

Block 17 — Magnetic Flux Density and Forces

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Block 17 — Magnetic Flux Density and Forces	2
<i>Learning objectives</i>	2
<i>Preparation at Home</i>	2
<i>90-minute plan</i>	2
<i>Conceptual overview</i>	2
<i>Core content</i>	2
Definition of the Magnetic Flux Density	3
Notice:	4
Materials	5
Diamagnetic Materials	6
Paramagnetic Materials	7
Ferromagnetic Materials	8
Applications of the Lorentz Force: Two parallel Conductors	9
<i>Common pitfalls</i>	10
<i>Exercises</i>	11
Exercise E6 Cylindrical Coil (written test, approx. 6 % of a 120-minute written test, SS2021)	11
Exercise E3 Magnetic Flux Density (written test, approx. 6 % of a 120-minute written test, SS2021)	11
Exercise E4 Toroidal Coil (written test, approx. 5 % of a 120-minute written test, SS2021)	13
Exercise E12 Lorentz Force (hard!) (written test, approx. 10 % of a 120-minute written test, SS2021)	14
Task 3.2.1 Magnetic Field Strength around a horizontal straight Conductor	18
Task 3.3.1 magnetic Flux Density	18
Task 3.3.2 Electron in Plate Capacitor with magnetic Field	19
<i>Embedded resources</i>	22

Block 17 — Magnetic Flux Density and Forces

Learning objectives

After this 90-minute block, you can

- ...

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

90-minute plan

1. Warm-up (x min):
 1.
2. Core concepts & derivations (x min):
 1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. ...

Core content

We know from [block11](#) that a static charge Q_1 generate a static electric field D .

Before in [block09](#), we developed that a static electric field $E = \frac{1}{\epsilon_0} D$ effects a force F_C on a static charge Q_2

From the last chapter ([block16](#)) we got, that moving charges $\frac{dQ_1}{dt} = I_1$ generate a static magnetic field H .

So, how does an acting magnetic field effects a force on a moving charge $\frac{dQ_2}{dt} = I_2$?

Definition of the Magnetic Flux Density

To derive the forces, we do a step back to the images of field lines.

In [figure 1 1](#)) the field lines of a single current-carrying wire is shown.

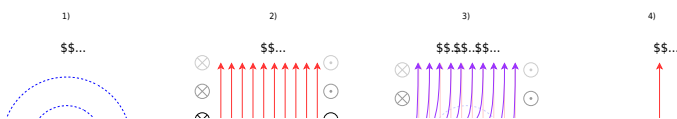
[figure 1 2](#)) depicts the homogenous field of a coil.

When a current-carrying wire is within the homogenous field, we get the superimposed picture of both fields ([figure 1 3](#)).

This leads to an enrichment of magnetic field on the left and an depletion on the right.

With the knowledge, that the field lines usually do not like to stay next to each other, one can conclude that there will be a force to the right.

Fig. 1: Force in magnetic field



When no current is flowing through the conductor the force is equal to zero.

The following is detectable:

1. $|\vec{F}| \sim I$: The stronger the current, the stronger the force F .
2. $|\vec{F}| \sim l$: As longer the conductor length l , as stronger the force F gets.
3. $|\vec{F}| \sim H$: As more current through the coil, as stronger the H -field. And a stronger the H -field leads to stronger force F .

To summarize:
$$F \sim H \cdot l \cdot I$$

The proportionality factor is μ_0 , the **magnetic field constant, permeability** or **vacuum permeability**: $\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$.

$$F = \mu_0 \cdot H \cdot l \cdot I$$

When adding an iron core into the coil the force F gets stronger. Therefore, we include a material-dependent constant μ_r , the so-called relative permeability

$$F = \mu_0 \mu_r \cdot H \cdot I \cdot l$$

The new field quantity is B the **magnetic flux density**:

$$\boxed{\vec{B} = \mu \cdot \vec{H}} \quad | \quad \text{with } \mu = \mu_0 \mu_r$$

Investigating the vectorial behaviour leads to the cross-product, and to the so-called **Lorentz force**

$$\boxed{\vec{F}_L = I \cdot \vec{l} \times \vec{B}}$$

With $\vec{l}$ pointing in the direction of the positive current I . The absolute value can be calculated by

$$\boxed{|\vec{F}_L| = I \cdot l \cdot B \cdot \sin(\angle \vec{l}, \vec{B})}$$

For the orientation of the vectors, another right-hand rule can be applied.

