# Exam Winter Semester 2022

## Student Group

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# Exam Winter Semester 2022

**Additional permitted Aids**

- non-programmable calculator,
- formulary (2 DIN A4 pages)

**Hits**

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

## Tasks

**Exercise E1 Resistance of a Wire by Resistivity**
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

A heating element made of a Nichrome wire with a round cross-section is used in an electric oven.
Nichrome is a common Nickel-Chrome alloy for heating elements.
The Nichrome wire has a resistivity of $1.10\cdot 10^{-6} ~\Omega \rm{m}$.
The heating element is $3 ~\rm{m}$ long and has a diameter of $3.57 ~\rm{mm}$.

1. Calculate the resistance $R$ of the heating element.

Solution

\begin{align*} R &= \rho \cdot \frac{l}{A} && | \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} && \\ R &= 1.10\cdot 10^{-6} ~\Omega \rm{m} \cdot \frac{4 \cdot 3~\rm{m}}{(3.57\cdot 10^{-3}~\rm{m})^2 \cdot \pi} && \\ \end{align*}

Result

\begin{align*} R=&1.10-33 ~\Omega \end{align*}

2. The heating element is used to heat the oven to a temperature of $180 ~°\rm{C}$. For this, a power dissipation (= heat flow) of $P=40 ~\rm{W}$ is necessary.
Calculate the current $I$ needed to operate it.

Solution

\begin{align*} P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 ~\rm{W}}{0.33 ~\Omega}} \end{align*}

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resistivity, power, exam ee1 ws2022

**Exercise E2 Temperature-dependent Resistance**
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

A thermistor is used in a refrigeration system. The thermistor has a resistance of $10 ~\rm{k}\Omega$ at $+25 ~°\rm{C}$.
Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$
The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

2. [illegible] Give an explanation for your answer.

3. Additionally, explain [illegible]

Solution

$$\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C} ) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}$$

Result

$$\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}$$

The power of the electrical energy out of the circuit and generate heat. Therefore, a resistance to the heat flow.

Resistors transfer electric $P = U \cdot I = R \cdot I^2 = {{U^2}\over{R}}$ is equivalent to might heat up the refrigeration system.

Therefore, with constant $U$ and increasing $R$ the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

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temperature dependent resistance, power, heat, exam ee1 ws2022

**Exercise E3 Pure Resistor Network Simplification**
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following circuit with $R_1=200 ~\Omega$, $R_2=R_3=100 ~\Omega$ and the switch $S$ is given.

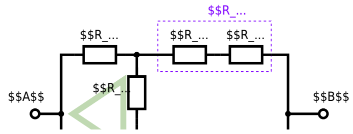
2. The switch shall now be closed. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution

$$\begin{align*} R_{\rm eq} &= 142.8 ~\Omega \\ \end{align*}$$

Result

Now a wye-delta transformation is necessary.

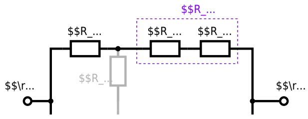



Since $R_2=R_3$ and based on the equations for the transformation, the transformed $R_Y$ is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution




The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega )&&||(100 ~\Omega + 100 ~\Omega ) &&\\ R_{\rm eq} &= (500 ~\Omega )&&||(200 ~\Omega )&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}

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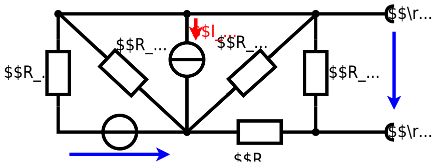
network simplification, exam ee1 ws2022

**Exercise E4 Equivalent linear Source**
**(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

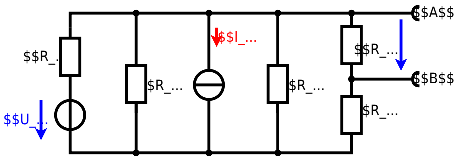
\begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}



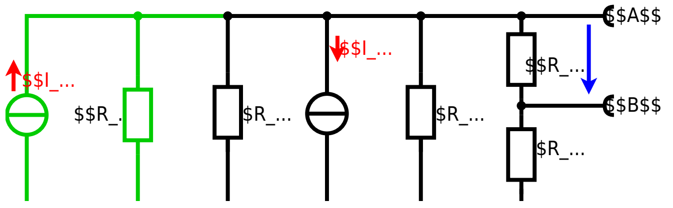

Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$. \begin{align*} R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

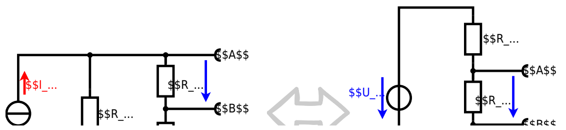
The best thing is to re-think the wiring like rubber bands and adjust them:




The linear voltage source of \$U_2\$ and \$R_1\$ can be transformed into a current source \$I_2={{U_2}\over{R_1}}\$ and \$R_1\$:




