

4 Analysis of Networks

Student Group

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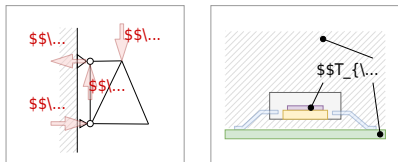
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4 Analysis of Networks

Fig. 1: examples for networks



Network analysis plays a central role in electrical engineering. It is so important because it can be used to simplify what at first sight appear to be complicated circuits and systems to such an extent that they can be understood and results derived from them.

In addition, networks also occur in other areas, for example, the momentum flux through a truss or the heat flux through individual hardware elements ([figure 1](#), or [an example for heat flow through electronics](#)). The concepts shown below can also be applied to these networks.

On the [wiki page for network analysis](#) the different methods are described very well in a compact way

Learning Objectives

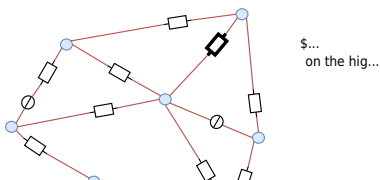
By the end of this section, you will be able to:

1. determine the number of nodes, number of (tree and connecting) branches, and number of meshes.
2. construct a complete tree from an electrical network.
3. understand the mesh current method and node potential method.
4. understand and be able to apply the superposition procedure.

4.1 Preliminary Work for Network Analysis

Preparation of the Circuit

Fig. 2: Preparing the circuit



Before the network analysis can be tackled, the circuit must be suitably prepared (cf. [figure 2](#)):

1. Clarify what is given and what is sought
2. Draw a circuit
3. Add voltage and current arrows. If not already given, then:
 1. First, draw current and voltage arrows at all sources according to the generator arrow

system.

2. Afterwards, define the current arrows at the remaining branches as you like.
3. Finally, draw the voltage arrows at the loads according to the load arrow system.
4. Select suitable current and voltage designations. If not already given, then:
 1. Count indices continuously, i.e. one number per element (source or load).
 2. Do not insert any signs in front of the designators in the circuit.

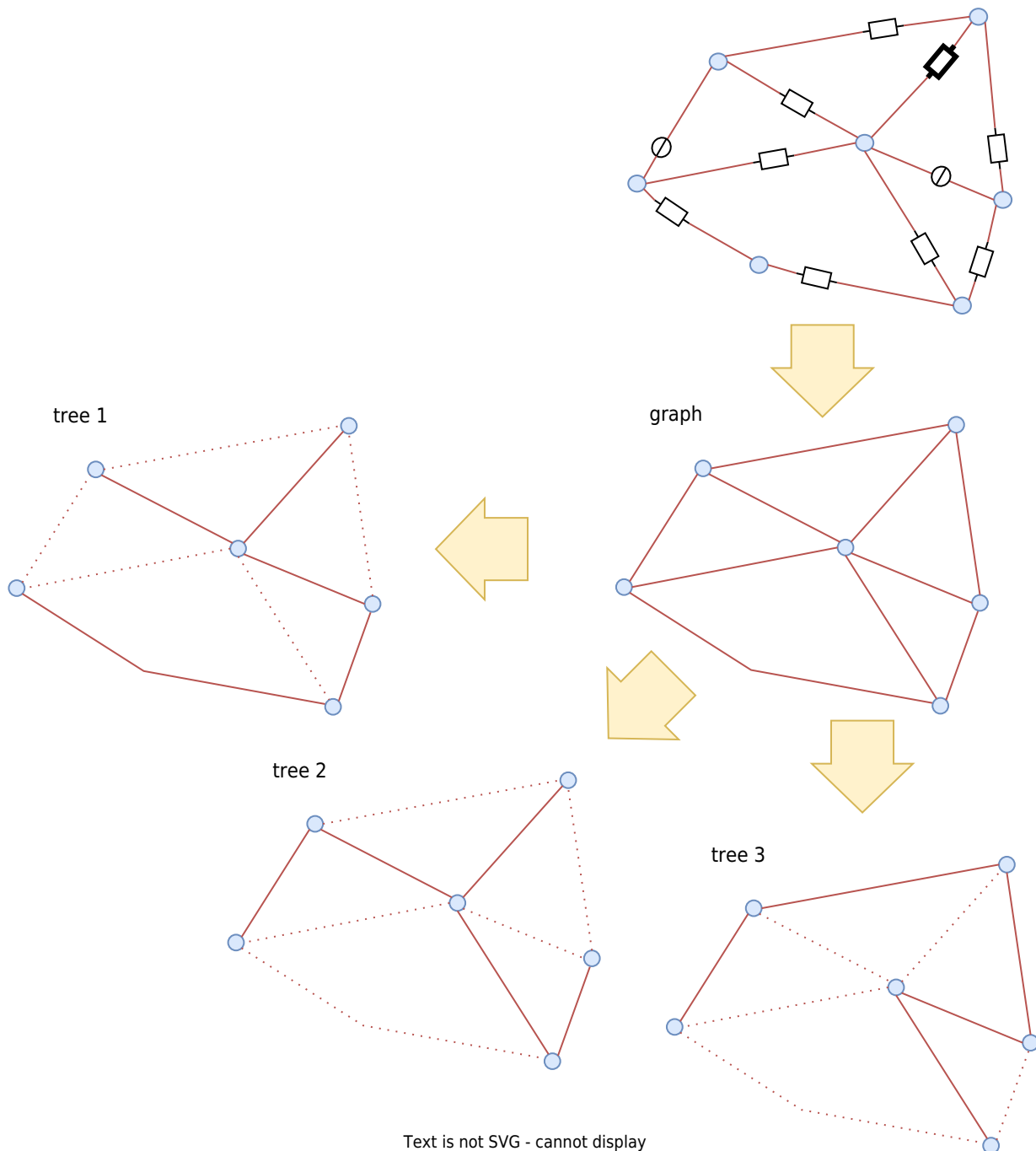
In real applications, it is useful to specify the number of variables ("what is wanted?"), parameters ("what can be adjusted?", e.g. potentiometer) and known quantities ("what is given?").

This makes it clear how many equations are needed. This seems to become difficult for larger networks - but a trick for this is presented below.

It often helps to draw the drawing several times (at least in your head) to have enough space for the identifiers (cf. [figure 2](#) below).

Graphs and Trees

Fig. 3: Graph of a network



In the chapter [2. simple dc_circuits](#) the terms nodes, branches, and loops have already been explained. These will now be expanded here to better explain the various network analysis methods in the following. In [figure 3](#) the **graph** of the example network is drawn. We had already seen this one too, but without knowing that this is called a graph!

But the important thing is: In this graph, only the (real) nodes are drawn. Nodes are by definition the connection of more than two branches. Accordingly, the connection between R_{10} and R_7 is not a node ¹⁾ ! For this reason, also the blue circle as a sign for nodes is omitted here.

A concept that has not yet appeared is that of the complete tree. For this, some (mathematical) graph theory is needed. There, too, the terms nodes and loops are used as before. A **tree** here is a special kind of graph. The graph in [figure 3](#) shows several loops.

Now a tree is characterized precisely by the fact that it contains no loops. Three different trees are drawn in the picture. From a given network, many different trees can be created (depending on the number of nodes).

Among the different trees, there are now some in which each node connects two or fewer loops.²⁾ These are called **complete trees** (occasionally also [Hamiltonian path](#)). Complete trees can also be understood as this shows a path through the network where all nodes are visited only exactly once.

