Exam Winter Semester 2022

Student Group

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Additional permitted Aids

- non-programmable calculator,
- formulary (2 DIN A4 pages)

Hits

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

The heating element is of a Nichrome wire with a temperature of $180 ~\rm{°C}$. Electric power dissipation (= heat flow) of $P=40 ~\rm{W}$ is necessary.

Nichrome is a common Nickel-Chromium alloy for heating elements.

The Nichrome wire has a resistivity of $1.10\cdot 10^{-6} ~\Omega \rm{m}$.

The heating element is $3 ~\rm{m}$ long and has a diameter of $3.57 ~\rm{mm}$.

1. Calculate the resistance R of the heating element.

Result

$$\begin{align*} R = 1.10\cdot 10^{-3}~\Omega \end{align*}$$

2. Calculate the current I needed for operation.

Solution

$$\begin{align*} P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 ~\rm{W}}{0.33 ~\Omega}} \end{align*}$$

Solution

$$\begin{align*} R &= \rho \cdot \frac{l}{A} && | \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} && \\ R &= 1.10\cdot 10^{-6} ~\Omega \rm{m} \cdot \frac{4 \cdot 3~\rm{m}}{(3.57\cdot 10^{-3}~\rm{m})^2 \cdot \pi} && \\ \end{align*}$$

resistivity, power, exam ee1 ws2022

Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. Additionally, explain 2. Give a 3. Fantier the the cold has a resistance of $10~\rm{k}\Omega$ at $*25~^\circ\rm{C}$.

Result

Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$

Result The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}

The power transfer is the circuit and generate heat RThis fore, a equivalents to the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C}) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}

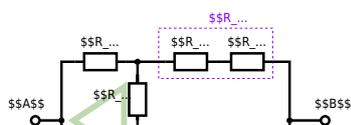temperature dependent resistance, power, heat, exam ee1 ws2022

Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall now be $R_1=200 ~\Omega$, $R_2=R_3=100~\Omega$ and the switch between $\rm A$ and $\rm B$.

Result

Solution

\begin{align*} R_{\rm eq} &= 132.8 ~\Omega \\ \end{align*}

Now a wye-delta transformation is necessary.

$$R_{...}$$

$$R_{...}$$ $$R_{...}$$ $$R_{...}$$

$$A$$ $$R_{...}$$ $$B$$

Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
\begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series:
\begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution
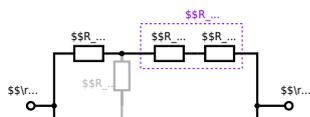
The equivalent resistor is given by a parallel configuration of resistors in series:
\begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega)&&||(100 ~\Omega + 100 ~\Omega) &&\\ R_{\rm eq} &= (500 ~\Omega)&&||(200 ~\Omega)&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}

network simplification, exam ee1 ws2022

Exercise E4 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

\begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}
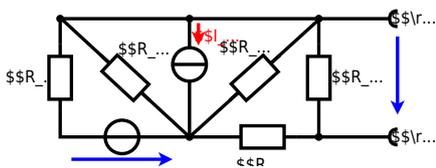
Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$.
\begin{align*}
R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*}
Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:

$$R\_$$

$R\_...$

$$I\_...$$

$R\_...$

$$R\_...$$

$$A$$

$$B$$

$R\_...$

$$U\_$$
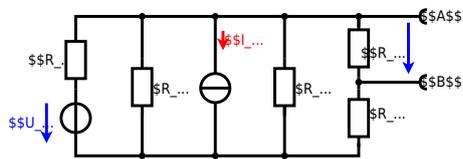
The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and R_1:

$$\begin{matrix} & & & & \circ\ \$\$A\$\$ \\ \uparrow\$\$I\_... & \$\$R\_ & & \downarrow\$\$I\_... & \$\$R\_... \downarrow \\ & \$\$R\_ & \$R\_... & \$R\_... & \circ\ \$\$B\$\$ \\ & & & & \$R\_... \end{matrix}$$
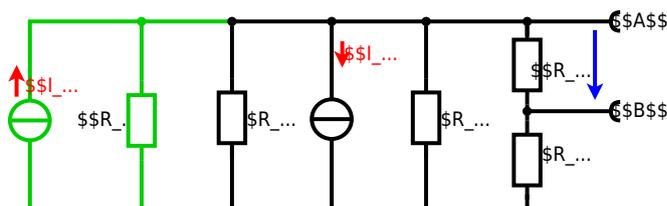
Now a lot of them can be combined. The resistors R_1, R_3, R_5 are in parallel, like also I_2 and I_4: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:

