

Exam Summer Semester 2024

Student Group

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Exam Summer Semester 2024

Additional permitted Aids

- non-programmable calculator,
- formulary (4 one-sided DIN A4 pages)

Hits

- The duration of the exam is 120 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Electrostatics I

(written test, approx. 8 % of a 120-minute written test, SS2024)

2. Which two charges are placed on the two spheres of the picture below? The value of the previous charge is $q_1 = 1 \text{ nC}$. Which value needs q_2 to have to get a resulting force of 0 N on q_1 ?

Path: $q_1 = 1 \text{ nC}$

- $q_2 = -1 \text{ nC}$

Path: $q_2 = -1 \text{ nC}$

- $\vec{F}_1 = \left(\begin{array}{c} 917 \cdot 10^{-6} \end{array} \right) \text{ N}$

In the beginning, the two spheres are separated by $r = 1 \text{ m}$. We can calculate the resulting magnitude of the

force $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_2}{r^2} = \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{(1 \cdot 10^{-9} \text{ C}) \cdot (-1 \cdot 10^{-9} \text{ C})}{(1 \text{ m})^2} = -917 \cdot 10^{-6} \text{ N}$

$$|\vec{F}_1| = |\vec{F}_2| \Rightarrow \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} = \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2}$$

$$|\vec{F}_1| = |\vec{F}_2| \Rightarrow \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} = \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2}$$

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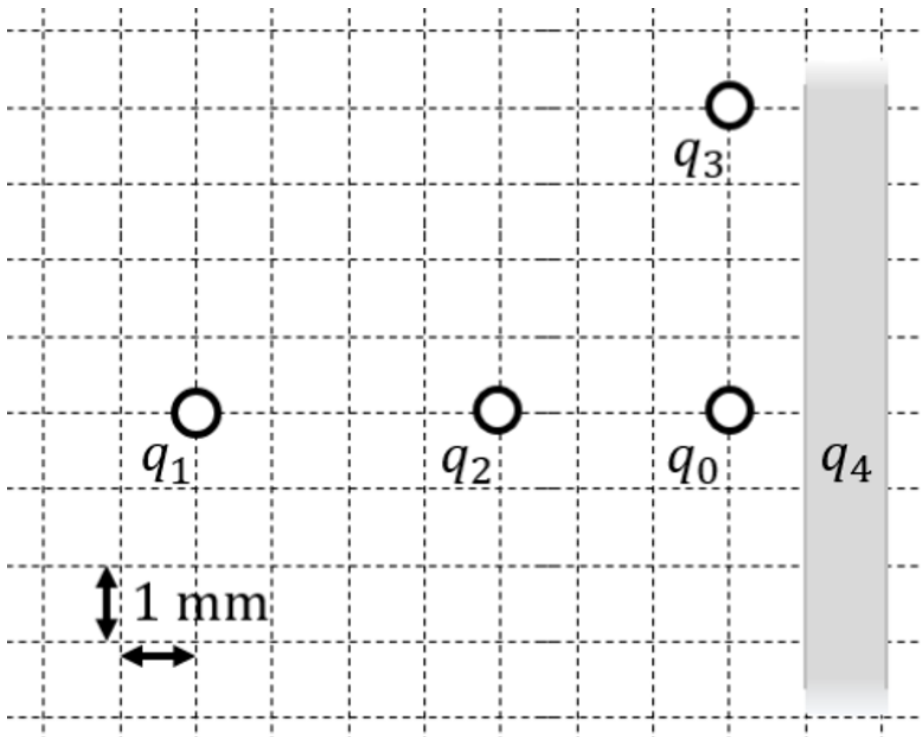
$$|\vec{F}_1| = |\vec{F}_2| \Rightarrow \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} = \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2}$$

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1. Calculate the single forces $\vec{F}_{01}$, $\vec{F}_{02}$, $\vec{F}_{03}$, on the charge q_0 !

Path

First, calculate the magnitude of the forces, like $\vec{F}_{01}$.

The force $\vec{F}_{01}$ is purely on the x -axis and therefore equal to

$$\begin{aligned} F_{01,x} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r_{01}^2} \\ &= \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{1 \cdot 10^{-9} \text{ C} \cdot 5 \cdot 10^{-9} \text{ C}}{(7 \cdot 10^{-3} \text{ m})^2} \\ &= 917. \cdot 10^{-6} \text{ Nm} \cdot \frac{(As)^2 \cdot Vm}{As \cdot m^2} = 917. \cdot 10^{-6} \text{ VAs} \cdot \frac{m}{m} = 917. \cdot 10^{-6} \text{ Ws} \cdot \frac{m}{m} \\ &= 917. \cdot 10^{-6} \text{ Nm} \quad \text{(to the right)} \end{aligned}$$

Similarly, we get for $\vec{F}_{02}$ and $\vec{F}_{03}$

$$\begin{aligned} \vec{F}_{02} &= F_{02,x} = -1997. \cdot 10^{-6} \text{ Nm} \quad \text{(to the right)} \\ \vec{F}_{03} &= F_{03,y} = -1123. \cdot 10^{-6} \text{ Nm} \quad \text{(to the top)} \end{aligned}$$

Exercise E1 Electrostatics I**(written test, approx. 8 % of a 120-minute written test, SS2024)**

2. Which net charge \$Q_0\$ of the test charge \$Q_0\$ is required to get \$F_{res} = 0\$ N? The values of the point charges are \$q_1 = 1 \text{ nC}\$, \$q_2 = 1 \text{ nC}\$, \$q_3 = 1 \text{ nC}\$, \$q_4 = 1 \text{ nC}\$, \$q_0 = 1 \text{ nC}\$. Which value needs \$E_4\$ to have to get a resulting force of \$0 \text{ N}\$ on \$q_0\$?