Notice:

Right-hand rule for the Lorentz Force:

- The causing current I is on the thumb. Since the current is not a vector, the direction is given by the direction of the conductor $\vec{l}$
- The mediating external magnetic field $\vec{B}$ is on the index finger
- The resulting force $\vec{F}$ on the conductor is on the middle finger

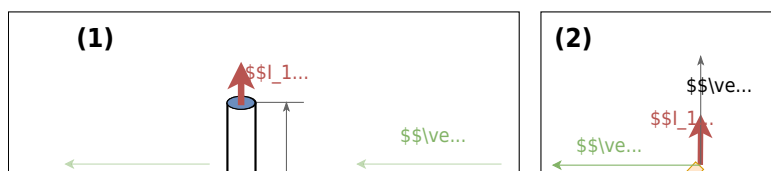
This is shown in [figure 2](#).

A way to remember the orientation is the mnemonic **FBI** (from middle finger to thumb):

- $\vec{F}$ force on middle finger
- $\vec{B}$ -Field on index finger
- Current I on thumb (direction with length $\vec{l}$)

To view the animation: [click here!](#)

Fig. 2: Force onto a single Conductor in a B-Field



Materials

The material can be divided into different types by looking at its relative permeability. [figure 3](#) shows the relative permeability in the **magnetization curve** (also called B - H -curve). In this diagram, the different effect (B -field on y -axis) based on the causing external H -field (on x -axis) for different materials is shown. The three most important material types shall be discussed shortly.

Fig. 3: Magnetization Curve of different materials

Diamagnetic Materials

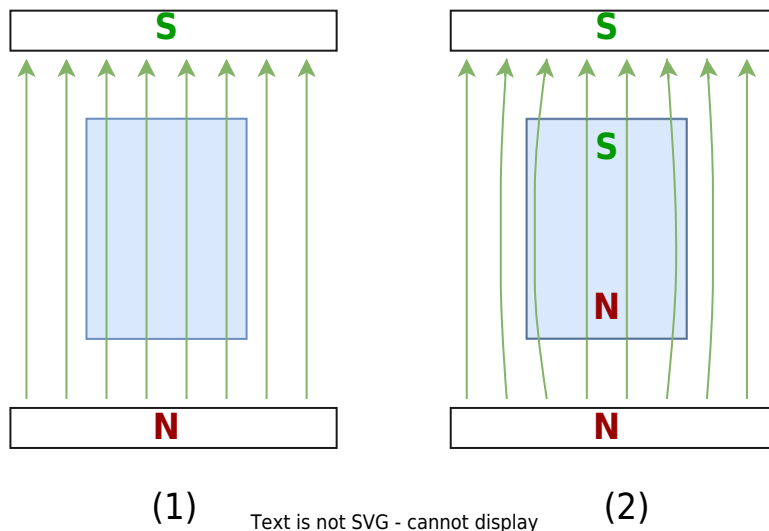
- Diamagnetic materials weaken the magnetic field, compared to the vacuum.
- The weakening is very low (see [table 1](#)).
- For diamagnetic materials applies $0 < \mu_{\text{r}} < 1$
- The principle behind the effect is based on quantum mechanics (see [figure 4](#)):
 - Without the external field no counteracting field is generated by the matter.
 - With an external magnetic field an antiparallel-orientated magnet is induced.
 - The reaction weakens the external field. This is similar to the weakening of the electric field by the dipoles of materials.
- Due to the repulsion of the outer magnetic field the material tends to move out of a magnetic field.

For very strong magnetic fields small objects can be levitated (see clip).