Tree 3 in [figure 3](#) is now just one of the possible complete trees of this network.

The branches in complete trees are now distinguished according to their membership:

- **tree branches** belong to the complete tree (solid lines in [figure 3](#)).
- **Connecting branches** do not belong to the complete tree (dotted lines in [figure 3](#)).

Why does the excursion to graph theory make sense now? The trick is that by defining the complete tree, all loops have just been removed. Conversely, a new (independent) loop can be created by each connecting branch. So if the number of independent loop equations m is sought, this is just equal to the number of connecting branches.

To do this, proceed as follows:

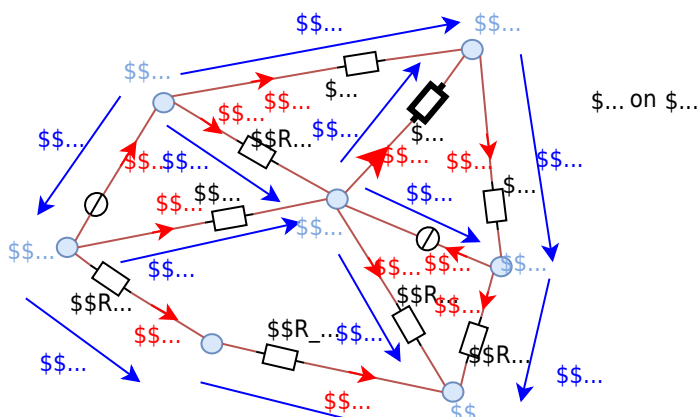
1. Determine the number of (real) nodes k .
2. Determine the number of branches z
3. The number of tree branches b is now $k-1$. (each node is traversed only once; at the last node there is no further branch).
4. The number of connecting branches v is given by "All branches minus tree branches": $v = z - b = z - k + 1$

Thus, the number of independent loop equations m is findable by counting the nodes k and branches z over $m = v = z - k + 1$.

This explanation can also be heard again in [this video](#) and is explained again clearly via [this video](#).

4.2 Branch Current Method

Fig. 4: example circuit



The branch current method (also called branch current method) now “simply times” (almost) all equations of the circuit. Specifically, for each node and each independent loop, the node and loop equations are written down:

$$\begin{pmatrix} 0 & +R_3 & 0 & 0 & 0 & -R_9 & 0 & 0 \\ 0 & +R_1 & -R_2 & -R_3 & 0 & 0 & 0 & 0 \\ 0 & +R_2 & 0 & +R_4 & 0 & 0 & 0 & 0 \\ 0 & -R_{11} & 0 & 0 & 0 & 0 & +R_5 & -R_6 \\ 0 & 0 & 0 & 0 & 0 & +R_6 & -R_7 & -U_{10} \\ 0 & -R_9 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_9 \\ I_{11} \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix} \quad \text{align*}$$

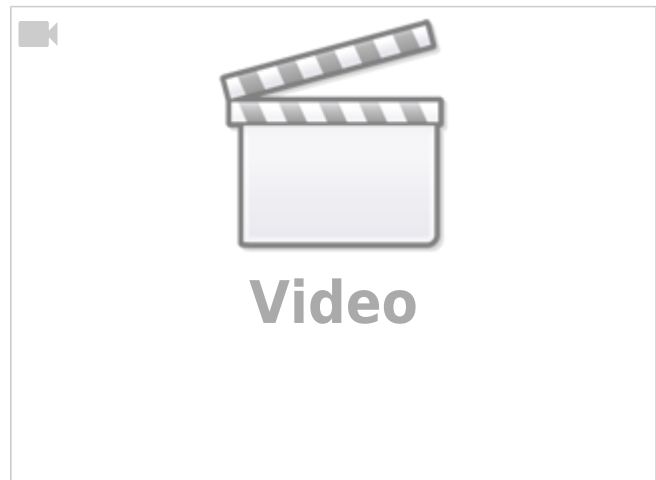
These matrices can be solved using, for example, the [Gaussian elimination](#).

Another example in videos

In video 1 you will learn:

1. writing down the given circuit and sizes
 2. drawing in and designating the nodes
 3. draw in and label the loops
 4. draw and label the mesh currents
 5. drawing in and designating the mesh voltages
- for a simple circuit

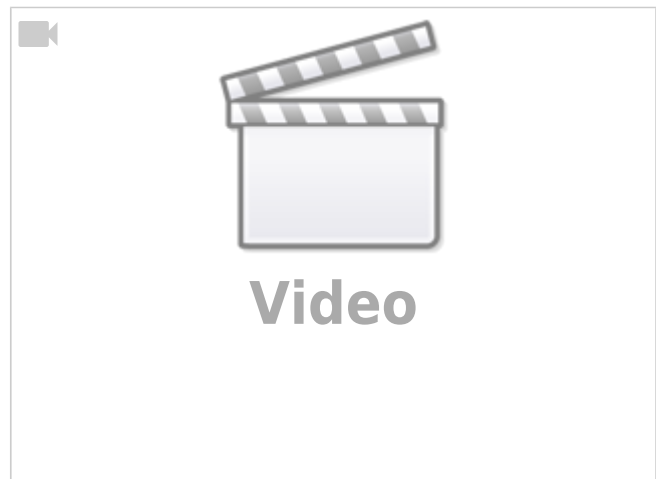
Mesh current analysis Example 1/3



In video 2 you will learn :

1. writing down the given circuit and sizes
 2. drawing in and designating the nodes
 3. draw in and label the loops
 4. draw and label the branch currents
 5. drawing in and designating the branch voltages
- for a more complex circuit

Branch current analysis 2/3

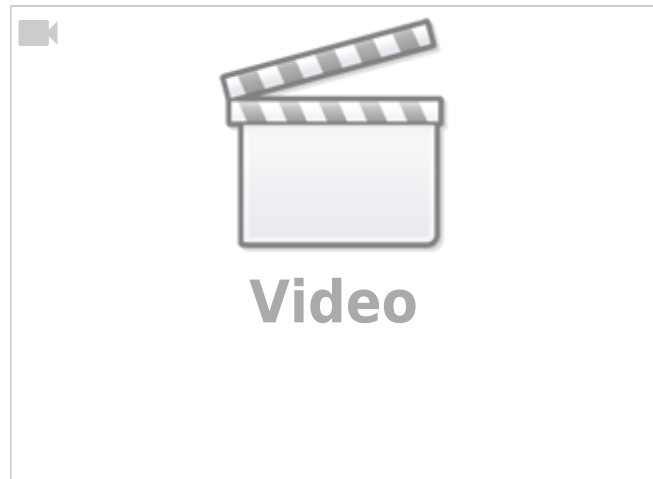


Video 3 describes the following steps:

1. write down the given circuit and sizes
2. draw in and designate the nodes
3. draw in and label the loops
4. draw and label the branch currents