Path: $q_0 = -1 \text{ nC}$

- $q_1 = -5 \text{ nC}$

Path: $E_4 = 500 \text{ V/m}$

$$\vec{F}_1 = \left(\begin{array}{c} +17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_2 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

In the beginning, the are 5 point charges, we cannot calculate the resulting magnitude of the force.

$$\vec{F}_3 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_4 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_0 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_1 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_2 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_3 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_4 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_0 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_1 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_2 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

$$\vec{F}_3 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

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$$\vec{F}_0 = \left(\begin{array}{c} -17 \\ 0 \\ 0 \end{array} \right) \text{ nN}$$

1. Calculate the single forces $\vec{F}_{01}$, $\vec{F}_{02}$, $\vec{F}_{03}$, on the charge q_0 !

Path

First, set up a coordinate system. Here, I choose x pointing to the right (positive values to the right) and y pointing upwards (positive values upwards).

Then, calculate the magnitude of the forces, like $\vec{F}_{01}$ (force on q_0 from q_1).

The force $\vec{F}_{01}$ is purely on the x-axis and therefore equal to

$$F_{01,x} = F_{01} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_0}{r_{01}^2}$$

$$= \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{1 \cdot 10^{-9} \text{ C} \cdot 5 \cdot 10^{-9} \text{ C}}{(7 \cdot 10^{-3} \text{ m})^2}$$

$$= \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{5 \cdot 10^{-18} \text{ C}^2}{49 \cdot 10^{-6} \text{ m}^2}$$

$$= \frac{1}{4\pi \cdot 8.854 \cdot 10^{-12} \text{ As/Vm}} \cdot \frac{5 \cdot 10^{-12} \text{ C}^2}{49 \text{ m}^2}$$

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$917. \cdot 10^{-6} \frac{\text{As}^2 \cdot \text{Vm}}{\text{As} \cdot \text{m}^2} = 917. \cdot 10^{-6} \frac{\text{VAs}}{\text{m}} = 917. \cdot 10^{-6} \frac{\text{Ws}}{\text{m}}$
 $\parallel \&= +917. \mu\text{N}$

Since both q_0 and q_1 have the same sign for their charges, they are repelling each other. Therefore, The force $\vec{F}_{01}$ points to the right (positive value).

Similarly, we get for $\vec{F}_{02}$ and $\vec{F}_{03}$

$$\vec{F}_{02} = F_{02,x} \&= -1997. \mu\text{N} \parallel \vec{F}_{03} = F_{03,y} \&= -1123. \mu\text{N}$$

Since q_0 and q_2 have the different sign for their charges, they are attract each other. Therefore, The force $\vec{F}_{02}$ points to the left (negative value). Since q_0 and q_3 have the different sign for their charges, they are attract each other. Therefore, The force $\vec{F}_{03}$ points downwards (negative value).

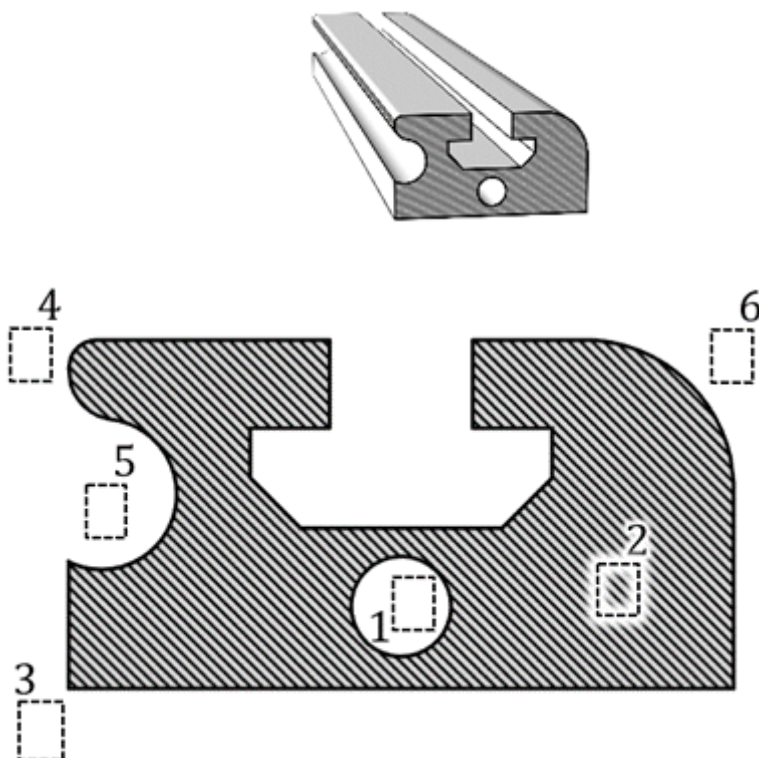
Exercise E1 Electrostatics II

(written test, approx. 10 % of a 120-minute written test, SS2024)

You must analyze an aluminum profile for usage in an environment critical for electrostatic discharge.

The figure on the right shows the cross-section of the aluminum element (hatched). During the application, it might get charged up. All areas in white consist of air (= dielectric).

Six designated areas are shown by dashed frames and numbers n, which are partly inside the object.



Arrange the designated areas clearly according to ascending field strengths $|\vec{E}_n|$ (absolute magnitude)! Indicate also, if designated areas have quantitatively the same field strength.

Result

$$|E_1|=|E_2|=0 < |E_5| < |E_6| < |E_4| < |E_3|$$

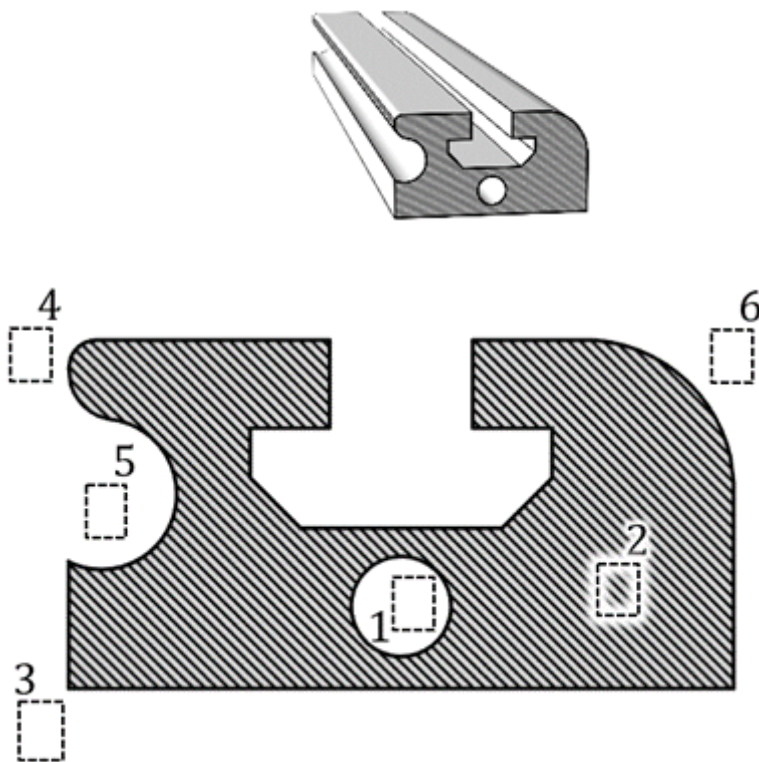
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Result