Material	Symbol	μ_{r}
Antimon	Sb	0.999 946
Copper	Cu	0.999 990
Mercury	Hg	0.999 975
Silver	Ag	0.999 981
Water	H_2O	0.999 946
Bismut	Bi	0.999 830

Tab. 1: Diamagnetic Materials

Fig. 4: Magnetic field in diamagnetic materials



Text is not SVG - cannot display

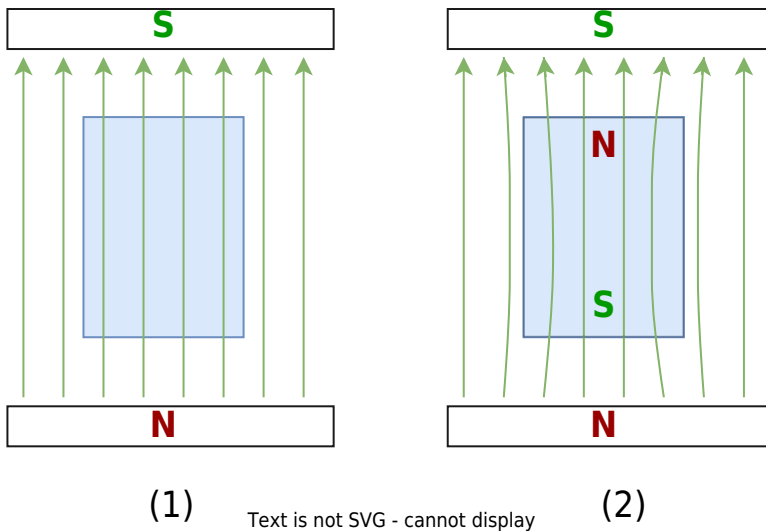
Paramagnetic Materials

- Paramagnetic materials strengthen the magnetic field, compared to the vacuum.
- The strengthening is very low (see [table 2](#)).
- For paramagnetic materials applies $\mu_{\text{r}} > 1$
- The principle behind the effect is again based on quantum mechanics (see [figure 5](#)):
 - Without the external field no counteracting field is generated by the matter.
 - With an external magnetic field internal “tiny magnets” based on the electrons in their orbitals are orientated similarly.
 - This reaction strengthens the external field.

Material	Symbol	μ_{r}
Aluminum	Al	1.000 022
Air		1.000 000 4
Oxygen	O_2	1.000 001 3
Platinum	Pt	1.000 36
Tin	Sn	1.000 003 8

Tab. 2: Paramagnetic Materials

Fig. 5: Magnetic field in paramagnetic materials



Ferromagnetic Materials

- Ferromagnetic materials strengthen the magnetic field strongly, compared to the vacuum.
- The strengthening can create a field multiple times stronger than in a vacuum (see [figure 6](#)).
- For ferromagnetic materials applies $\mu_{\text{r}} \gg 1$
- Ferromagnetic materials are characterized by the magnetization curve (see [figure 7](#))
 - Non-magnetized ferromagnets are located in the origin.
 - With an external field H the initial magnetization curve (in German: *Neukurve*, dashed in [figure 7](#)) is passed.
 - Even without an external field ($H=0$) and the internal field is stable.
The stored field without external field is called **remanence** $B(H=0) = B_{\text{r}}$ (or remanent magnetization).
 - In order to eliminate the stored field the counteracting **coercive field strength** H_{C} (also called coercivity) has to be applied.
 - The **saturation flux density** B_{sat} is the maximum possible magnetic flux density (at the maximum possible field strength H_{sat})

