

Block 17 — Magnetic Flux Density and Forces

Student Group

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Block 17 — Magnetic Flux Density and Forces

Learning objectives

After this 90-minute block, you can

- ...

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

90-minute plan

1. Warm-up (x min):
 1.
2. Core concepts & derivations (x min):
 1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. ...

Core content

We know from [block11](#) that a static charge Q_1 generate a static electric field D .

Before in [block09](#), we developed that a static electric field $E = \frac{1}{\epsilon_0} D$ effects a force F_C on a static charge Q_2

From the last chapter ([block16](#)) we got, that moving charges $\frac{dQ_1}{dt} = I_1$ generate a static magnetic field H .

So, how does an acting magnetic field effects a force on a moving charge $\frac{dQ_2}{dt} = I_2$?

Definition of the Magnetic Flux Density

To derive the forces, we do a step back to the images of field lines.

In figure 1 a) the field lines of a single current-carrying wire is shown.

figure 1 b) depicts the homogenous field of a coil.

When a current-carrying wire is within the homogenous field, we get the superimposed picture of both fields.

This leads to an enrichment of magnetic field on the left and a depletion on the right.

With the knowledge, that the field lines usually do not like to stay next to each other, one can conclude that there will be a force to the right.

Fig. 1: Force in magnetic field



When no current is flowing through the conductor the force is equal to zero.

The following is detectable:

1. $|\vec{F}| \sim I$: The stronger the current, the stronger the force F .
2. $|\vec{F}| \sim l$: As longer the conductor length l , as stronger the force F gets.
3. $|\vec{F}| \sim H$: As more current through the coil, as stronger the H -field. And a stronger the H -field leads to stronger force F .

To summarize:
$$F \sim H \cdot l \cdot I$$

The proportionality factor is μ_0 , the **magnetic field constant** or **permeability** : $\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$.

$$F = \mu_0 \cdot H \cdot l \cdot I$$

When adding an iron core into the coil the force F gets stronger. Therefore, we include a material-dependent constant μ_r , the so-called relative permeability

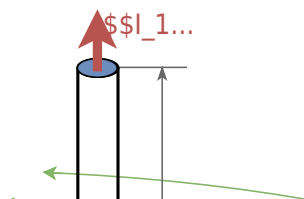
$$\begin{aligned} F &= \mu_0 \mu_r H \cdot I \cdot l \\ \boxed{F_L} &= B \cdot I \cdot l \end{aligned}$$

The new field quantity is B the **magnetic flux density**

To do so, the effect between two parallel conductors has to be examined closer.

The experiment consists of a part of two very long¹⁾ conductors with the different currents I_1 , I_2 in the distance r (see [figure 2](#)).

Fig. 2: Forces between two Conductors



When no current is flowing through the conductors the forces are equal to zero.

Once the currents flow in the same direction (e.g. $I_1 > 0$, $I_2 > 0$) attracting forces $\vec{F}_{12} = -\vec{F}_{21}$ appear.

The force $\vec{F}_{xy}$ shall be the force on the conductor x caused by conductor y . In the following the force $\vec{F}_{12}$ on the conductor 1 will be examined.

The following is detectable:

1. $|\vec{F}_{12}| \sim I_1$, $|\vec{F}_{12}| \sim I_2$: The stronger each current, the stronger the force F_{12} .
2. $|\vec{F}_{12}| \sim l$: As longer the conductor length l , as stronger the force F_{12} gets.
3. $|\vec{F}_{12}| \sim \frac{1}{r}$: A smaller distance r leads to stronger force F_{12} .

To summarize:
$$\vec{F}_{12} \sim I_1 \cdot I_2 \cdot \frac{l}{r}$$

The proportionality factor is arbitrarily chosen as:
$$\frac{\vec{F}_{12} \cdot r}{I_1 \cdot I_2 \cdot l} = \frac{\mu}{2\pi}$$

Here μ is the magnetic permeability and for vacuum ([vacuum permeability](#)):
$$\mu = \mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} = 1.257 \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$$

This leads to the **Ampere's Force Law**:
$$|\vec{F}_{12}| = \frac{\mu}{2\pi} \cdot \frac{I_1 \cdot I_2 \cdot l}{r}$$

Since we want to investigate the effect on the current I_1 , the following rearrangement can be done:
$$|\vec{F}_{12}| \cdot \frac{2\pi}{\mu} = \frac{I_2 \cdot l}{r} \cdot I_1 \quad || \cdot \frac{2\pi}{\mu} = B \cdot I_1 \cdot l$$

The properties of the field from I_2 acting on I_1 are summarized to B - the **magnetic flux density**.