$$|E_1|=|E_2|=0 < |E_5| < |E_6| < |E_4| < |E_3|$$

Exercise E1 Capacitor

(written test, approx. 12 % of a 120-minute written test, SS2024)

0. Calculate the capacitance of the capacitor described by the circuit diagram and the supply voltage. The result is applied.

The contaminant has $\epsilon_{\text{r,c}} > \epsilon_{\text{r,air}}$, while the distance between the plates remains the same. Give a generalized formula

Path 1
 $C_2 = f(A, d, x, \epsilon_{\text{r,c}}, \epsilon_{\text{r,air}})$.
 $Q = \frac{\epsilon_0 \epsilon_{\text{r,c}} A}{d-x} U + \frac{\epsilon_0 \epsilon_{\text{r,air}} A x}{d} U$
 There are two ways now. Either: $Q = C U = 1.1 \cdot 10^{-6} \text{ F} \cdot 3.3 \text{ V} = 3.6322 \cdot 10^{-6} \text{ C}$ Or: $Q = D A = 146 \cdot 10^{-6} \text{ C/m}^2 \cdot 25 \cdot 10^{-6} \text{ m}^2 = 3.65 \cdot 10^{-6} \text{ C}$
 The displacement field is given by: $D = \epsilon_0 \epsilon_{\text{r,c}} \frac{U}{d-x} + \epsilon_0 \epsilon_{\text{r,air}} \frac{U}{d}$
 $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$
 As a resulting capacity, C is now a series circuit of C_{air} and C_{c} .
 Therefore: $C = \frac{1}{\frac{1}{C_{\text{air}}} + \frac{1}{C_{\text{c}}}}$

With $C_{\text{air}} = \epsilon_0 \epsilon_{\text{r,air}} \frac{A}{d}$ and $C_{\text{c}} = \epsilon_0 \epsilon_{\text{r,c}} \frac{A}{d-x}$
 $C = \frac{\epsilon_0 A}{d} \left(\frac{\epsilon_{\text{r,air}}}{1} + \frac{\epsilon_{\text{r,c}}}{\frac{d-x}{d}} \right)$

In the following, such a sensor is given with:

This leads to: $C = \epsilon_0 A \left(\frac{\epsilon_{\text{r,air}}}{d} + \frac{\epsilon_{\text{r,c}}}{d-x} \right)$

- Plate area: $A = 25 \cdot 10^{-6} \text{ m}^2$
- Distance between both plates: $d = 200 \cdot 10^{-6} \text{ m}$
- Air between the plates: $\epsilon_{\text{r,air}} = 1$
- Supply voltage: $U = 3.3 \text{ V}$
- Boundary effects on the end of the layers shall be ignored in the following calculations.

$$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$$

1. Calculate the capacity C .

Path

$$C = \epsilon_0 A \left(\frac{\epsilon_{\text{r,air}}}{d} + \frac{\epsilon_{\text{r,c}}}{d-x} \right) = 8.854 \cdot 10^{-12} \text{ F/m} \cdot 25 \cdot 10^{-6} \text{ m}^2 \cdot \left(\frac{1}{200 \cdot 10^{-6} \text{ m}} + \frac{\epsilon_{\text{r,c}}}{d-x} \right)$$

$$10^{-6} \text{ m} \} \} \end{align*}$$

Exercise E1 Capacitor

(written test, approx. 12 % of a 120-minute written test, SS2024)

3. Calculate the capacity of the capacitor, which is filled by air and a contaminant with $\epsilon_r = 2.3$. Results are applied.

The contaminant has $\epsilon_r > \epsilon_{\text{air}}$, while the distance between the plates remains the same. Give a generalized formula

$$C = f(A, d, x, \epsilon_r, \epsilon_{\text{air}})$$

$$Q = \epsilon_0 \epsilon_r \frac{A}{d} U = 1.1 \text{ pF} \cdot U$$

There are two ways now. Either: $Q = C \cdot U = 1.1 \text{ pF} \cdot U$

Or: $Q = D \cdot A = \epsilon_0 \epsilon_r \frac{A}{d} U$

The displacement field is given by: $D = \epsilon_0 \epsilon_r \frac{U}{d}$

And the resulting capacity is now $C = \epsilon_0 \epsilon_r \frac{A}{d}$

Therefore: $C = \frac{1}{\frac{1}{C_{\text{air}}} + \frac{1}{C_{\text{c}}}}$

$$\begin{aligned} C_{\text{air}} &= \epsilon_0 \epsilon_{\text{air}} \frac{A}{d} \\ C_{\text{c}} &= \epsilon_0 \epsilon_r \frac{A}{d} \end{aligned}$$

In the following such a sensor is given with:

$$C = \epsilon_0 \epsilon_r \frac{A}{d}$$

- Plate area: $A = 25 \text{ mm}^2$
- Distance between both plates: $d = 200 \text{ } \mu\text{m}$
- Air between the plates: $\epsilon_{\text{air}} = 1$
- Supply voltage: $U = 3.3 \text{ V}$
- Boundary effects on the end of the layers shall be ignored in the following calculations.

