magnitude of the field strength is summed up along a path to arrive at another field-describing quantity.
Because of the similarity the so-called **magnetic potential difference V_m** between point 1 and 2 is introduced:

\begin{align*} V_m &= H \cdot s \quad \quad | \quad \text{applies to rotational symmetric problems only} \\ \end{align*}

\begin{align*} \boxed{ V_m = V_{m, 12} = \int_1^2 \vec{H} \; {\rm d}\vec{s} \\ V_m = \oint \vec{H} {\rm d}\vec{s} = \theta } \end{align*}

We need to take a loser look here. Any closed path in the static electric field leads to a potential difference of $U = \oint \vec{E} \; {\rm d}\vec{s} =0$.
BUT: closed paths in the static magnetic field leads to a magnetic potential difference which is **not mandatorily** 0! $ V_m = \oint \vec{H} {\rm d}\vec{s} = \theta$

Another new quantity is introduced: the **magnetic voltage θ**:

1. The magnetic voltage θ is the magnetic potential difference on a closed path.
2. Since the magnetic voltage θ is valid for exactly <u>one turn</u> along our single wire, θ is also equal to the current through the wire:
   \begin{align*} \theta = H \cdot s = I \quad \quad | \quad \text{applies only to the long, straight conductor} \end{align*}
3. The magnetic potential difference can take a fraction or a multiple of one turn and is therefore **not mandatorily** equal to I.
4. The magnetic voltage is generalized in the following box.

Notice:

The path integral of the magnetic field strength along an arbitrary closed path is equal to the free currents (= current density) through the surface enclosed by the path.

The magnetic voltage θ (and therefore the current) is the cause of the magnetic field strength.

This leads to the **Ampere's Circuital Law**

\begin{align*} \boxed{\oint_{s} \vec{H} {\rm d} \vec{s} = \theta } \end{align*}	The magnetic voltage θ can be given as • $\theta = I \quad \quad \quad \ $ for a single conductor • $\theta = N \cdot I \quad \:\; \, $ for a coil • $\theta = \sum_n \cdot I_n \quad$ for multiple conductors • $\theta = \iint_A \; \vec{S} {\rm d}\vec{A}$ for any spatial distribution (see block15)

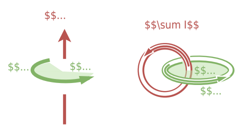
The unit of the magnetic voltage θ is **Ampere** (or **Ampere-turns**).

Notice:

${\rm d}\vec{s}$ and ${\rm d}\vec{A}$ in $\oint_{s} \vec{H} {\rm d} \vec{s} = \theta = \iint_A \; \vec{S} {\rm d}\vec{A}$ build a right-hand system.

1. Once the thumb of the right hand is pointing along ${\rm d}\vec{A}$, the fingers of the right hand show the correct direction for ${\rm d}\vec{s}$ for positive $\vec{H}$ and $\vec{S}$
2. Currents into the direction of the right hand's thumb count positive. Currents antiparallel to it count negative.

Fig. 1: Right hand rule



Common pitfalls

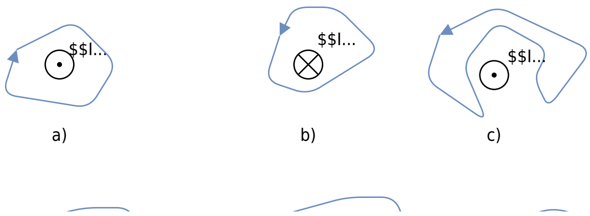
- …

Exercises

Task 3.2.3 Magnetic Potential Difference

Fig. 3: different trajectories around current-carrying conductors
Result a)

a) $V_{\rm m,a} = I_1 - I_2 = 2~\rm A - 4.5~\rm A = - 2.5~\rm A$



Given are the adjacent closed trajectories in the magnetic field of current-carrying conductors (see figure 3). Let $I_1 = 2~\rm A$ and $I_2 = 4.5~\rm A$ be valid.

In each case, the magnetic potential difference $V_{\rm m}$ along the drawn path is sought.

Path

- The magnetic potential difference is given as the **sum of the current through the area within a closed path**.
- The direction of the current and the path have to be considered with the righthand rule.