Mesh current analysis 3/3

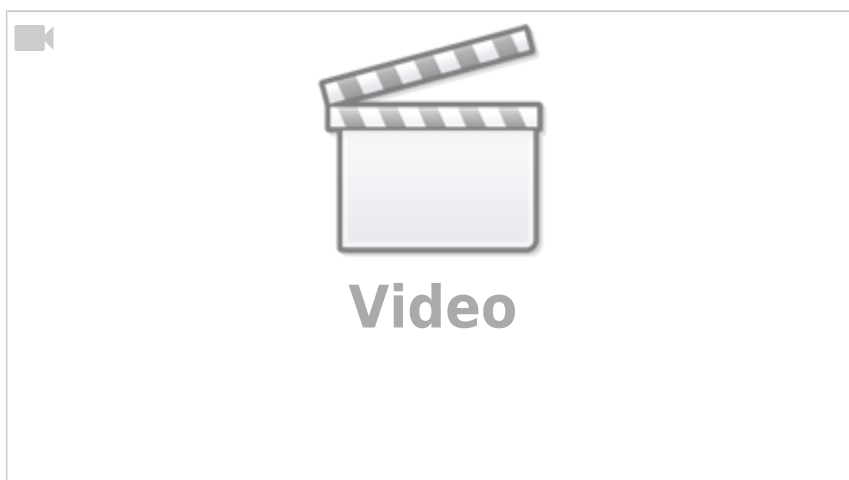
5. draw in and designate the branch voltages
6. set up node equations and loop equations
7. convert to matrix notation
8. solve the matrix



In the [Video 4](#) (not embedded here), will teach you how to solve a matrix by using a calculator:

9. inserting the numerical values into a calculator
10. calculating the matrix with a calculator

Exercise 4.2.1



4.3 Mesh Current Method

In the [mesh current method](#), only for all loops m each equation:

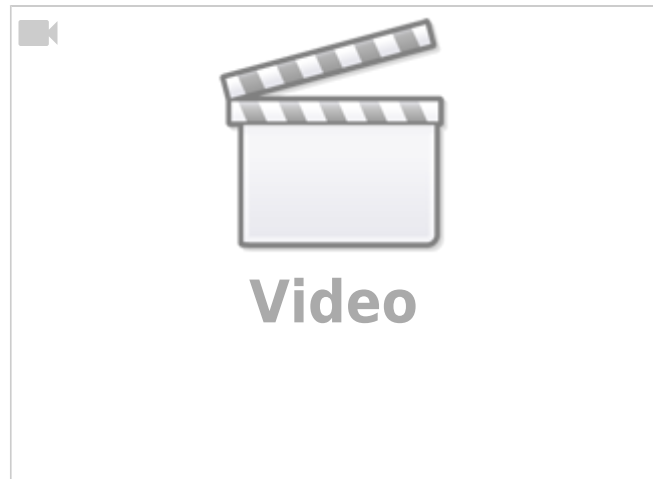
$\sum_{j=0}^{N_j} U_j = 0$ are considered. However, these are represented in the form $R \cdot I = U$.

The advantage here is that the number of equations to be solved is reduced to the number of independent loop currents.

These can also be considered matrix equations and can be solved with the rules of (mathematical) art.

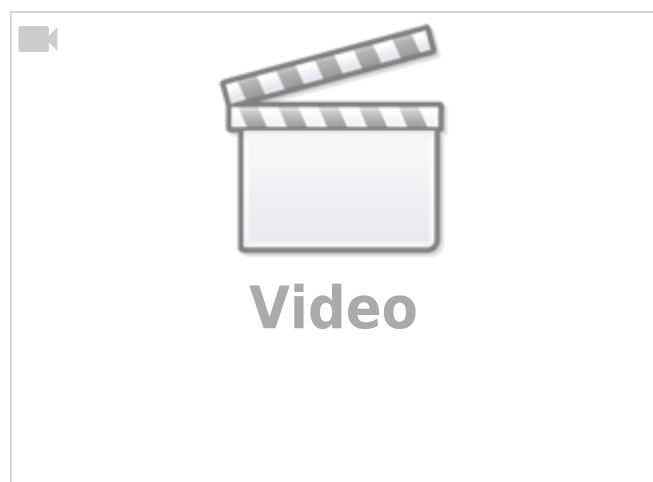
In video 1, the mesh current method is applied mesh current method

using an example.



Also in video 2, the mesh current method is applied using an example.

mesh current method



In the [Video 3](#) (not embedded here) shows in detail how the loop current method can be derived.

4.4 Nodal Potential Method

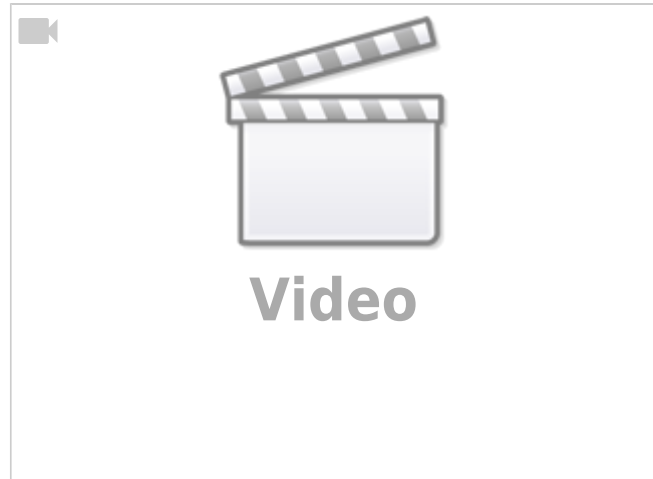
In the [nodal potential method](#), only the equation: $\sum_{i=0}^{N_i} I_i = 0$ are considered for all nodes k respectively. However, these are expressed in the form $\frac{1}{R} \cdot U = I$ and $G \cdot U = I$ respectively.

The advantage here is that the number of equations to be solved is reduced to the number of existing nodes (minus 1).

These can also be considered matrix equations and can be solved with the rules of (mathematical) art.

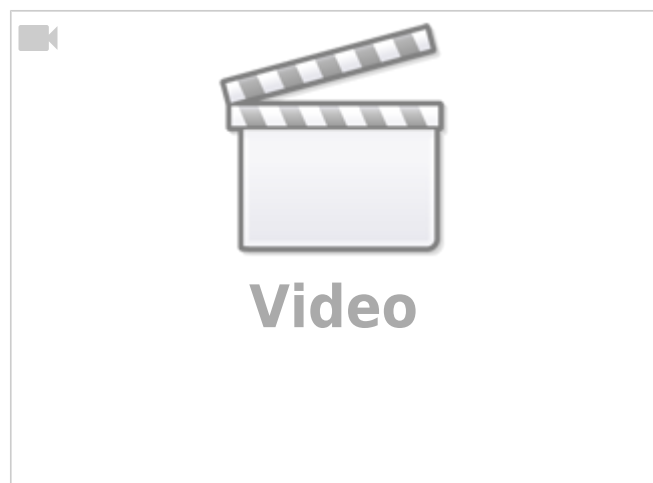
In Video 1, the idea behind node potential analysis is simply explained.

Simple example of node potential analysis



Video 2 also uses the nodal potential method with an example.

more complex example of a node potential analysis



In the [Video 3](#) (not embedded here) shows in detail how the nodal potential method can be derived.

4.5 Superposition Method / Superposition Principle

The superposition principle shall first be illustrated by some examples:

Example 1 - from an interview of a consulting company

Task: Three students are to fill a pool. If Alice has to fill it alone, she would need 2 days. Bob would need 3 days and Carol would need 4 days. How long would it take all three to fill a pool if they helped together?