$$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$$

1. Calculate the capacity C .

Path

$$C = \epsilon_0 \epsilon_r \frac{A}{d} = 8.854 \cdot 10^{-12} \text{ F/m} \cdot 2.3 \cdot \frac{25 \cdot 10^{-6} \text{ m}^2}{200 \cdot 10^{-6} \text{ m}}$$

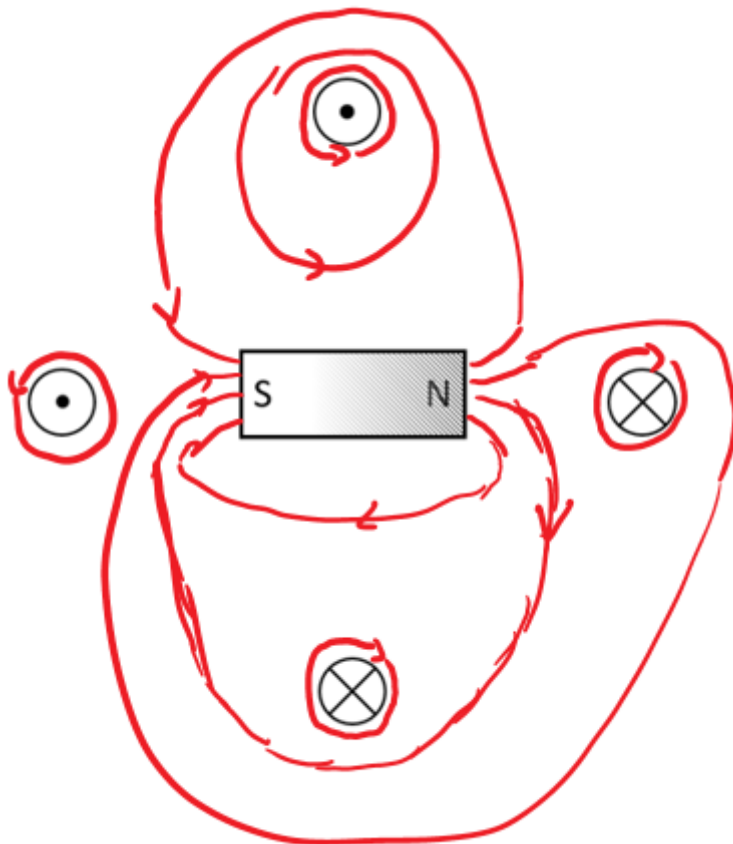
Exercise E1 Magnetic Field Lines

(written test, approx. 6 % of a 120-minute written test, SS2024)

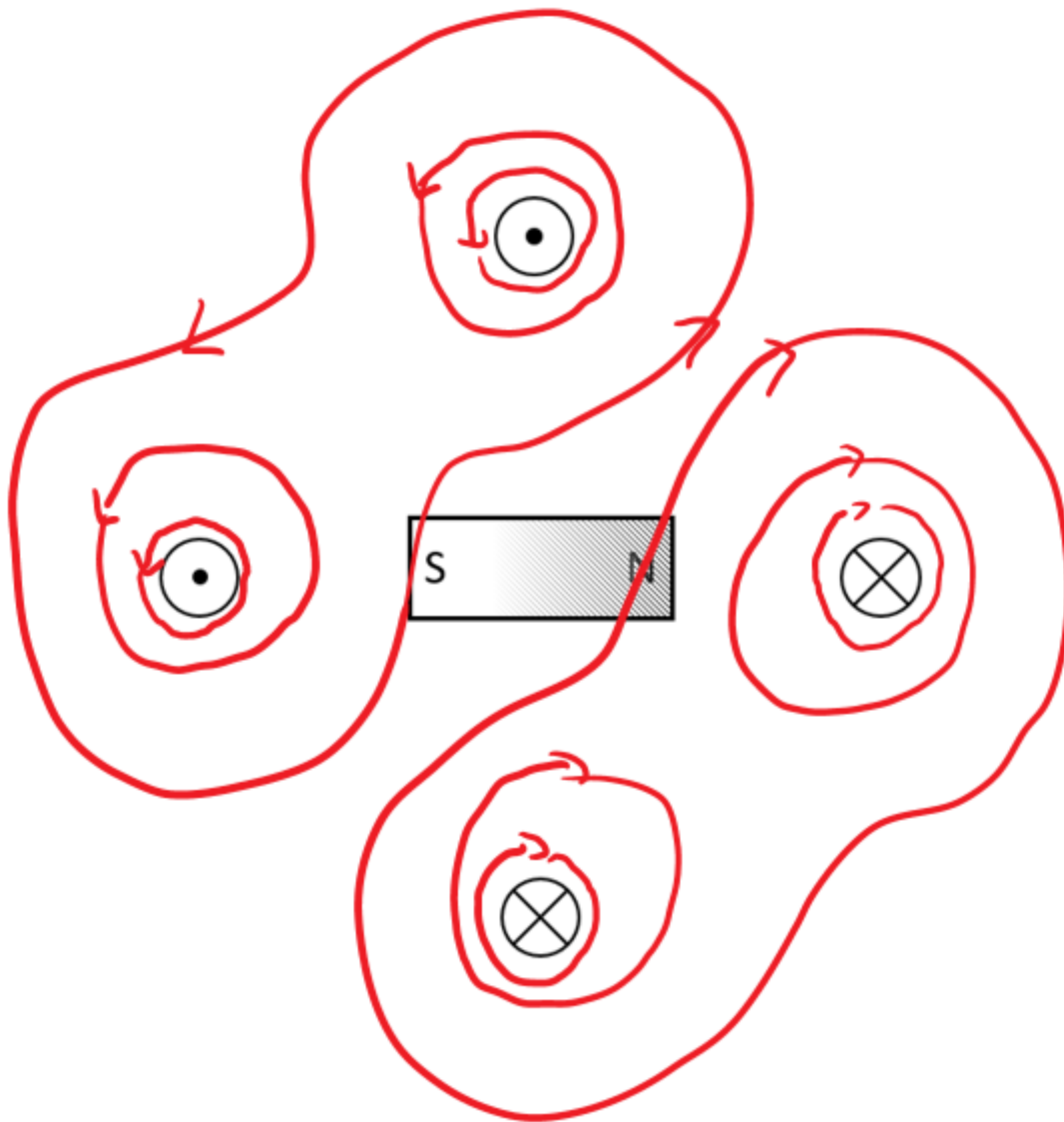
The following setup shall be given. A permanent magnet affects the H-field, based on the fundamental definition of the H-field.