B has the unit:
$$[B] = \frac{[F]}{[I] \cdot [l]} = 1 \frac{\text{N}}{\text{Am}} = 1 \frac{\text{VA}}{\text{m}} = 1 \frac{\text{Vs}}{\text{m}^2} \quad || \quad = 1 \text{ T} \quad (\text{Tesla})$$

This formula can be generalized with the knowledge of the directions of the conducting wire $\vec{l}$, the magnetic field strength $\vec{B}$ and the force $\vec{F}$ using vector multiplication too:

$$\vec{F}_L = l \cdot \vec{l} \times \vec{B}$$

The absolute value can be calculated by

$$|\vec{F}_L| = l \cdot |\vec{l}| \cdot |\vec{B}| \cdot \sin(\angle \vec{l}, \vec{B})$$

The force is often called **Lorentz Force** F_L . For the orientation, another right-hand rule can be applied.

Notice:

Right-hand rule for the Lorentz Force:

- The causing current I is on the thumb. Since the current is not a vector, the direction is

given by the direction of the conductor $\vec{I}$

- The mediating external magnetic field $\vec{B}$ is on the index finger
- The resulting force $\vec{F}$ on the conductor is on the middle finger

This is shown in [figure 2](#).

A way to remember the orientation is the mnemonic **FBI** (from middle finger to thumb):

- $\vec{F}$ force on middle finger
- $\vec{B}$ -Field on index finger
- Current I on thumb (direction with length $\vec{I}$)

To view the animation: [click here!](#)

Fig. 2: Force onto a single Conductor in a B-Field



Lorentz Law and Lorentz Force

The true Lorentz force is not the force on the whole conductor but the single force onto an (elementary) charge.

To find this force the previous force onto a conductor can be used as a start. However, the formula will be investigated infinitesimally for small parts $d\vec{l}$ of the conductor:

$$\vec{F}_{L} = I \cdot d\vec{l} \times \vec{B}$$

The current is now substituted by $I = \frac{dQ}{dt}$, where dQ is the small charge packet in the length $d\vec{l}$ of the conductor.

$$\vec{F}_{L} = \left\{ \frac{dQ}{dt} \right\} \cdot d\vec{l} \times \vec{B}$$

Mathematically not quite correct, but in a physical way true the following rearrangement can be done:

$$\begin{aligned} \vec{F}_{\text{L}} &= \left\{ \frac{dQ}{dt} \cdot \vec{l} \right\} \times \vec{B} \\ \vec{B} &= \frac{dQ}{dt} \cdot \left\{ \frac{\vec{l}}{dt} \right\} \times \vec{B} \\ &= \frac{dQ}{dt} \cdot \left\{ \frac{\vec{l}}{dt} \right\} \times \vec{B} \end{aligned}$$

Here, the part $\left\{ \frac{dQ}{dt} \cdot \vec{l} \right\}$ represents the speed $\vec{v}$ of the small charge packet dQ .

$$\vec{F}_{\text{L}} = dQ \cdot \vec{v} \times \vec{B}$$

The **Lorenz Force** on a finite charge packet is the integration:

$$\boxed{\vec{F}_{\text{L}} = Q \cdot \vec{v} \times \vec{B}}$$

Notice:

- A charge Q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force of $\vec{F}_{\text{L}}$.
- The direction of the force is given by the right-hand rule.

Please have a look at the German contents (text, videos, exercises) on the page of the [KIT-Brückenkurs >> Lorentz-Kraft](#). The last part “Magnetic field within matter” can be skipped.

Common pitfalls

- ...

Exercises

Exercise E6 Cylindrical Coil

(written test, approx. 6 % of a 120-minute written test, SS2021)

A) the magnetic flux (2 points)

Result

- Length $l = 30 \text{ cm}$,

Path • Winding diameter $d = 390 \text{ mm}$,

- Number of windings $N = 240$,

• Current in the inductor $I = 500 \text{ mA}$,

- Material inside: Air

- $\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$

The magnetic field strength is $B = \mu_0 \cdot I \cdot N / l$:

The proportion of the magnetic voltage outside the coil can be neglected. Determine the following for the inside of the coil:

$$\Phi = B \cdot A$$

B) the magnetic field strength (2 points)

a) What is the highest magnetic flux density through the line in your body? (3 points)

Path

The magnetic field strength for a conducting wire is given as:

$$H = \frac{I}{2\pi \cdot r}$$

The magnetic flux density B is given as: $B = \mu_0 \cdot \mu_r \cdot H$

Here, the maximum current is $\hat{I} = 100 \text{ A}$ and the distance to the cable is $r = \sqrt{(0.1 \text{ m})^2 + (0.4 \text{ m})^2} = 0.412... \text{ m}$.

$$\text{Therefore: } B = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1 \cdot \frac{100 \text{ A}}{2\pi \cdot 0.412... \text{ m}}$$

Exercise E4 Toroidal Coil**(written test, approx. 5 % of a 120-minute written test, SS2021)**

A magnetic field with a flux density of at least 50 mT is to be achieved in a ring-shaped coil (toroidal coil).