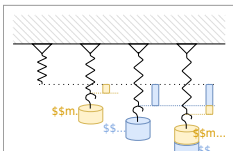
The question sounds far off-topic at first but is directly related. The point is that to solve it, filling the pool is assumed to be linear. So Alice will fill $\frac{1}{2}$, Bob $\frac{1}{3}$, and Carol $\frac{1}{4}$ of the pool per day. So on the first day, $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6 + 4 + 3}{12} = \frac{13}{12}$ of the pool filled. So the three of them need $\frac{12}{13}$ of a day.

However, this solution path is only possible because in linear systems the partial results can be

added.

Example 2 - Spring Force and Displacement

Fig. 5: mechanical spring



Task: A mechanical, linear spring is displaced due to masses m_1 and m_2 in the Earth's gravitational field (see [figure 5](#)). What is the magnitude of the deflection if both masses are attached simultaneously?

Again, a linear law is used here:
$$\vec{s} = f(\vec{F}) = -D \cdot \vec{F}$$

The (seemingly trivial) approach applies here:
$$\begin{aligned} \vec{s}_{1+2} &= f(\vec{F}_1 + \vec{F}_2) \\ &= -D \cdot (\vec{F}_1 + \vec{F}_2) \\ &= -D \cdot \vec{F}_1 - D \cdot \vec{F}_2 \\ &= f(\vec{F}_1) + f(\vec{F}_2) \\ &= \vec{s}_1 + \vec{s}_2 \end{aligned}$$

Notice:

In a physical system in which effect and cause are linearly related, the effect of each cause can first be determined separately. The total effect is then the sum of the individual effects.

For electrical engineering this principle was described by [Hermann_von_Helmholtz](#):

The currents in the branches of a linear network are equal to the sum of the partial currents in the branches concerned caused by the individual sources.

Thus, in the superposition method, the current (or voltage) sought in a circuit with multiple

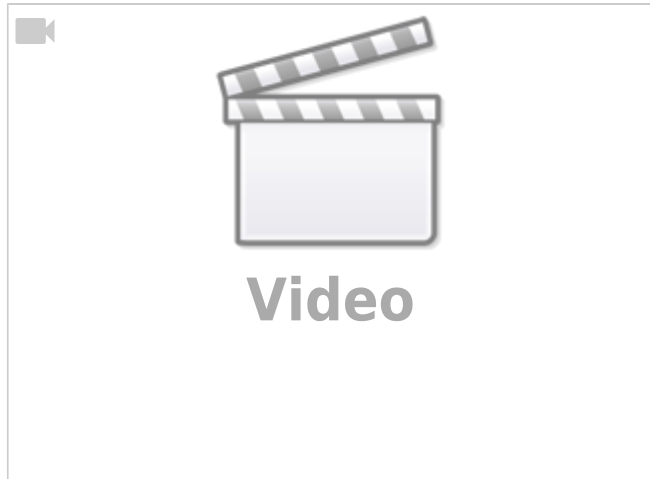
Simple view of the superposition principle

sources can be viewed as a superposition of the resulting currents (or voltages) of the individual sources.

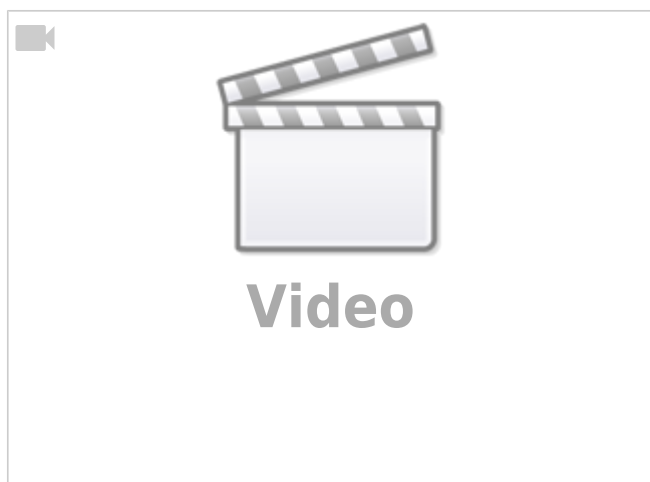
The “recipe” for the overlay is as follows:

1. Choose the next source x
2. Replace all ideal sources with their respective equivalent resistors:
 1. ideal voltage sources by short circuits
 2. ideal current sources by an open line
3. Calculate the partial currents sought in the branches considered.
4. Go to the next source $x=x+1$ ³⁾, and to point 2, as long as the partial currents of all sources have not been calculated.
5. Add up the partial currents in the branches under consideration, observing the correct sign.

This procedure is explained again in more detail using examples in the two videos on the right.

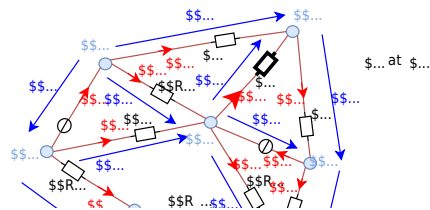


A more complex example of the superposition method

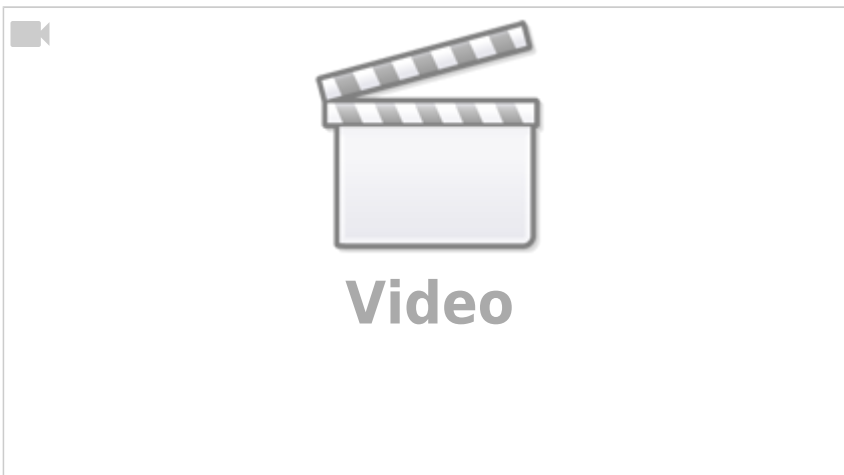


Example

Fig. 6: example circuit with superposition



Introduction to Nodal, Mesh and Superposition Method



Exercises

Exercise E4.5.1 Converting a bipolar signal to a unipolar signal (not from written test)