Now a lot of them can be combined. The resistors $R_1$, $R_3$, $R_5$ are in parallel, like also $I_2$ and $I_4$: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:



Here, the $U_{24}$ is calculated by $I_{24}$ as the following: \begin{align*} U_{24}

&= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit, there is a voltage divider given by $R_{135}$, $R_6$, and $R_7$.
Therefore the voltage between $A$ and $B$ is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance $R_i$ the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || ( R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| ( 7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

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dc network analysis, pure resistor network simplification, delta wye transformation, exam ee1 ws2022

**Exercise E5 Charging Capacitors**
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

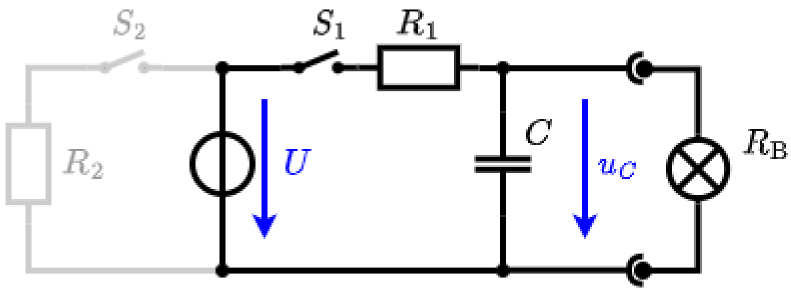
[illegible] The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch $S_1$ is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Result / Solution [illegible]

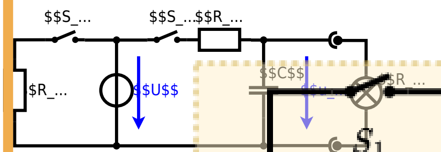
Hint: To solve this, first create an equivalent linear voltage source from $U$, $R_1$, and $R_\rm B$.

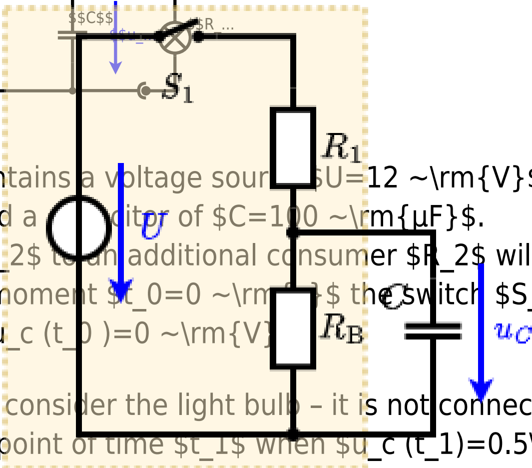
[illegible]

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting $R_2$.





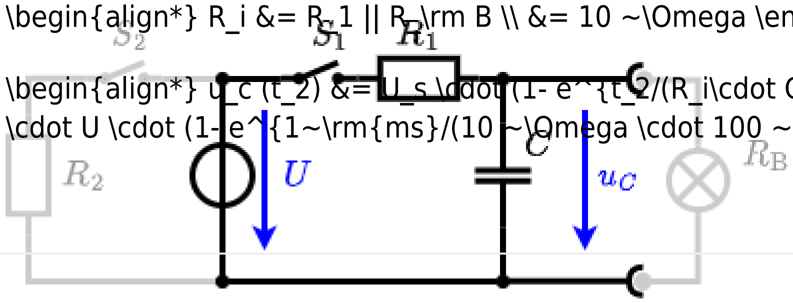

The circuit contains a voltage source $U=12 ~\rm{V}$, a switch $S_1$, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=100 ~\rm{µF}$.
The switch $S_2$ to an additional consumer $R_2$ will be considered to be open for the first tasks. At the moment $t_0=0 ~\rm{s}$ the switch $S_1$ is closed, the voltage across the capacitor is $u_c (t_0 )=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit. Calculate the point of time $t_1$ when $u_c (t_1)=0.5\cdot U$.

Solution

An equivalent linear voltage source can be given with $U$, $R_1$, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit). \begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}

\begin{align*} u_c (t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}




So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to $t$ \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}

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charging capacitors, dc network analysis, pure resistor network simplification, delta wye transformation, exam ee1 ws2022

**Exercise E6 Analyzing complex Impedances**
**(written test, approx. 14 % of a 60-minute written test, WS2022)**

components. ($R$ and $\underline{X}_1$) shall be given.

$$\underline{Z} = \left( {{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega$$

Solution

1. Calculate the physical values of the two components.

Solution

$$C = 103 ~\rm{µF}$$

$$|\underline{I}| = 10.67 ~\rm{A} \qquad \varphi_i = -87.06°$$

Solution

$$\underline{Z} = \left( {{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega = \left( {{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4{\rm j}}} + 5{\rm j} \right) ~\Omega = \left( {{2}\over{9+16}} \cdot (3-4{\rm j}) + 5{\rm j} \right) ~\Omega = 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega = R + {\rm j} X_L$$

With the complex part comes the physical value:

$$X_L = \omega L \qquad L = {{X_L}\over{2\pi \cdot f}} = {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}}$$

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.

Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$

$$X_C = {{1}\over{\omega \cdot C}} = X_L \qquad C = {{1}\over{\omega \cdot X_L}} = {{1}\over{2\pi \cdot 300 ~\rm{Hz} \cdot 4.68 ~\Omega}}$$

$$\underline{I} = {{\underline{U}}\over{\underline{Z}}} = {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} = {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }}$$

The absolute value $|\underline{I}|$ can be calculated as

$$|\underline{I}| = {{|\underline{U}|}\over{|\underline{Z}|}} = {{50 ~\rm{V}}\over{| 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega |}} = {{50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}}$$

The phase $\varphi_i$ can be calculated as

$$\varphi_i = \arctan \left( {{\Im()}\over{\Re()}} \right) = \arctan \left( {{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right)$$

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complex impedance, exam ee1 ws2022

**Exercise E7 Impedances at different Frequencies**
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

1. A resistor $R_1$ shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

Solution

$$R_1 = 1.00 ~\Omega$$

Solution

$$R_2 = 10.0 ~\Omega$$

A series circuit means that the current is constant on every component.
The equivalent impedance for $R$ and $L$ combined is given by: \begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\&= R_2 + {\rm j} \cdot \omega L \end{align*} Since ${\rm j} \cdot \omega L $ is perpendicular to $R_2$ this can be simplified to: \begin{align*} \left| {{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left( {{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}
This can be rearranged to get $R_2$: \begin{align*} R_2 &= \sqrt{ \left( {{U }\over{I }} \right)^2 - X_{L2}^2 } \\ &= \sqrt{ \left( {{1~{\rm V}}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450 ~{\rm kHz} \cdot 4.7 ~{\rm µH})^2 } \\ \end{align*}

Parallel circuit means that the voltage is the same on $R_3$ and $C_3$: \begin{align*} R_3 \cdot \underline{I}_{3R} &= \underline{X}_{3C} \cdot \underline{I}_{3C} \end{align*} So it gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since $R_3$ is perpendicular to $ {\rm j}\cdot {X}_{3C}$, too).
Therefore, the resulting current of the parallel circuit is given as: \begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ {I}_{3C} &= \sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2} \end{align*}
Back to the first formula: \begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}

\begin{align*} X_{C1} &= {{1}\over{2\pi\cdot f \cdot C_1}} \\&= {{1}\over{2\pi\cdot 40 ~{\rm MHz} \cdot 10 ~{\rm nF}}} \end{align*}

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complex impedance, exam ee1 ws2022

**Exercise E8 Complex Impedance Circuit**
**(written test, approx. 15 % of a 60-minute written test, WS2022)**

A circuit is supplied by a sinusoidal voltage source $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ with an internal resistance of $10 ~\Omega$.
This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

1. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.
2. Calculate the complex impedances $\underline{Z}_L$, $\underline{Z}_C$ and $\underline{Z}$.
3. Calculate the current.
4. Draw the pointer diagram of $\underline{Z}$, $\underline{Z}_L$, $\underline{Z}_C$ and $R$ in a diagram. Choose an appropriate scaling factor and write it down.

Solution

Result

\begin{align*} Z_L &= 31 ~\Omega \\ Z_C &= 48.2 ~\Omega \\ Z &= 19.8 ~\Omega \end{align*}

\begin{align*} I = 107.3 ~{\rm mA} \end{align*}

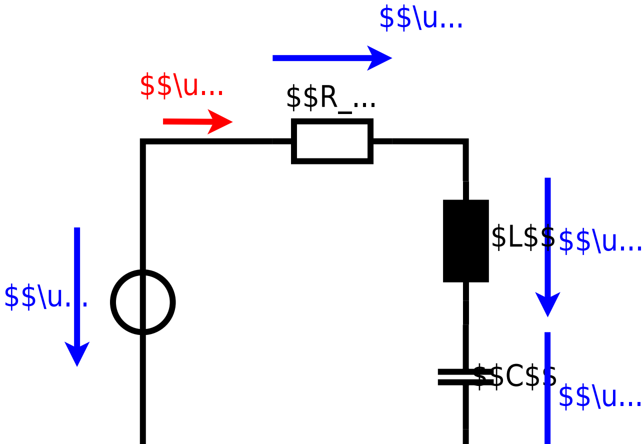
\begin{align*} Z &= {{\hat{U}}\over{\hat{I}}} \\ \hat{I} &= {{\hat{U}}\over{Z}} \\ \end{align*}
With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$: \begin{align*} I &= {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ &= {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}} \\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}\\ \end{align*}

\begin{align*} Z_L &= 2\pi \cdot f \cdot L \\ &= 2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}\\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}}}\\ \end{align*}

\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}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