$$\begin{matrix} & & \circ\$\$A\$\$ & & & \$\$R\_... \\ \uparrow\$\$I\_... & \$\$R\_... \downarrow & & \iff & \$\$U\_\downarrow & \circ\$\$A\$\$ \\ & \$\$R\_... & \circ\$\$B\$\$ & & & \$\$R\_... \downarrow \\ & & & & & \circ\$\$B\$\$ \end{matrix}$$

Here, the U_{24} is calculated by I_{24} as the following: \begin{align*} U_{24}

$$&= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}$$

On the right side of the last circuit, there is a voltage divider given by R_{135}, R_6, and R_7.
Therefore the voltage between A and B is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || (R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| (7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

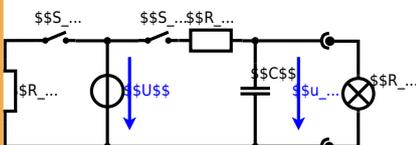dc network analysis, pure resistor network simplification, delta wye transformation, exam ee1 ws2022

Exercise E5 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The equivalent voltage source is now derived by R_1, R_5... charging capacitor again...

The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of C ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.
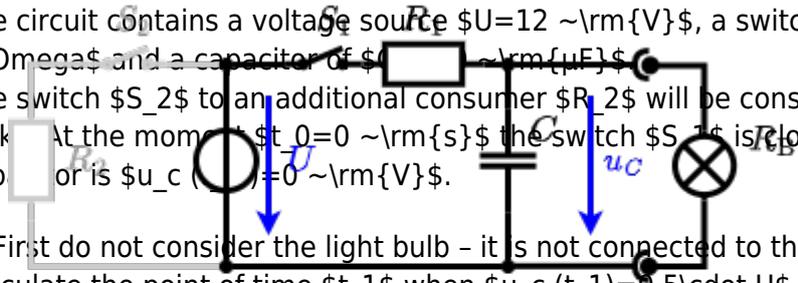
1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= \ln(0.5) \\ t &= \tau \cdot \ln(0.5) \\ t &= R_1 \cdot C \cdot \ln(0.5) \end{align*}

An equivalent linear voltage source can be given with U, R_1, and $R_\rm B$ as seen in yellow.
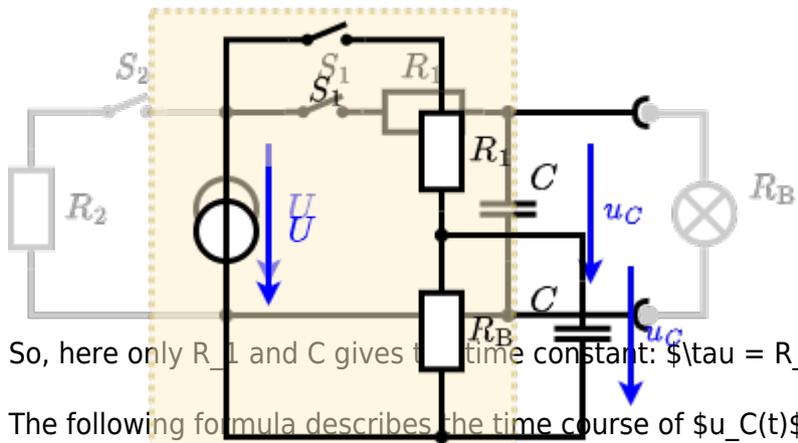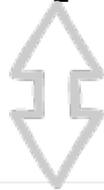Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit).
\begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}

\begin{align*} u_c (t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}

[charging capacitors](#), [dc network analysis](#), [pure resistor network simplification](#), [delta wye transformation](#), [exam ee1 ws2022](#)

Exercise E6 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phase voltage source value $\underline{U} = 50(x)~\rm{V}$ if $f = 30 ~\rm{kHz}$ describes (a.) to the Result components.(R and $\underline{X}_1$) shall be given.
After analysis, the following formula for the impedance can be extracted \begin{align*} in phase. Calculate the $\underline{Z}_2$ power \left({{2}\over{3 + 4~\rm j}} + 5{\rm j} \right) \Omega \end{align*}

Calculate the physical values of the two components.