Fig. 6: Magnetic field in paramagnetic materials

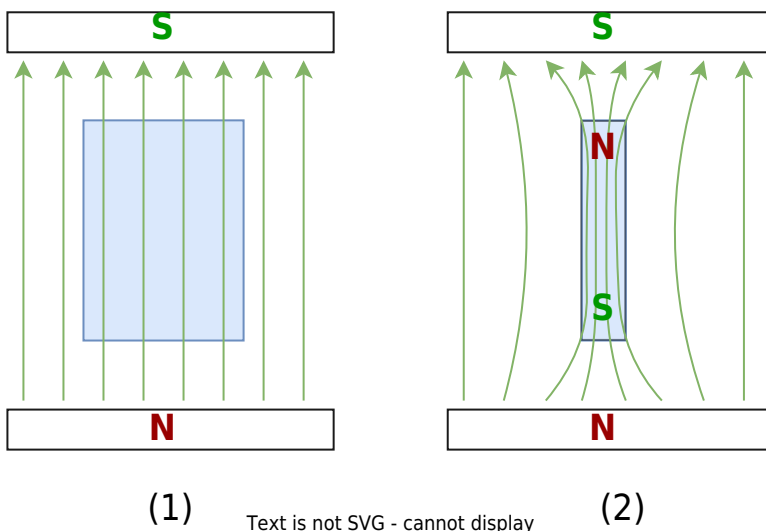
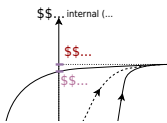


Fig. 7: Magnetization Curve

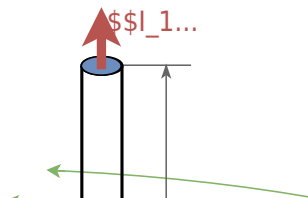


Applications of the Lorentz Force: Two parallel Conductors

The Lorentz force can be applied to two parallel conductors.

The experiment consists of a part of two very long¹⁾ and parallel conductors with the currents I_1 , I_2 and the distance r (see [figure 2](#)).

Fig. 2: Forces between two Conductors



Here, we get for the B field caused by I_2 :
$$B_2 = \mu \cdot H_2 = \mu \cdot \frac{I_2}{2\pi \cdot r}$$

We insert this into the formula of the Lorentz force
$$\vec{F}_L = I \cdot \vec{l} \times \vec{B}$$

This leads to the so-called **Ampere's Force Law**, applied on long and parallel conductors:

$$\boxed{|\vec{F}_{12}| = \frac{\mu}{2\pi} \cdot \frac{I_1 \cdot I_2}{r} \cdot l}$$

Common pitfalls

- ...

Exercises

Exercise E6 Cylindrical Coil

(written test, approx. 6 % of a 120-minute written test, SS2021)

A) The magnetic flux (2 points)

Result

- Length $l = 30 \text{ cm}$,

Path • Winding diameter $d = 390 \text{ mm}$,

- Number of windings $N = 240$,

• Current through the inductor $I = 500 \text{ mA}$,

- Material inside: Air

• $\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$

The magnetic field strength is $B = \mu_0 \mu_r H$:

The proportion of the magnetic voltage outside the coil can be neglected. Determine the following for the inside of the coil:

$\Phi = B \cdot A$

a) the magnetic field strength (2 points)

Result: $H = \frac{NI}{l} = \frac{240 \cdot 0.5}{0.3} = 400 \text{ A/m}$

$B = \mu_0 \mu_r H = 4\pi \cdot 10^{-7} \cdot 1 \cdot 400 = 0.0005026 \text{ T}$

$A = \pi r^2 = \pi \left(\frac{d}{2} \right)^2$

Path

Therefore: $\Phi = B \cdot \pi \left(\frac{d}{2} \right)^2$

$\Phi = 0.00006004 \text{ Vs}$

Putting in the numbers: $\Phi = 0.0005026 \text{ T} \cdot \pi \left(\frac{0.39 \text{ m}}{2} \right)^2 = 0.00006004 \text{ Vs}$

Putting in the numbers: $H = \frac{240 \cdot 0.5 \text{ A}}{0.3 \text{ m}} = 400 \text{ A/m}$

Exercise E3 Magnetic Flux Density

(written test, approx. 6 % of a 120-minute written test, SS2021)

A) The circuit is operated for 10 minutes at the frequency $f = 50 \text{ Hz}$ with an

Result: $I = 100 \text{ A}$

Now stand next to it and think about what this value means (3 points, independent)

The figure below shows the top view of the laboratory with the supply line between A and B .