- Four conductors are located perpendicular to the plane of the diagram
- Result: All of them conduct a current with the same magnitude, but not in the same direction.
- A permanent magnet is located in between the conductors.

- The H-field is defined by currents $\sum I = \int H \, ds$.
- In the permanent magnet, there are no free currents.
- The bound currents (of the permanent magnet) create also an H field.
- This exits on the north pole and enters the magnet on the south pole (similar to the B-field)
- $H = B/\mu$
- The H-field from task 1 gets distracted



st 10 field lines of the H-field
and density for the shown



Exercise E1 Magnetic Field Lines

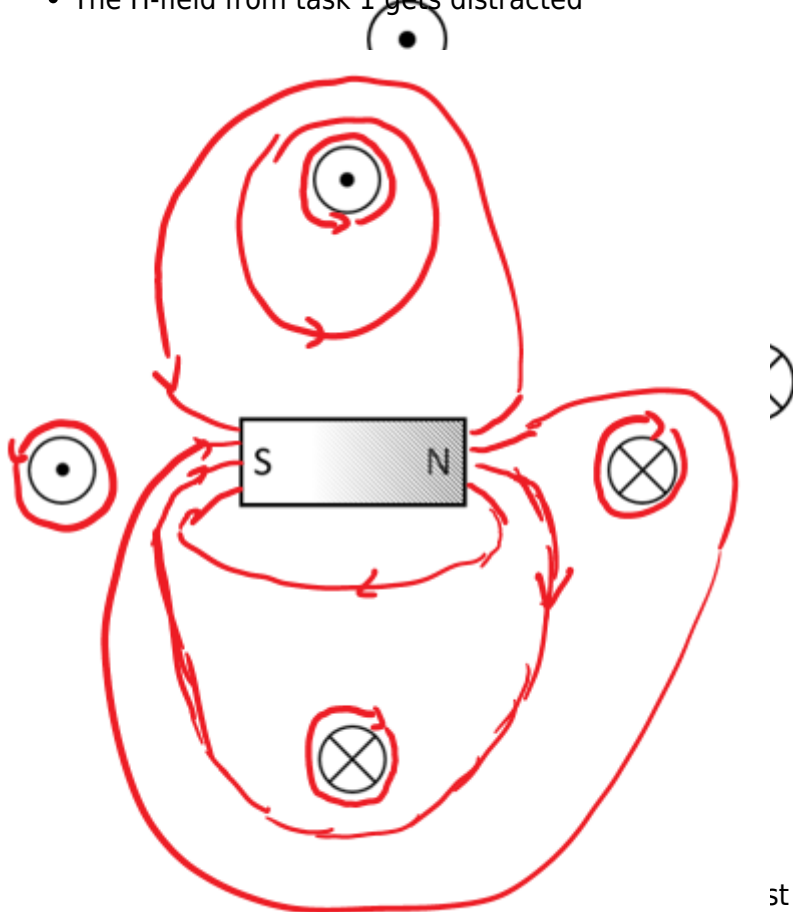
(written test, approx. 6 % of a 120-minute written test, SS2024)

The following setup is given. A permanent magnet affects the H-field, based on the fundamental definition of the H-field.

- Four conductors are located perpendicular to the plane of the diagram
- Result: All of them conduct a current with the same magnitude, but not in the same direction.
- A permanent magnet is located in between the conductors.

- The H-field is defined by currents $\sum I = \oint \vec{H} \cdot d\vec{s}$.
- In the permanent magnet, there are no free currents.
- The bound currents (of the permanent magnet) create also an H field.
- This exits on the north pole and enters the magnet on the south pole (similar to the B-field)_

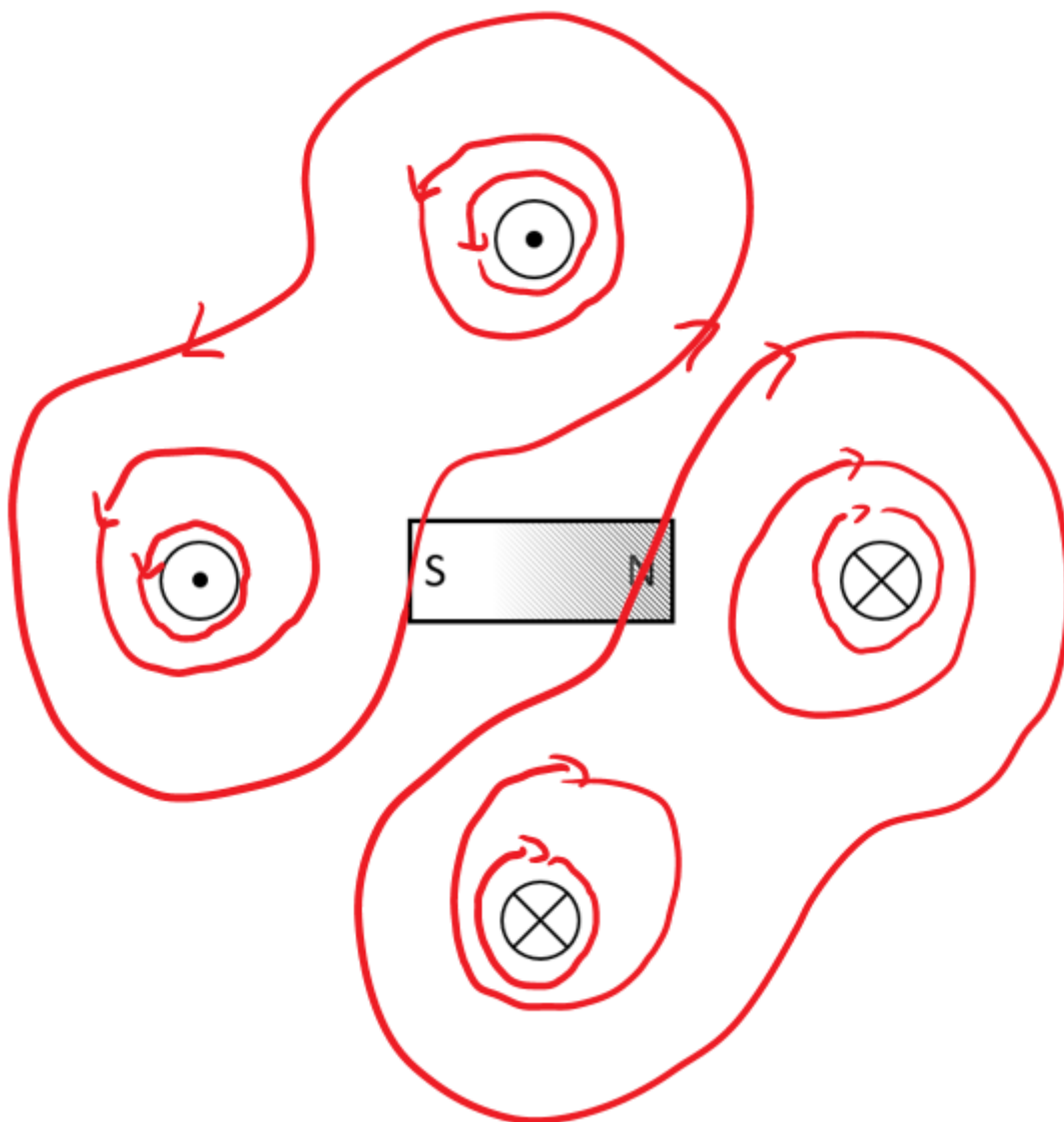
- $H = B/\mu$
- The H-field from task 1 gets distracted