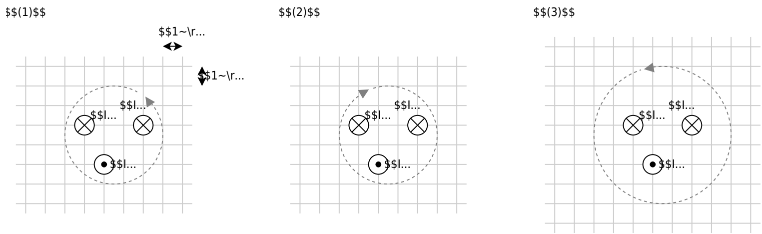
Exercise E1 Magnetic Voltage
(written test, approx. 6 % of a 120-minute written test, SS2021)

The following images show cross-sections of electrical cables.
A closed path is shown as a dashed line. The magnetic voltage θ on these paths shall be analyzed.
The following values are given for the currents:

- $I_1 = 5 {~\rm A}$
- $I_2 = 2 {~\rm A}$
- $I_3 = 1 {~\rm A}$
- $I_4 = 4 {~\rm A}$

Result

$$\begin{align*} \theta_{(1)} &= -4 {~\rm A} \\ \theta_{(2)} &= 0 {~\rm A} \\ \theta_{(3)} &= -5 {~\rm A} \\ \end{align*}$$



Specify which magnetic voltages $\theta_{(1)}$, $\theta_{(2)}$, and $\theta_{(3)}$ result. Note the direction of the path in each case!

Path

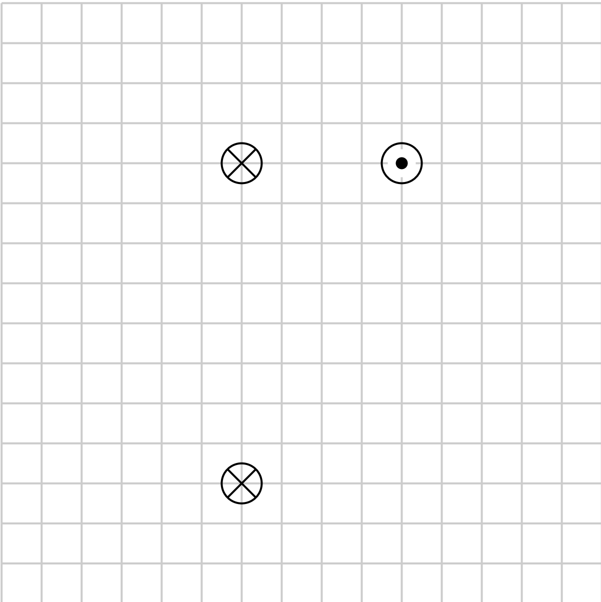
For the resulting current the direction of the path has to be considered with the right-hand rule:

- $I_{(1)} = +I_2 - I_1 - I_3 \quad \rightarrow \quad \theta_{(1)} = 2 {~\rm A} - 5 {~\rm A} - 1 {~\rm A} $
- $I_{(2)} = +I_3 + I_4 - I_1 \quad \rightarrow \quad \theta_{(2)} = 1 {~\rm A} + 4 {~\rm A} - 5 {~\rm A} $
- $I_{(3)} = +I_3 - I_4 - I_2 \quad \rightarrow \quad \theta_{(3)} = 1 {~\rm A} - 4 {~\rm A} - 2 {~\rm A} $

Exercise E2 Magnetic Field Lines

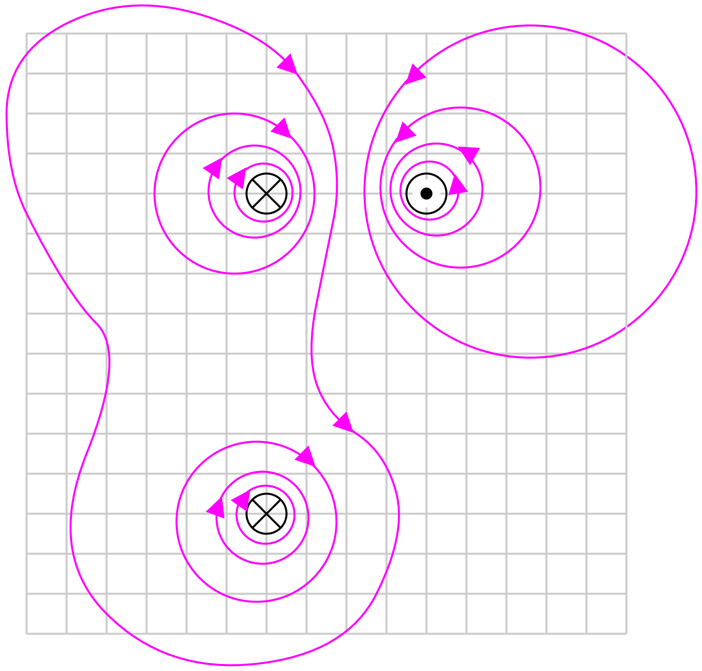
(written test, approx. 4 % of a 120-minute written test, SS2021)

Several parallel conductors are projecting out of the plane.
The same current $|I|$ flows through all the conductors in different directions (see image below).
Sketch at least 10 field lines of the magnetic field strength $\vec{H}$ in such a way that the different properties of the field lines (e.g. direction and density) can be seen.