Solution

\begin{align*} C = 103 ~\rm{µF} \end{align*}

\begin{align*} R &= 0.24 ~\Omega & \varphi_i &= -87.06° \end{align*}

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance.

Therefore, the component must be a capacitor with the same absolute value of impedance $\underline{X} = \underline{X}_C$.

\begin{align*} \underline{Z} &= \left({{2}\over{3+4\,{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{3+4\,{\rm j}}}\cdot{{3-4\,{\rm j}}\over{3-4\,{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2\cdot(3-4\,{\rm j})}\over{9+16}} + 5{\rm j} \right) ~\Omega \\ &= \left(0.24 - 0.32{\rm j} + 5{\rm j} \right) ~\Omega \\ &= 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega \\ &= R + {\rm j}\,X_L \end{align*}

With the complex part comes the physical value: \begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \\ \end{align*}

The phase φ_i can be calculated as \begin{align*} \varphi_i &= \arctan \left({{\Im()}\over{\Re()}} \right) \\ &= \arctan \left({{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

complex impedance, exam ee1 ws2022

Exercise E7 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

A R series circuit values $L=34.7 ~\rm{µH}$ used in AC voltage circuits of $U = 3.0 ~\rm{V}$, with $f=450 ~\rm{kHz}$ which generates a current $I=5.0 ~\rm A$ through R_3 resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

Solution

\begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}

\begin{align*} R_2 &= 70.0 ~\Omega \\ \end{align*}

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by \begin{align*} \underline{Z} &= R_2 + \underline{X}_{C1} \\ &= R_2 + {{1}\over{{\rm j} \cdot \omega \cdot C_1}} \\ &= {{1}\over{2\pi \cdot f \cdot C_1}} \\ \end{align*}

Since $\underline{X}_{C}$ is perpendicular to R_2, this can be simplified to: \begin{align*} |\underline{U}| &= R_2 \\ |\underline{X}_{L2}|\cdot{\rm j} \cdot \left({{U}\over{I}}\right)^2 &= {R_2}^2 + {X_{L2}}^2 \end{align*}

Therefore, the resulting current of the parallel circuit is given as: \begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ \end{align*}

This can be rearranged to: \begin{align*} R_2 &= \sqrt{ \left({{U}\over{I}}\right)^2 - {X_{L2}}^2 } \\ &= \sqrt{ \left({{21}\over{60~\rm mA}}\right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}

Back to the first formula: \begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot

{I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &=
{{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 -
|{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}

[complex impedance](), [exam ee1 ws2022]()

Exercise E8 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

A. Calculate the complex impedance and analyze the $\underline{Z}_L$, $\underline{Z}_C$ and $\underline{Z}_R$ of the voltage source $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$.

This linear source is connected with an inductor of $330 ~{\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

1. Draw the circuit diagram of the given electrical circuit. Label all components, voltages, and currents.

Solution

Result

\begin{align*} Z_L &= 1807.3 ~{\rm \Omega} \\ Z_C &= 48.2 ~\Omega \\ Z &= 19.8 ~\Omega \end{align*}

\begin{align*} Z &= {{\hat{U}}\over{\hat{I}}} \\ \hat{I} &= {{\hat{U}}\over{Z}} \\ \end{align*}

\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}\\ \end{align*}

With $\hat{I} = {{1}\over{\sqrt{2}}} \cdot \hat{U}$: \begin{align*} \hat{I} &= {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ &= {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.8 ~\Omega}}\cdot \\ \end{align*}

\begin{align*} Z_L &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}}}\\ \end{align*}

\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}

$$\u...$$

$$\u...$$

$$R_...$$

$L$$ $$\u...$$

$$\u...$$

$$C$$ $$\u...$$

[complex impedance](#), [exam ee1 ws2022](#)