The coil has 60 turns, wound around soft iron with $\mu_r = 1200$.

The average field line length in the coil should be $l = 12 \text{ cm}$.

Result: $I_{\text{min}} = 4 \text{ A}$



What is the minimum current that must flow through a single winding?

Path

The magnetic field strength of a toroidal coil is given as:

$$H = \frac{N \cdot I}{l}$$

Based on the flux density the magnetic field strength can be derived by $B = \mu_0 \mu_r H$.

By this, the formula can be rearranged:

$$H = \frac{N \cdot I}{l} \quad || \quad \frac{B}{\mu_0 \mu_r} = H \\ \frac{N \cdot I}{l} = \frac{B \cdot l}{\mu_0 \mu_r \cdot N}$$

$$\text{Putting in the numbers: } I = \frac{0.05 \text{ T} \cdot 0.12 \text{ m}}{4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1'200 \cdot 60} = 0.6631... \frac{\text{T} \cdot \text{m}}{\frac{\text{Vs}}{\text{Am}}} = 0.6631... \frac{\text{Vs}}{\text{m}^2} \cdot \text{m} \cdot \frac{\text{Vs}}{\text{Am}}} = 0.6631... \text{ A}$$

Exercise E12 Lorentz Force (hard!)

(written test, approx. 10 % of a 120-minute written test, SS2021)

A) The picture below shows a straight high-voltage line, which of the directions shown makes the Resultant component of $F = (1'200 \text{ N})$ on the Resulting force is? (Independent)

A homogeneous geomagnetic field is assumed. The magnetic field strength has a vertical component of $B_v = 40 \text{ } \mu\text{T}$ and a horizontal component of $B_h = 20 \text{ } \mu\text{T}$.

Only a $1'500 \text{ A}$ is perpendicular to $\vec{B}_v$ and to $\vec{B}_h$ and points in the right direction by the right-hand rule.

The picture on the right shows the line (black), the field strength components, and the angle in front and top view for illustration purposes.

- a) Calculate the force that results from the current flow on the entire conductor.
First, calculate the vertical and horizontal components and combine them accordingly.

Path
Top View

Path

The force on the transmission line can be calculated via the Lorentz force

$$\vec{F} = I \cdot (\vec{l} \times \vec{B})$$

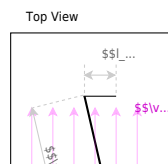
- The horizontal component $\vec{F}_{\text{h}}$ of the force is based on the vertical component $\vec{B}_{\text{v}}$ of the magnetic field.
- The vertical component $\vec{B}_{\text{v}}$ of the magnetic field is not shown in the image but is pointing into the ground.
- Considering the right-hand rule (and the cross product), the vertical field $\vec{B}_{\text{v}}$ has to be perpendicular to $\vec{B}_{\text{h}}$ and to $\vec{l}$. The right-hand rule has to be applied.

The **horizontal component** is given by

$$\begin{aligned} F_{\text{h}} &= I \cdot (I \cdot B_{\text{v}}) \quad \&= 1'200 \text{ A} \cdot 300 \\ &\quad \cdot 10^3 \text{ m} \cdot 40 \cdot 10^{-6} \frac{\text{Vs}}{\text{m}^2} \quad \&= 14'400 \\ &\quad \text{VA} = 14'400 \text{ W} = 14'400 \text{ N} \end{aligned}$$

For the **vertical component** the angle α has to be considered.

For the maximum F_{v} the angle α has to be 90° , therefore the $\sin$ has to be used.



$$F_{\text{v}} = I \cdot I \cdot B_{\text{h}} \cdot \sin \alpha \quad \&= 1'200 \text{ A} \cdot 300 \cdot 10^3 \text{ m} \cdot 40 \cdot 10^{-6} \frac{\text{Vs}}{\text{m}^2} \cdot \sin 20^\circ \quad \&= 2'462.545... \text{ N}$$

For the **overall force** F the Pythagorean theorem has to be used:

$$F = \sqrt{F_{\text{v}}^2 + F_{\text{h}}^2} \quad \&= \sqrt{(14'400 \text{ N})^2 + (2'462.545... \text{ N})^2} \quad \&= 14'609.04... \text{ N}$$

Embedded resources

Explanation (video): ...

¹⁾

ideally: infinite long; in reality much longer, than the distance between them

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