Result

- high density of field lines near the conductors
- direction of the field lines given by the right-hand rule
- magnetic field has closed field lines
- resulting field given by superposition of field lines



Exercise E1 Magnetic Field Lines
(written test, approx. 6 % of a 120-minute written test, SS2024)

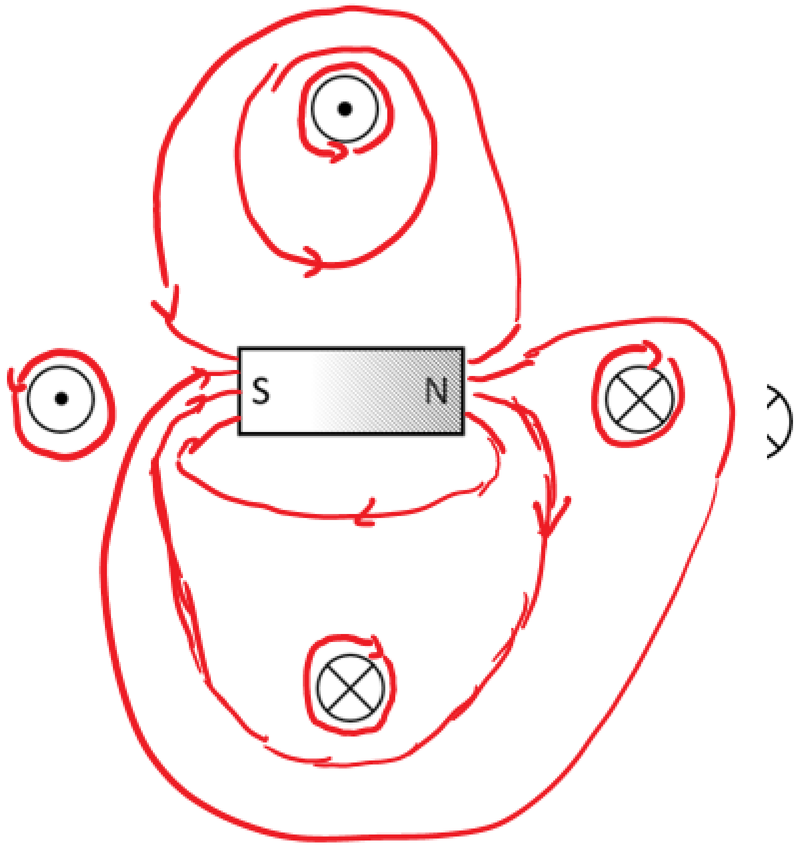
The following setup shall be given:

- Four conductors are located perpendicular to the plane of the diagram
- All of them conduct a current with the same magnitude, but not in the same direction.
- A permanent magnet is located in between the conductors.