Path: $B = 0.12 \text{ mT}$

$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$, $\mu_r = 1$

The formula for the magnetic field strength can be rearranged: $H =$

$$\frac{I}{2\pi \cdot r} \quad r = \frac{I}{2\pi \cdot H}$$

Again, the magnetic flux density B is given as: $B = \mu_0 \mu_r H$

Therefore:
$$r = \mu_0 \mu_r \left\{ \frac{I}{2\pi \cdot B} \right\} = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \left\{ \frac{100 \text{ A}}{2\pi \cdot 100 \cdot 10^{-6} \text{ T}} \right\}$$

a) What is the highest magnetic flux density through the line in your body? (3 points)

Path

The magnetic field strength for a conducting wire is given as:

$$H = \frac{I}{2\pi \cdot r}$$

The magnetic flux density B is given as: $B = \mu_0 \mu_r H$

Here, the maximum current is $\hat{I} = 100 \text{ A}$ and the distance to the cable is $r = \sqrt{(0.1 \text{ m})^2 + (0.4 \text{ m})^2} = 0.412... \text{ m}$.

Therefore:
$$B = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1 \cdot \frac{100 \text{ A}}{2\pi \cdot 0.412... \text{ m}}$$

Exercise E4 Toroidal Coil**(written test, approx. 5 % of a 120-minute written test, SS2021)**

A magnetic field with a flux density of at least 50 mT is to be achieved in a ring-shaped coil (toroidal coil).

The coil has 60 turns, wound around soft iron with $\mu_r = 1200$.

The average field line length in the coil should be $l = 12 \text{ cm}$.

Result: $I_{\text{min}} = 4 \text{ A}$



What is the minimum current that must flow through a single winding?

Path

The magnetic field strength of a toroidal coil is given as:

$$H = \frac{N \cdot I}{l}$$

Based on the flux density the magnetic field strength can be derived by $B = \mu_0 \mu_r H$.

By this, the formula can be rearranged:

$$H = \frac{N \cdot I}{l} \quad || \quad \frac{B}{\mu_0 \mu_r} = H \\ \frac{N \cdot I}{l} = \frac{B}{\mu_0 \mu_r} \quad || \quad I = \frac{B \cdot l}{\mu_0 \mu_r \cdot N}$$

Putting in the numbers:

$$I = \frac{0.05 \text{ T} \cdot 0.12 \text{ m}}{4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} \cdot 1'200 \cdot 60} = 0.6631... \frac{\text{T} \cdot \text{m}}{\frac{\text{Vs}}{\text{Am}}} = 0.6631... \frac{\text{Vs}}{\text{m}^2} \cdot \text{m} \cdot \frac{\text{Vs}}{\text{Am}}} = 0.6631... \text{ A}$$

Exercise E12 Lorentz Force (hard!)

(written test, approx. 10 % of a 120-minute written test, SS2021)

A) The picture below shows a straight high-voltage line, which of the dimensions shown makes the Resultant component of $F = 1'200 \text{ N}$ on the resulting force is? (Independent)

A homogeneous geomagnetic field is assumed. The magnetic field strength has a vertical component of $B_v = 40 \text{ } \mu\text{T}$ and a horizontal component of $B_h = 20 \text{ } \mu\text{T}$.

Only a 500 A is perpendicular to $\vec{B}_v$ and to $\vec{B}_h$ and points in the right direction by the right-hand rule.

The picture on the right shows the line (black), the field strength components, and the angle in front and top view for illustration purposes.

- a) Calculate the force that results from the current flow on the entire conductor.
First, calculate the vertical and horizontal components and combine them accordingly.

Path
Top View

Path

The force on the transmission line can be calculated via the Lorentz force

$$\vec{F} = I \cdot (\vec{l} \times \vec{B})$$

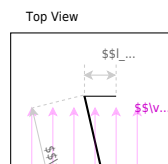
- The horizontal component $\vec{F}_{\text{h}}$ of the force is based on the vertical component $\vec{B}_{\text{v}}$ of the magnetic field.
- The vertical component $\vec{B}_{\text{v}}$ of the magnetic field is not shown in the image but is pointing into the ground.
- Considering the right-hand rule (and the cross product), the vertical field $\vec{B}_{\text{v}}$ has to be perpendicular to $\vec{B}_{\text{h}}$ and to $\vec{l}$. The right-hand rule has to be applied.