Amplifier, the input channel developed a ± 10 mV trip (A and B ± 2 mV, ± 5 mV, ± 10 mV, ± 20 mV, ± 30 mV, ± 40 mV, ± 50 mV, ± 60 mV, ± 70 mV, ± 80 mV, ± 90 mV, ± 100 mV, ± 110 mV, ± 120 mV, ± 130 mV, ± 140 mV, ± 150 mV, ± 160 mV, ± 170 mV, ± 180 mV, ± 190 mV, ± 200 mV, ± 210 mV, ± 220 mV, ± 230 mV, ± 240 mV, ± 250 mV, ± 260 mV, ± 270 mV, ± 280 mV, ± 290 mV, ± 300 mV, ± 310 mV, ± 320 mV, ± 330 mV, ± 340 mV, ± 350 mV, ± 360 mV, ± 370 mV, ± 380 mV, ± 390 mV, ± 400 mV, ± 410 mV, ± 420 mV, ± 430 mV, ± 440 mV, ± 450 mV, ± 460 mV, ± 470 mV, ± 480 mV, ± 490 mV, ± 500 mV, ± 510 mV, ± 520 mV, ± 530 mV, ± 540 mV, ± 550 mV, ± 560 mV, ± 570 mV, ± 580 mV, ± 590 mV, ± 600 mV, ± 610 mV, ± 620 mV, ± 630 mV, ± 640 mV, ± 650 mV, ± 660 mV, ± 670 mV, ± 680 mV, ± 690 mV, ± 700 mV, ± 710 mV, ± 720 mV, ± 730 mV, ± 740 mV, ± 750 mV, ± 760 mV, ± 770 mV, ± 780 mV, ± 790 mV, ± 800 mV, ± 810 mV, ± 820 mV, ± 830 mV, ± 840 mV, ± 850 mV, ± 860 mV, ± 870 mV, ± 880 mV, ± 890 mV, ± 900 mV, ± 910 mV, ± 920 mV, ± 930 mV, ± 940 mV, ± 950 mV, ± 960 mV, ± 970 mV, ± 980 mV, ± 990 mV, ± 1000 mV, ± 1010 mV, ± 1020 mV, ± 1030 mV, ± 1040 mV, ± 1050 mV, ± 1060 mV, ± 1070 mV, ± 1080 mV, ± 1090 mV, ± 1100 mV, ± 1110 mV, ± 1120 mV, ± 1130 mV, ± 1140 mV, ± 1150 mV, ± 1160 mV, ± 1170 mV, ± 1180 mV, ± 1190 mV, ± 1200 mV, ± 1210 mV, ± 1220 mV, ± 1230 mV, ± 1240 mV, ± 1250 mV, ± 1260 mV, ± 1270 mV, ± 1280 mV, ± 1290 mV, ± 1300 mV, ± 1310 mV, ± 1320 mV, ± 1330 mV, ± 1340 mV, ± 1350 mV, ± 1360 mV, ± 1370 mV, ± 1380 mV, ± 1390 mV, ± 1400 mV, ± 1410 mV, ± 1420 mV, ± 1430 mV, ± 1440 mV, ± 1450 mV, ± 1460 mV, ± 1470 mV, ± 1480 mV, ± 1490 mV, ± 1500 mV, ± 1510 mV, ± 1520 mV, ± 1530 mV, ± 1540 mV, ± 1550 mV, ± 1560 mV, ± 1570 mV, ± 1580 mV, ± 1590 mV, ± 1600 mV, ± 1610 mV, ± 1620 mV, ± 1630 mV, ± 1640 mV, ± 1650 mV, ± 1660 mV, ± 1670 mV, ± 1680 mV, ± 1690 mV, ± 1700 mV, ± 1710 mV, ± 1720 mV, ± 1730 mV, ± 1740 mV, ± 1750 mV, ± 1760 mV, ± 1770 mV, ± 1780 mV, ± 1790 mV, ± 1800 mV, ± 1810 mV, ± 1820 mV, ± 1830 mV, ± 1840 mV, ± 1850 mV, ± 1860 mV, ± 1870 mV, ± 1880 mV, ± 1890 mV, ± 1900 mV, ± 1910 mV, ± 1920 mV, ± 1930 mV, ± 1940 mV, ± 1950 mV, ± 1960 mV, ± 1970 mV, ± 1980 mV, ± 1990 mV, ± 2000 mV, ± 2010 mV, ± 2020 mV, ± 2030 mV, ± 2040 mV, ± 2050 mV, ± 2060 mV, ± 2070 mV, ± 2080 mV, ± 2090 mV, ± 2100 mV, ± 2110 mV, ± 2120 mV, ± 2130 mV, ± 2140 mV, ± 2150 mV, ± 2160 mV, ± 2170 mV, ± 2180 mV, ± 2190 mV, ± 2200 mV, ± 2210 mV, ± 2220 mV, ± 2230 mV, ± 2240 mV, ± 2250 mV, ± 2260 mV, ± 2270 mV, ± 2280 mV, ± 2290 mV, ± 2300 mV, ± 2310 mV, ± 2320 mV, ± 2330 mV, ± 2340 mV, ± 2350 mV, ± 2360 mV, ± 2370 mV, ± 2380 mV, ± 2390 mV, ± 2400 mV, ± 2410 mV, ± 2420 mV, ± 2430 mV, ± 2440 mV, ± 2450 mV, ± 2460 mV, ± 2470 mV, ± 2480 mV, ± 2490 mV, ± 2500 mV, ± 2510 mV, ± 2520 mV, ± 2530 mV, ± 2540 mV, ± 2550 mV, ± 2560 mV, ± 2570 mV, ± 2580 mV, ± 2590 mV, ± 2600 mV, ± 2610 mV, ± 2620 mV, ± 2630 mV, ± 2640 mV, ± 2650 mV, ± 2660 mV, ± 2670 mV, ± 2680 mV, ± 2690 mV, ± 2700 mV, ± 2710 mV, ± 2720 mV, ± 2730 mV, ± 2740 mV, ± 2750 mV, ± 2760 mV, ± 2770

Solution

For the internal resistance of the microcontroller, input applies: $R_{\text{uC}} \rightarrow \infty$

The sensor resistance is $R_S = 15 \text{ k}\Omega$

For conditioning, the input signal is to be fed via the series resistor R_3 to the center. We can choose R_3 arbitrarily. Here I choose a nice value to get integer values for potential of a voltage divider $R_1 = R_2$ with $R_1 = 1$ to a supply voltage $U_0 = 1 \text{ mV}$.

```

forall (a: int) (R: Prop) (h: (forall (b: int) (R))) : a = 2 && (forall (b: int) (R)) <=> (forall (b: int) (R)) && (forall (b: int) (R)) //

```

The following simulation shows you just the situation. Be aware that the resistor and voltage values are not correct. $\frac{R_2}{R_1 + R_2} = \frac{15 \text{ } \Omega}{1 \text{ } \Omega + 15 \text{ } \Omega} = 11.8 \text{ } \Omega$

Based on the 2014 Series, the following continues from the supply catalog (on form S) to

$$V_{\text{rms}} = \frac{1}{\sqrt{2}} V_m = \frac{1}{\sqrt{2}} \times 15 = 10.6 \text{ V}$$

.. Find the relationship between $\$P_1$, $\$P_2$, and $\$P_3$ using superposition.
This led to the formula based on the Kirchhoff's nodal rule!

- Determine suitable values for $\$R_1\$, \$R_2\$ and $\$R_3\$$

$$\begin{aligned} & \left\{ \frac{U_0}{R_1} + \frac{U_1}{R_2} + \frac{U_2}{R_3} \right\} - U_0 \\ & \left\{ \frac{U_0}{R_1} + \frac{U_1}{R_2} + \frac{U_2}{R_3} \right\} - U_0 \end{aligned}$$

 • What values for $\$R_1\$, \$R_2\$ and $\$R_3\$ from the 524 series can be used to do this?$$$

The formula can be rearranged, with all terms containing U_{O} on the left side:

$$\begin{aligned} \frac{U_{\text{O}}}{R_1} + \frac{U_{\text{O}}}{R_2} + \frac{U_{\text{O}}}{R_4} &= \frac{U_{\text{S}}}{R_1} + \frac{U_{\text{I}}}{R_4} \\ U_{\text{O}} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right) &= \frac{U_{\text{S}}}{R_1} + \frac{U_{\text{I}}}{R_4} \end{aligned}$$

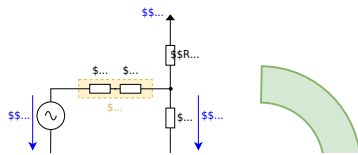
Using superposition, we create two separate circuits where one source is considered.