... D
qua
area.

st 10 field lines of the H-field
and density for the shown

Result



Exercise E1 Fields of an coax Cable

(written test, approx. 12 % of a 120-minute written test, SS2024)

2. For the graph of the magnitude of B as a function of r (see diagram) the cross-section of a coaxial cable is shown. The inner conductor has a radius of 0.1 mm and carries a current of 3.3 mA out of the page. The outer conductor has an inner radius of 0.55 mm and carries a current of 10 nC into the page. The diagram shows the magnitude of the magnetic field B as a function of the radial distance r from the center of the cable. The field is zero for $r < 0.1 \text{ mm}$ and $r > 0.55 \text{ mm}$. The field is constant for $0.1 \text{ mm} < r < 0.55 \text{ mm}$. The field is zero for $r > 0.55 \text{ mm}$.

- Inner conductor: $+3.3 \text{ mA}$, $+10 \text{ nC}$ (current into the plane of the diagram)
- for $(0.1 \text{ mm} | 0)$: $B_{\text{inner}} = 3.28 \cdot 10^{-6} \text{ A/m}^2$
- Outer conductor: -3.3 mA , -10 nC (current out of the plane of diagram)
- for $(0.55 \text{ mm} | 0)$: $B_{\text{outer}} = 0.985 \cdot 10^{-6} \text{ A/m}^2$

The magnitude of the electric displacement field D can be calculated by: $\int D \cdot d\mathbf{A} = Q$.

In general, the B -field is proportional to $\frac{1}{r}$ for the situation here, for any position radial to the center, the surrounding area is the surface of a cylindrical shape (here for simplicity without the round endings).

For the charges Q within the surface of the conductor, the electric field E is such that $\int E \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$. This leads to: $D(x) = \frac{Q}{A}$ and $E(x) = \frac{Q}{\epsilon_0 A}$.

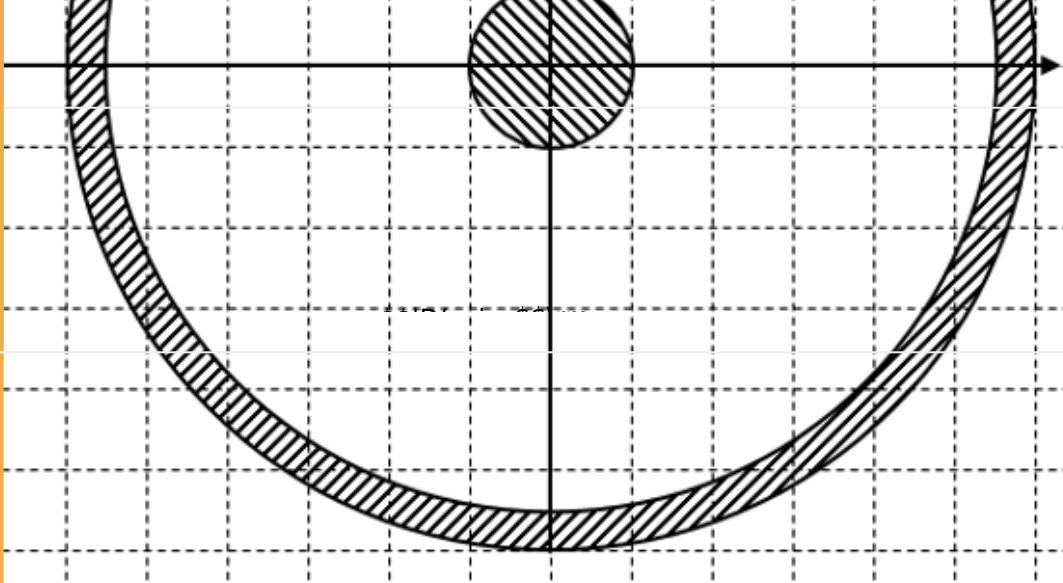
This is proportional to the area within this radius. Therefore, The formula $H = \frac{I}{2 \pi \cdot r}$ gets $H(x) = \frac{I}{2 \pi \cdot x} \cdot \pi$. So we get for $D_{\text{rm i}}$ at $\text{rm}(0.1 \text{ mm} | 0)$ and $D_{\text{rm o}}$ at $\text{rm}(0.55 \text{ mm} | 0)$. For x within the outer conductor one also gets a linear proportionality with a similar approach.

$$D_{\text{rm i}} = \frac{Q}{2 \pi \cdot r_{\text{rm i}} \cdot l} = \frac{10 \cdot 10^{-9} \text{ C}}{2 \pi \cdot \{0.1 \cdot 10^{-3} \text{ m}\} \cdot 0.5 \text{ m}}$$

$$D_{\text{rm o}} = \frac{Q}{2 \pi \cdot r_{\text{rm o}} \cdot l} = \frac{10 \cdot 10^{-9} \text{ C}}{2 \pi \cdot \{0.55 \cdot 10^{-3} \text{ m}\} \cdot 0.5 \text{ m}}$$

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the D -field is positive.