The **horizontal component** is given by

$$\begin{aligned} F_{\text{h}} &= I \cdot (I \cdot B_{\text{v}}) \quad \&= 1'200 \text{ A} \cdot 300 \\ &\quad \cdot 10^{-3} \text{ m} \cdot 40 \cdot 10^{-6} \frac{\text{Vs}}{\text{m}^2} \quad \&= 14'400 \\ &\quad \frac{\text{VA}}{\text{m}} = 14'400 \frac{\text{Ws}}{\text{m}} = 14'400 \text{ N} \end{aligned}$$

For the **vertical component** the angle α has to be considered.

For the maximum F_{v} the angle α has to be 90° , therefore the $\sin$ has to be used.



$$\begin{aligned} F_{\text{v}} &= I \cdot I \cdot B_{\text{h}} \cdot \sin \alpha \quad \&= 1'200 \\ &\quad \text{A} \cdot 300 \cdot 10^{-3} \text{ m} \cdot 40 \cdot 10^{-6} \frac{\text{Vs}}{\text{m}^2} \\ &\quad \cdot \sin 20^\circ \quad \&= 2'462.545... \text{ N} \end{aligned}$$

For the **overall force** F the Pythagorean theorem has to be used:

$$\begin{aligned} F &= \sqrt{F_{\text{v}}^2 + F_{\text{h}}^2} \quad \&= \sqrt{(14'400 \text{ N})^2 + (2'462.545... \text{ N})^2} \\ &\quad \&= 14'609.04... \text{ N} \end{aligned}$$

Task 3.2.1 Magnetic Field Strength around a horizontal straight Conductor

What is the magnetic field strength H_1 at a point P_1 , which is outside the conductor. The conductor has a constant section and the current I_0 flows through it. The radius of the conductor is $r_L = 4 \text{ mm}$.

What is the magnetic field strength H_1 at a point P_1 , which is outside the conductor at a distance $r_1 = 250 \text{ mm}$ from the conductor? $H(3 \text{ mm})$

The H -field is given as: $H(r) = \frac{I_0}{2\pi r}$

But now I is not I_0 anymore, but only a fraction, so ΔI . I_0 is evenly distributed over the cross-section of the conductor. The cross-sectional area is given as $A = \pi r_L^2$ of the closed path around the area.

- Here, the relevant current is **not** the given one. There is only a fraction of the current flowing through the part of the conductor inside the area.
- The H -field is given as: the current I through an area divided by the cross-sectional area of the closed path around the area. This shall give you the formula (when not already known)

Therefore, the relevant current is $I(r) = \frac{\Delta I}{A} \cdot A(r) = \frac{I_0}{A} \cdot \pi r_2^2$

$$H(r) = \frac{I(r)}{2\pi r} = \frac{I_0 \cdot \pi r_2^2}{2\pi r \cdot \pi r_L^2} = \frac{I_0}{2\pi} \cdot \frac{r_2^2}{r \cdot r_L^2}$$

Task 3.3.1 magnetic Flux Density

What is the magnetic flux density B in a toroidal coil with $N = 1000$ windings and a toroidal diameter for the average field line of $d = 1 \text{ m}$?

For comparison, the same flux density shall be created inside a toroidal coil with $10'000$ windings and a toroidal diameter for the average field line of $d = 1 \text{ m}$.

The B -field is given as: $B = \mu \cdot H$

Now B has to be given as $B = \mu \cdot \frac{I \cdot N}{l}$

This can be rearranged to the current $I = \frac{B \cdot l}{\mu \cdot N}$

The B -field can be calculated by the H -field.

- The H -field is given as: the current I through an area divided by the "specific" length l of the closed path around the area. This shall give you the formula (when not already known)
- The current is number of windings times I .

Task 3.3.2 Electron in Plate Capacitor with magnetic Field

An electron enters a plate capacitor on a trajectory parallel to the plates. It shall move with the velocity $\vec{v}$ in the plate capacitor parallel to the plates. The plates have a potential difference U and a distance d . In the vacuum in between the plates, there is also a magnetic field $\vec{B}$ present.