[illegible]

The last formula was just the result we also got by the superposition but by more thinking.

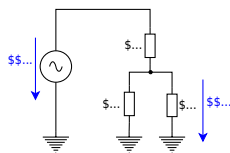
So, sometimes there is an easier way...

- Unluckily, there is no simple way to know before, what way is the easiest.
- Luckily, all ways lead to the correct result.



$$\begin{aligned} U_{\text{O}}^{(1)} &= U_{\text{S}} \cdot \frac{\{R_2 \parallel R_4\}}{R_1 + R_2 \parallel R_4} = U_{\text{S}} \cdot \frac{\left\{ \frac{R_2 R_4}{R_2 + R_4} \right\}}{R_1 + \left\{ \frac{R_2 R_4}{R_2 + R_4} \right\}} \\ &= U_{\text{S}} \cdot \frac{R_2 R_4}{(R_2 + R_4) + R_2 R_4} \\ &= U_{\text{S}} \cdot \frac{R_2 R_4}{R_1 R_2 + R_1 R_4 + R_2 R_4} \end{aligned}$$

Circuit 2 : only consider U_{I} , ignore U_{S}



$$\begin{aligned} U_{\text{O}}^{(2)} &= U_{\text{I}} \cdot \frac{\{R_1 || R_2\}}{R_4 + R_1 || R_2} = U_{\text{I}} \cdot \frac{\left\{ \frac{R_1 R_2}{R_1 + R_2} \right\}}{R_4 + \left\{ \frac{R_1 R_2}{R_1 + R_2} \right\}} \\ &= U_{\text{I}} \cdot \frac{\{R_1 R_2\}}{(R_1 + R_2) + R_1 R_2} \\ &= U_{\text{I}} \cdot \frac{\{R_1 R_2\}}{R_4 R_1 + R_4 R_2 + R_1 R_2} \end{aligned}$$

Superposition: Let's sum it up!

These two intermediate voltages for the single sources have to be summed up as $U_{\text{O}} = U_{\text{O}}^{(1)} + U_{\text{O}}^{(2)}$.

When deeper investigated, one can see that the denominator for both $U_{\text{O}}^{(1)}$ and $U_{\text{O}}^{(2)}$ is the same.

We can also simplify further when looking at often-used sub-terms (here: R_2)

$$\begin{aligned} U_{\text{O}} &= \left\{ \frac{1}{R_4 R_1 + R_4 R_2 + R_1 R_2} \right\} \\ &\cdot (U_{\text{S}} \cdot R_2 R_4 + U_{\text{I}} \cdot R_1 R_2) \\ &= U_{\text{S}} \cdot R_2 R_4 + U_{\text{I}} \cdot R_1 R_2 \\ &\cdot \left\{ \frac{1}{R_4 R_1 + R_4 R_2 + R_1 R_2} \right\} \\ &= U_{\text{S}} \cdot R_2 R_4 + U_{\text{I}} \cdot R_1 R_2 \cdot \frac{1}{R_4 R_1 + R_4 R_2 + R_1 R_2} \end{aligned}$$

The formula (1) is the general formula to calculate the output voltage U_{O} for a changing input voltage U_{I} , where the supply voltage U_{S} is constant.

Now, we can use the requested boundaries:

1. For the minimum input voltage $U_{\text{I}} = -15 \text{ V}$, the output voltage shall be $U_{\text{O}} = 0 \text{ V}$
2. For the maximum input voltage $U_{\text{I}} = +15 \text{ V}$, the output voltage shall be $U_{\text{O}} = 3.3 \text{ V}$

This leads to two situations:

Situation I : $U_{\text{I,min}} = -15 \text{ V}$ shall create $U_{\text{O,min}} = 0 \text{ V}$

We put $U_{\text{A}} = 0 \text{ V}$ in the formula (1):
$$0 = U_{\text{S}} \cdot R_4 + U_{\text{I,min}} \cdot R_1 - U_{\text{I,min}} \cdot R_1 + U_{\text{S}} \cdot R_4 \cdot \frac{R_1}{R_4} = -\frac{U_{\text{S}}}{U_{\text{I,min}}} = k_{14} \quad \text{tag 2}$$

So, with formula (2), we already have a relation between R_1 and R_4 .

Yeah $\square$

The next step is situation 2

Situation II : $U_{\text{I,max}} = +15 \text{ V}$ shall create $U_{\text{O,max}} = 3.3 \text{ V}$

We use formula (2) to substitute $R_1 = k_{14} \cdot R_4$ in formula (1), and:
$$U_{\text{O,max}} \cdot (k_{14} \cdot \frac{R_4^2}{R_2} + R_4 + k_{14} \cdot R_4) = U_{\text{S}} \cdot R_4 + U_{\text{I,max}} \cdot k_{14} \cdot R_4 \cdot \frac{R_4}{R_2} + U_{\text{O,max}} \cdot (k_{14} \cdot \frac{R_4}{R_2} + 1 + k_{14}) = U_{\text{S}} + U_{\text{I,max}} \cdot k_{14} \cdot \frac{R_4}{R_2} + 1 + k_{14} = \frac{U_{\text{S}} + U_{\text{I,max}} \cdot k_{14}}{U_{\text{O,max}}} \cdot \frac{R_4}{R_2} = \frac{U_{\text{S}} + U_{\text{I,max}} \cdot k_{14}}{U_{\text{O,max}}} \cdot \frac{R_4}{R_2} - (1 + k_{14}) \cdot \frac{R_4}{R_2} = \frac{U_{\text{S}} + U_{\text{I,max}} \cdot k_{14}}{k_{14} \cdot U_{\text{O,max}}} - \frac{1 + k_{14}}{k_{14}} \quad \text{tag 3}$$

So, another relation for R_4 and R_2 . $\square$

So, to get values for the relations, we have to put in the values for the input and output voltage conditions. For k_{14} we get by formula (2):
$$k_{14} = \frac{R_1}{R_4} = -\frac{5 \text{ V}}{-15 \text{ V}} = \frac{1}{3} \quad \text{end{align*}}$$

This value k_{14} we can use for formula (3):
$$\frac{R_4}{R_2}$$

$$\frac{1}{R_2} = \frac{5 \text{ V} + 15 \text{ V} \cdot \frac{1}{3}}{3.3 \text{ V} \cdot \frac{1}{3}} - \frac{1 + \frac{1}{3}}{\frac{1}{3}} \quad \& \approx 5.09 \text{ k}\Omega$$

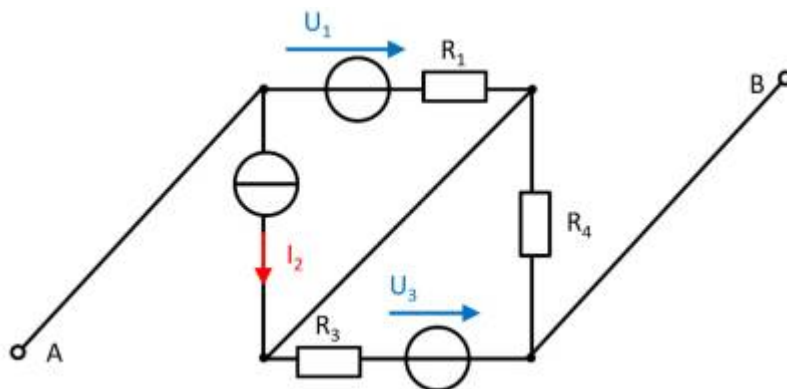
We could now - theoretically - arbitrarily choose one of the resistors, e.g., R_2 , and then calculate the other two.