2. Discuss how the permanent magnet affects the H-field, based on the fundamental definition of the H-field.

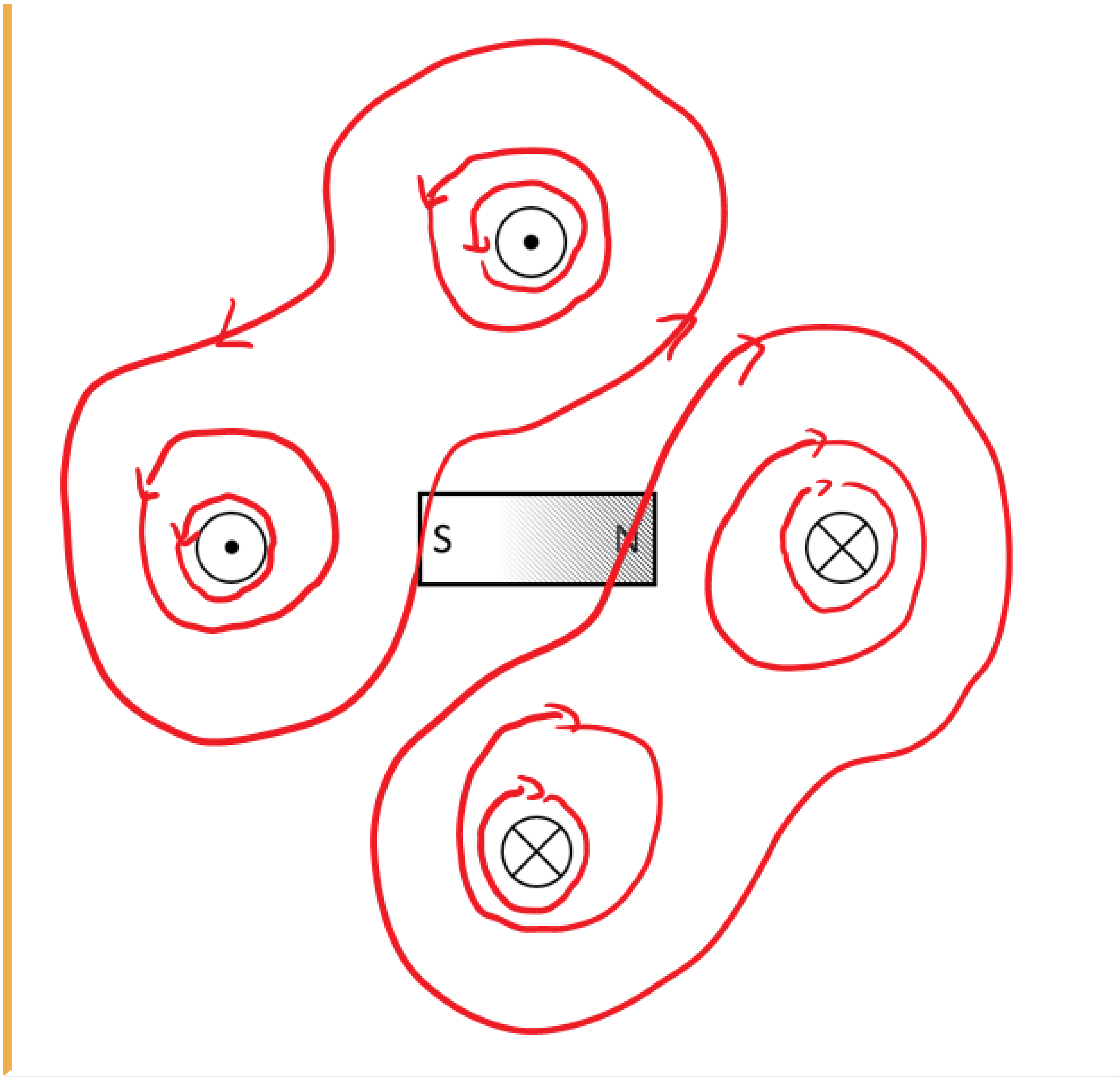
Result

- The H-field is defined by currents $\sum I = \int H {\rm d}s$.
- In the permanent magnet, there are no free currents.
- The bound currents (of the permanent magnet) create also an H field.
- This exits on the north pole and enters the magnet on the south pole (similar to the B-field)_
- $H = B/\mu$
- The H-field from task 1 gets distracted



.. Do not consider the permanent magnet at first. Draw at least 10 field lines of the H-field qualitatively. Give a a correct representation of their direction, and density for the shown area.

Result



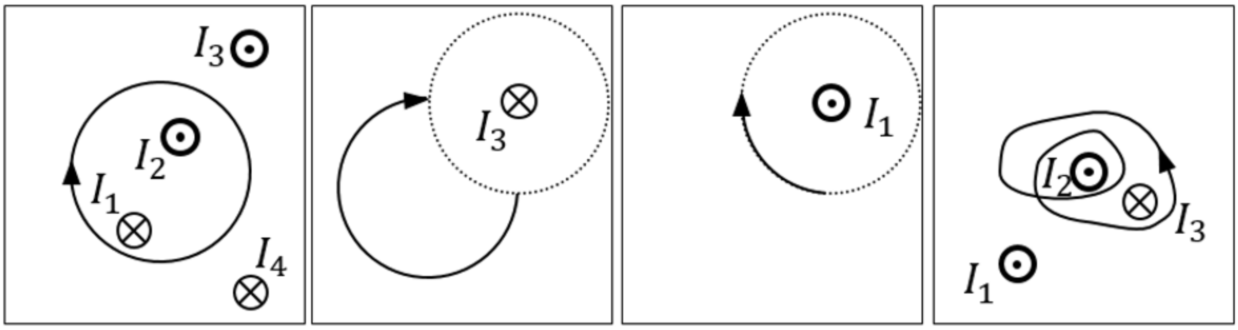
Exercise E2 Magnetic Potential
(written test, approx. 8 % of a 120-minute written test, SS2024)

Calculate the magnetic potential difference $V_\rm m$ for the following paths as shown by the solid lines.
Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1|=2 ~\rm A$
- $|I_2|=5 ~\rm A$
- $|I_3|=11 ~\rm A$
- $|I_4|=7 ~\rm A$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



1. 2. 3. 4.

Result

Based on the right-hand rule and the part of a full revolution the following results:

1. Task: $+I_1 - I_2 = -3 ~\rm A$
2. Task: $+{{1}\over{4}} I3 = 11/4 ~\rm A$ (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)
3. Task: $-{{1}\over{4}} I1 = -0.5 ~\rm A$
4. Task: $+2 \cdot I_2 - 1 \cdot I_3 = -1 ~\rm A$

Embedded resources

Explanation (video): …