Fig. 9: Electron in B- and E-Field



Calculate the velocity depending on the other parameters $\vec{v} = f(U, |\vec{B}|, d)$!

Path

- Think about the two forces on the electron from the fields - gravity is ignored.
Write their definitions down.
- With which relationship between these two forces does the electron move through the plate capacitor parallel to the plates?
So the trajectory neither gets bent up nor down.
- What is the relationship between the E -field in the plate capacitor and the electric voltage U ?

Solution

Within the electric field, the Coulomb force acts on the electron:

$$\vec{F}_C = q_e \cdot \vec{E}$$

Within the magnetic field, also the Lorentz force acts on the electron:

$$\vec{F}_L = q_e \cdot \vec{v} \times \vec{B}$$

The absolute value of both forces must be equal to compensate each other:

$$|\vec{F}_C| = |\vec{F}_L| \quad |q_e \cdot \vec{E}| = |q_e \cdot \vec{v} \times \vec{B}| \quad |q_e| \cdot |\vec{E}| = |q_e| \cdot |\vec{v}| \cdot |\vec{B}| \quad |\vec{E}| = |\vec{v}| \cdot |\vec{B}|$$

Since $\vec{v}$ is perpendicular to $\vec{B}$ the right side is equal to $|\vec{v}| \cdot |\vec{B}| = v \cdot B$.

Additionally, for the plate capacitor $|\vec{E}| = U/d$.

Therefore, it leads to the following:

$$\frac{U}{d} = v \cdot B \quad v = \frac{U}{B \cdot d}$$

Result

$$v = \frac{U}{B \cdot d}$$

Task 1

1. Which hand can be used to infer magnetic field direction from currents?

Tip for 1

- ☐ The right hand
- ☐ The left hand

For the current, you use which hand?

2. In the derivation from 1. how are the fingers to be assigned?

Tip for 2

- ☐ Thumb for current direction, remaining

fingers for magnetic field direction

- ☐ Thumb for magnetic field direction, remaining fingers for current direction
- ☐ both possibilities are correct

3. Two conductors carrying current are parallel and close to each other. The current in both is flowing in the same direction. What force effect can be seen?

- ☐ none
- ☐ The conductors attract
- ☐ The conductors repel

Tip for 3

4. Two conductors carrying current are at right angles to each other. Current flows through both of them. What force effect can be seen? See 3rd video.

- ☐ none
- ☐ The conductors attract
- ☐ The conductors repel

5. What is the magnetic field inside the earth or a permanent magnet?

- ☐ from the magnetic north pole to the south pole
- ☐ from the magnetic south pole to the north pole
- ☐ the inside is free of field

Tip for 4

6. At which location of a current-carrying coil are the field lines densest?

- ☐ at the magnetic north pole
- ☐ at the magnetic south pole
- ☐ inside the coil
- ☐ at both poles

Check Answers

You Scored % - /

- Imagine a coil with a winding pictorially, or draw it on.
- Now think of a generated field through this to it. What direction must the current flow, that causes the field? Does this fit the rule of thumb?
- Then try it the other way round: If a current is given, where do the field lines go in and where out? What poles are created there?

- Picture the two wires, or draw them on.
- In which direction would the outer field run in each case?
- The field is a linear vector field. So the total field can be created from several individual fields by adding them together. Does adding the field in between make it larger, or smaller?

- First imagine the parallel wires again. What happens when the current flows in the same direction and what happens when the current flows in opposite directions? Are the resulting forces equal in magnitude?
- The reversal of the direction of the current can now also be produced by turning the wire instead of changing the current - just so that the wires are perpendicular to each other in the meantime when turning.
- With parallel wires and different current directions, the amount-wise same force arises. So, this is also true for every angle in between (in detail given by integration of the force over single wire pieces).
- But then there must be a point at which the force becomes 0.

Tip for 5

- The magnetic field lines must be closed.

- Compare the field curve between the coil and permanent magnet.