But we must consider another boundary, a boundary for R_S . The maximum voltage and the maximum current are given for the sensor. By this, we can calculate R_S :

$$R_S = \frac{U_{\text{OC}}}{I_{\text{SC}}} = \frac{U_{\text{S,max}}}{I_{\text{S,max}}} = \frac{15 \text{ V}}{1 \text{ mA}} = 15 \text{ k}\Omega$$

Therefore, $R_4 = R_S + R_3$ must be larger than this.

Aufgabe 4.5.2: open circuit voltage via superposition (exam task, approx. 12 % of a 60-minute exam, WS2020)



A circuit is given with the following parameters

$$R_1 = 5 \text{ }\Omega$$

$$U_1 = 2 \text{ V}$$

$$I_2 = 1 \text{ A}$$

$$R_3 = 20 \text{ }\Omega$$

$$U_3 = 8 \text{ V}$$

$$R_4 = 10 \text{ }\Omega$$

Determine the open circuit voltage between A and B using the principle of superposition.

Tips

- What do the individual circuits look like, by which the effects of the individual sources can be calculated?
Which equivalent resistor must be used to replace a current or voltage source when calculating the individual effects?
- Where are the open-circuit voltages applied when looking at the individual

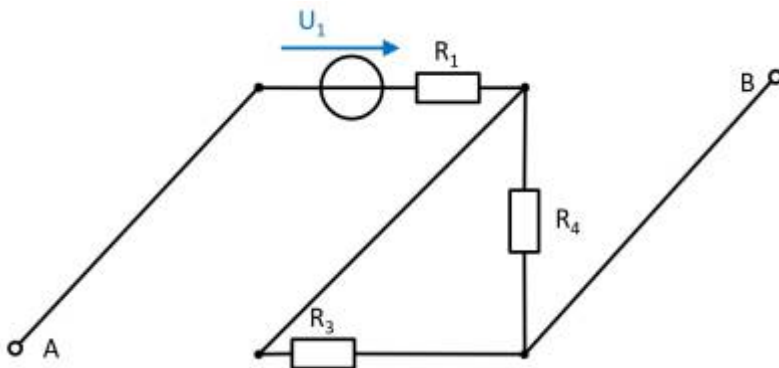
components?

Solution

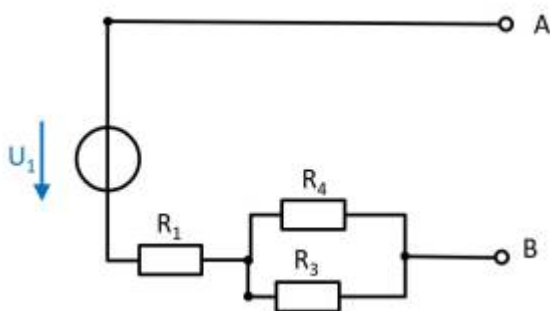
First, the individual circuits must be created, from which the effect of the individual sources between points A and B can be determined.

(Voltage) source U_1

- substitute the current source I_2 with a short-circuit
- substitute the voltage source U_3 with an open circuit



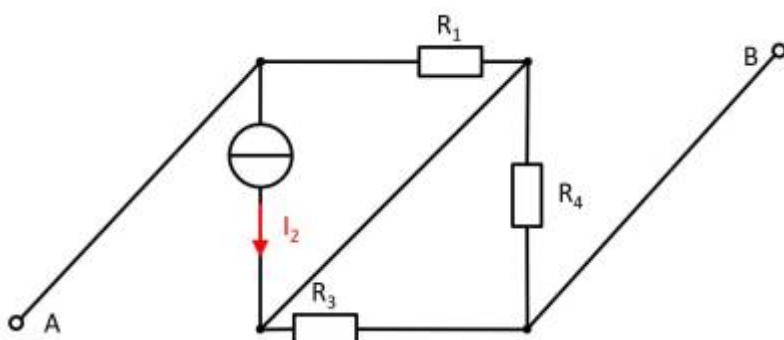
The components can be moved in order to understand the circuit a bit better.



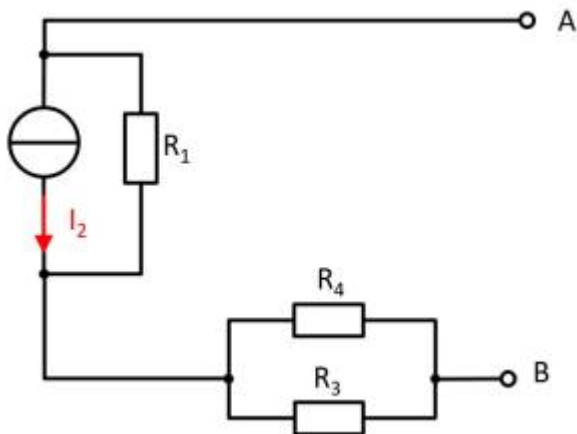
For the open circuit, no current is flowing through any resistor. Therefore, the effect is:
 $U_{AB,1} = U_1$

(current) source I_2

- substitute the voltage source U_1 with an open circuit
- substitute the voltage source U_3 with an open circuit



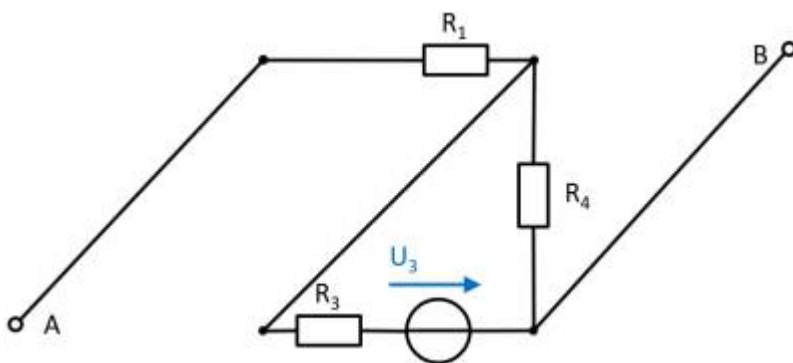
Also here, the components can be shifted for a better understanding:



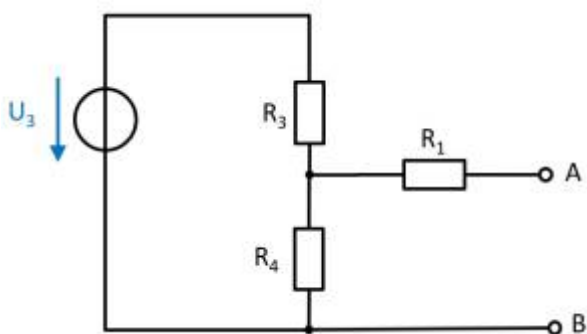
Here, the current source I_2 creates a voltage drop $U_{AB,2}$ on the resistor R_2 :
 $U_{AB,2} = -R_1 \cdot I_2$

(Voltage) source U_3

- substitute the voltage source U_1 with an open circuit
- substitute the current source I_2 with a short-circuit



Again, rearranging the circuit might help for an understanding:



In this case, between the unloaded outputs A and B there will be an unloaded voltage divider given by R_3 and R_4 . On R_1 there is no voltage drop since there is no current flow out of the unloaded outputs.