But here, again only the magnitude was questioned!



.. What is the magnitude of the magnetic field strength H at $\text{rm}(0.1 \text{ mm} | 0)$ and $\text{rm}(0.55 \text{ mm} | 0)$?

Path

The magnitude of the magnetic field strength H can be calculated by: $H = \frac{I}{2 \pi \cdot r}$

So, we get for $H_{\text{rm i}}$ at $\text{rm}(0.1 \text{ mm} | 0)$, and $H_{\text{rm o}}$ at $\text{rm}(0.55 \text{ mm} | 0)$:

$$H_{\text{rm i}} = \frac{I}{2 \pi \cdot r_{\text{rm i}}} = \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.1 \cdot 10^{-3} \text{ m}\}} \quad H_{\text{rm o}} = \frac{I}{2 \pi \cdot r_{\text{rm o}}} = \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.55 \cdot 10^{-3} \text{ m}\}}$$

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the H -field on the right side points downwards.

Therefore, the sign of the H -field is negative.

But here, only the magnitude was questioned!

Exercise E1 Fields of an coax Cable**(written test, approx. 12 % of a 120-minute written test, SS2024)**

2. With the graph of the magnitude of the electric field strength E (in V/m) as a function of the radius r (in mm) of the coax cable, the diagram shows the cross-section of the coax cable with the origin $|0\rangle$ in the center of the diagram. Use the dimensions and labels for the diagram.

- Path
- Inner conductor: $+3.3 \text{ nA}$, $+10 \text{ nC}$ (current into the plane of the diagram)
 - Outer conductor: -3.3 nA , 0 nC (current out of the plane of diagram)
 - for $(0.1 \text{ mm} | 0)$: $E_{\text{r i}} = 528 \text{ V/m}$
 - for $(0.55 \text{ mm} | 0)$: $E_{\text{r o}} = 6.985 \text{ V/m}$

The magnitude of the electric displacement field D can be calculated by: $\oint D \cdot d\mathbf{A} = Q_{\text{enc}}$.

In general, the E -field is proportional to $\frac{1}{r}$ for the situation between both conductors. Here, for any position r (at the center, the surrounding area is the surface of a cylindrical shape (here for simplicity without the round endings).

For the E -field, the area of the surface of the cylinder is $A = 2\pi r \cdot L$. This leads to: $E(r) = \frac{Q_{\text{enc}}}{2\pi r \cdot L}$. This is proportional to the area within this radius. Therefore, the formula $H = \frac{I}{2\pi r}$ is also valid for the E -field.

So, we get for $E_{\text{r i}}$ at $r = 0.1 \text{ mm}$, and $E_{\text{r o}}$ at $r = 0.55 \text{ mm}$. This leads to a formula proportional to $\frac{1}{r}$.

For r within the outer conductor one also gets a linear proportionality with a $\frac{1}{r}$ factor. $E_{\text{r i}} = \frac{Q_{\text{enc}}}{2\pi r \cdot L} = \frac{10 \cdot 10^{-9} \text{ C}}{2\pi \cdot 0.1 \cdot 10^{-3} \text{ m} \cdot 0.5 \cdot 10^{-3} \text{ m}} = 528 \text{ V/m}$. $E_{\text{r o}} = \frac{Q_{\text{enc}}}{2\pi r \cdot L} = \frac{10 \cdot 10^{-9} \text{ C}}{2\pi \cdot 0.55 \cdot 10^{-3} \text{ m} \cdot 0.5 \cdot 10^{-3} \text{ m}} = 6.985 \text{ V/m}$.

Hint: For the direction, one has to consider the sign of the enclosed charge. By this, we see that the D -field is positive.

But here, again only the magnitude was questioned!

.. What is the magnitude of the magnetic field strength H at $(0.1 \text{ mm} | 0)$ and $(0.55 \text{ mm} | 0)$?

Path

The magnitude of the magnetic field strength H can be calculated by: $H = \frac{I}{2\pi r}$

So, we get for $H_{\text{r i}}$ at $(0.1 \text{ mm} | 0)$, and $H_{\text{r o}}$ at $(0.55 \text{ mm} | 0)$.

$\sim \text{mm} \mid 0) \$:$

$$\begin{aligned} H_{\text{rm i}} &= \frac{1}{2 \pi \cdot r_{\text{rm i}}} \quad \&= \quad \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.1 \cdot 10^{-3} \sim \text{m}\}} \quad \&= \quad \frac{1}{2 \pi \cdot r_{\text{rm o}}} \quad \&= \quad \frac{+3.3 \text{ A}}{2 \pi \cdot \{0.55 \cdot 10^{-3} \sim \text{m}\}} \end{aligned}$$

Hint: For the direction, one has to consider the right-hand rule. By this, we see that the H -field on the right side points downwards.

Therefore, the sign of the H -field is negative.

But here, only the magnitude was questioned!

Exercise E1 Lorentz Force

(written test, approx. 8 % of a 120-minute written test, SS2024)

2. Describe the system of the lift train, that is to say, the magnetic levitation system. To do this, you have to draw a schematic diagram of the system. The diagram should show the resulting force vector and the magnetic field of the levitation system (see image).