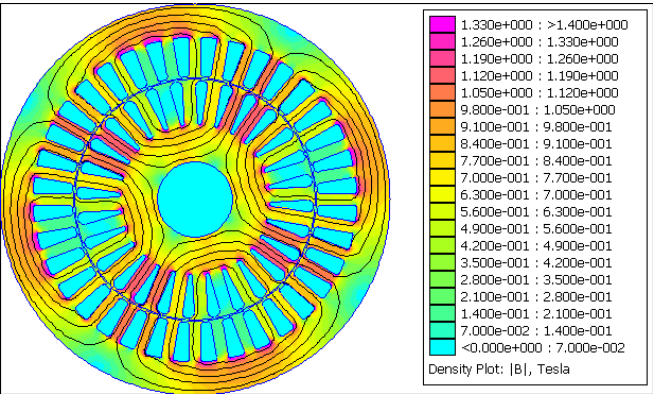
Tip for 6

- In video 1 you can see the course outside and inside the coil.

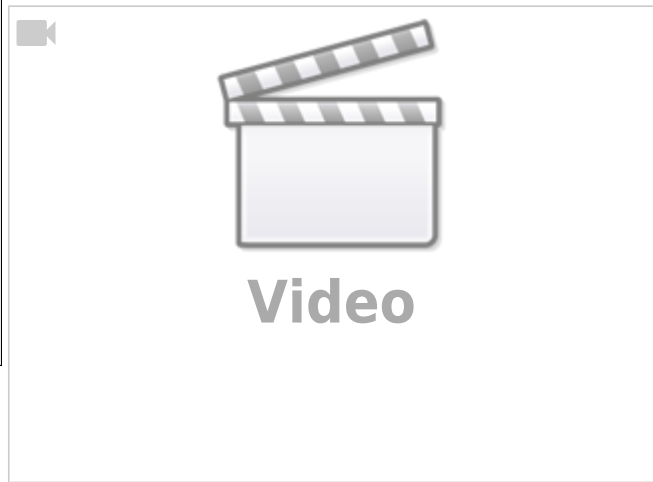
Embedded resources

Please have a look at the German contents (text, videos, exercises) on the page of the [KIT-Brückenkurs >> Lorentz-Kraft](#). The last part “Magnetic field within matter” can be skipped.

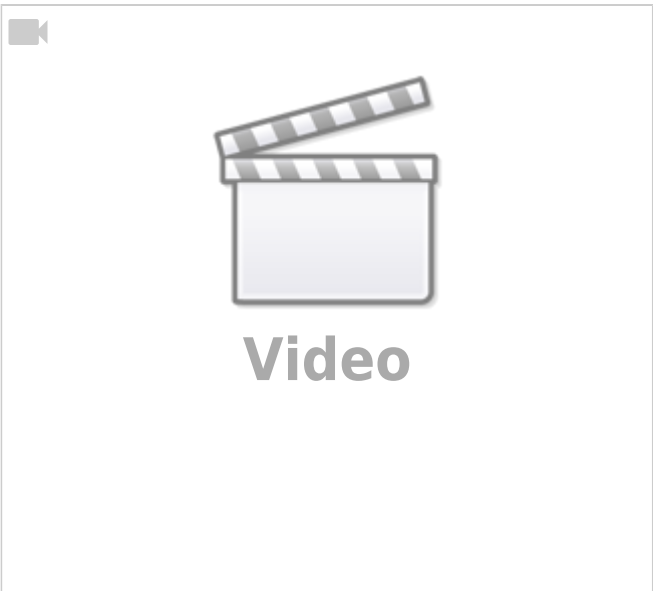
The rotating flux (density) in the stator of a motor, source/CC-BY 3.0 see 



A living insect (“diamagnet”) floats in a very strong magnetic field



Explanation of diamagnetism and paramagnetism



In Oxygen magnetic?

1)
ideally : infinite long; in reality much longer, than



Video

the
dist
anc
e
bet
wee
n
the
m

From:
<https://wiki.mexle.te.hs-heilbronn.de/> - **MEXLE Wiki**

Permanent link:
https://wiki.mexle.te.hs-heilbronn.de/electrical_engineering_and_electronics_1/block17?rev=1764693574

Last update: **2025/12/02 17:39**