Therefore:

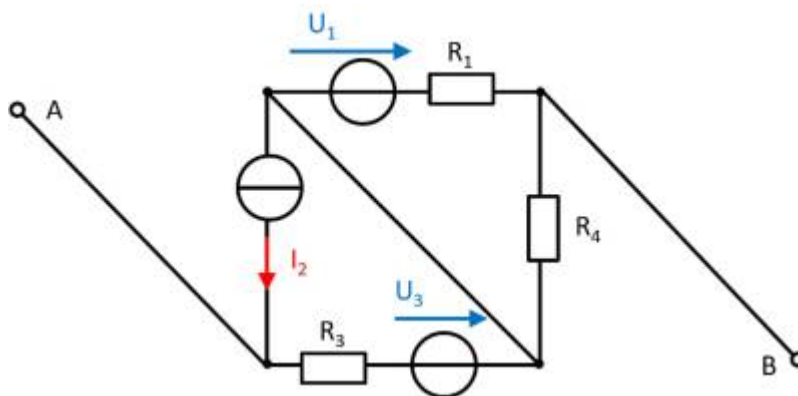
$$U_{AB,3} = \frac{R_4}{R_3 + R_4} \cdot U_3$$

resulting voltage

$$U_{\text{AB}} = U_1 - R_1 \cdot I_2 + \frac{R_4}{R_3 + R_4} \cdot U_3$$

Final value

$$U_{\text{AB}} = 2 \text{ V} - 5 \Omega \cdot 1 \text{ A} + \frac{10 \Omega}{20 \Omega + 10 \Omega} \cdot 8 \text{ V} \\ U_{\text{AB}} = -0.333 \text{ V} \rightarrow -0.3 \text{ V}$$

Exercise 4.5.3 -Variation: open circuit voltage via superposition (exam task, approx. 12 % of a 60-minute exam, WS2020)


A circuit is given with the following parameters

$$R_1 = 5 \Omega$$

$$U_1 = 2 \text{ V}$$

$$I_2 = 1 \text{ A}$$

$$R_3 = 20 \Omega$$

$$U_3 = 8 \text{ V}$$

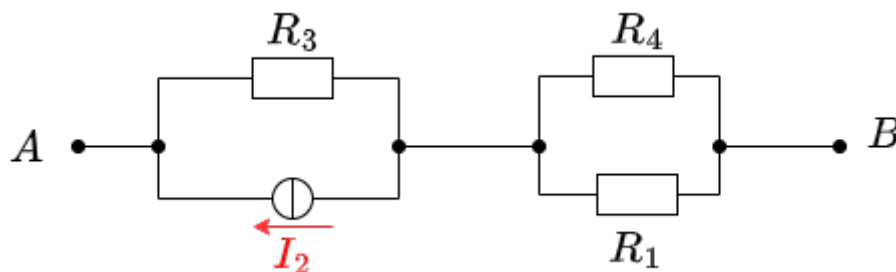
$$R_4 = 10 \Omega$$

Determine the open circuit voltage between A and B using the principle of superposition.

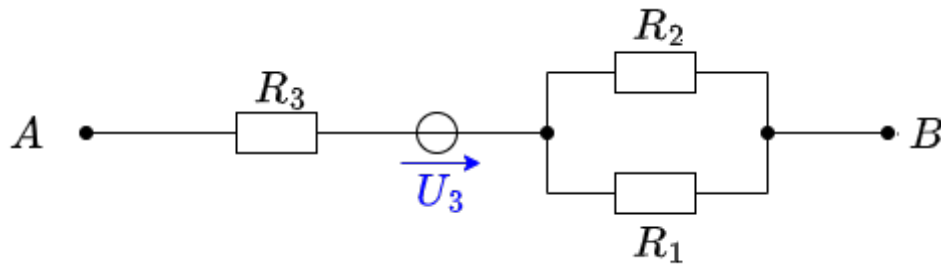
Solution

Case 1: For this case is $I_2 = 0 \text{ A}$ and $U_3 = 0 \text{ V}$. The voltage is at R_4 .

$$U_{\text{AB},1} = \frac{R_4}{R_1 + R_4} U_1 = \frac{10\,\Omega}{5\,\Omega + 10\,\Omega} \cdot 2\,\text{V} = 1.33\,\text{V}$$
 Case 2: For this case is $U_1 = 0\,\text{V}$ and $U_3 = 0\,\text{V}$. The voltage is at R_3 .



$$U_{\text{AB},2} = R_3 I_2 = 20\,\Omega \cdot 1\,\text{A} = 20\,\text{V}$$
 Case 3: For this case is $U_1 = 0\,\text{V}$ and $I_2 = 0\,\text{A}$. The voltage comes from the source U_3 .



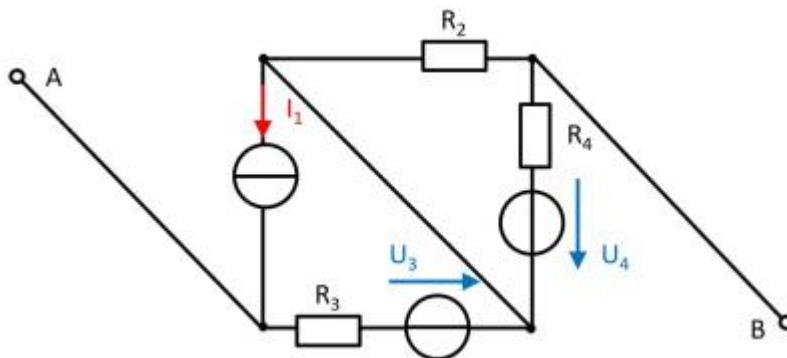
$$U_{\text{AB},3} = 8 \text{ V}$$
 Superposition means adding the voltages of all three cases.

$$U_{\text{AB}} = U_{\text{AB},1} + U_{\text{AB},2} + U_{\text{AB},3} = 1.33 \text{ V} + 20 \text{ V} + 8 \text{ V}$$

Final value

$$U_{\text{AB}} = 29.333... \text{ V} \rightarrow 29.3 \text{ V}$$

Exercise 4.5.4 - Variation: open circuit voltage via superposition (exam task, approx. 12 % of a 60-minute exam, WS2020)



A circuit is given with the following parameters

$$I_1 = 2 \text{ A}$$

$$R_2 = 5 \text{ } \Omega$$

$$R_3 = 20 \text{ } \Omega$$

$$U_3 = 1 \text{ V}$$

$$R_4 = 10 \text{ } \Omega$$

$$U_4 = 3 \text{ V}$$

Determine the open circuit voltage between A and B using the principle of superposition.

1)

sometimes such connections are called "fake nodes"

2)

Here we now depart from the previous electrotechnical notion of node (= connecting more than 2 branches). The mathematical notion of a node does not have this restriction

3)

$x=x+1$ is not meant mathematically, but procedurally as in the programming language C

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