Result

Path

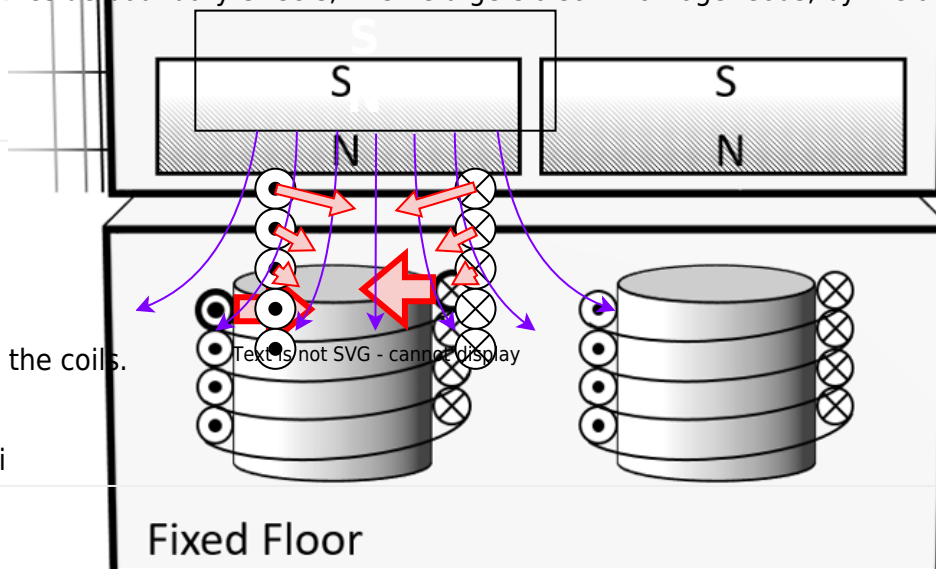
Since the result in the diagram has to be perpendicular to B -field and conductor, the force has to point to the left or the right.

For a homogeneous B -field ("constant magnetic field of the shuttle"), the Lorentz forces cancel each other out.

The Lorentz force can only have a lifting effect in an inhomogeneous field.

In this case, the sum of the forces results in a repulsing force, see image.

Beside boundary effects, The field gets also inhomogeneous, by the additional field of



- current $I = 1.6 \sim \text{A}$
- magnetic field of the shuttle is homogeneous with $B = 0.5 \sim \text{T}$

1. Calculate the magnitude of the resulting force on one coil!

Path

The Lorentz force on a conductor the length l and the current I in a B -field is

$$|\vec{F}_L| = I \cdot l \cdot B \cdot \cos(\angle \vec{B}, \vec{l})$$

$$= I \cdot (N \cdot 2\pi r) \cdot B \cdot \cos(\angle \vec{B}, \vec{l}) = 1.6 \text{ A} \cdot (500 \cdot 2\pi \cdot 40 \cdot 10^{-3} \text{ m}) \cdot 0.5 \text{ T} \cdot \cos 90^\circ$$

Exercise E2 Lorentz Force

(written test, approx. 8 % of a 120-minute written test, SS2024)

2. Do not build a system for the lift train, that is, that is, the magnetic field of the fixed floor is homogeneous, the resulting force vector is zero in the image for all sides of the shuttle (see image).

Result

Path

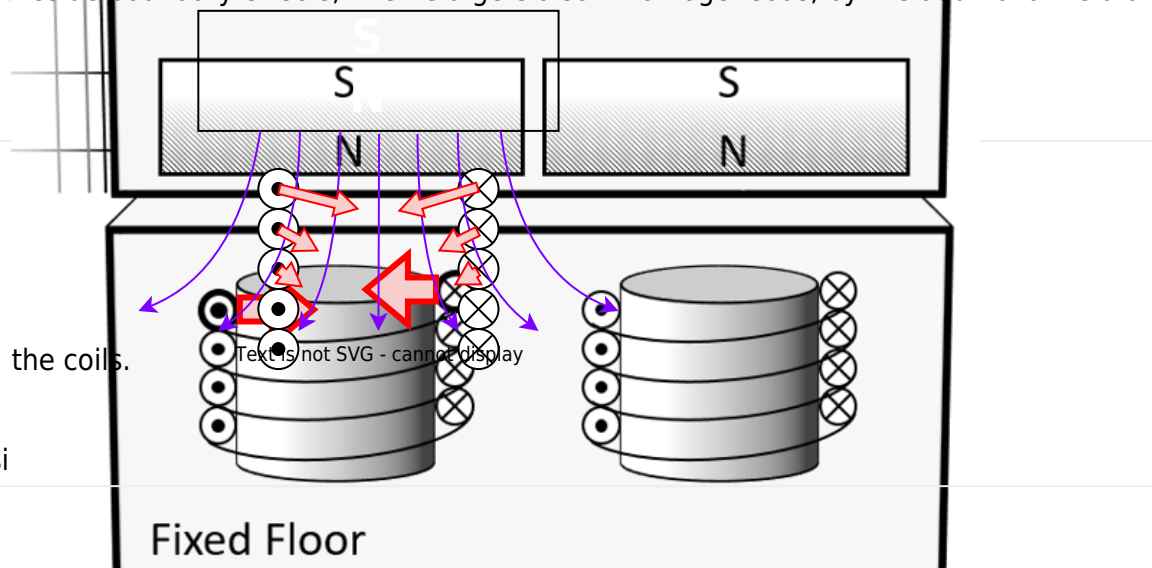
Since the shuttle is to be perpendicular to B -field and conductor, the force has to point to the left or the right.

From a homogeneous B -field ("constant magnetic field of the shuttle"), the Lorentz forces cancel each other out.

The Lorentz force can only have a lifting effect in an inhomogeneous field.

In this case, the sum of the forces results in a repulsing force, see image.

Beside boundary effects, The field gets also inhomogeneous, by the additional field of



- current $I = 1.6 \text{ A}$
- magnetic field of the shuttle is homogeneous with $B = 0.5 \text{ T}$

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Path

The Lorentz force on a conductor the length l and the current I in a B -field is

$$|\vec{F}_L| = I \cdot l \cdot B \cdot \cos(\angle \vec{B}, \vec{l})$$

$$= I \cdot (N \cdot 2\pi r) \cdot B \cdot \cos(\angle \vec{B}, \vec{l}) = 1.6 \text{ A} \cdot (500 \cdot 2\pi \cdot 40 \cdot 10^{-3} \text{ m}) \cdot 0.5 \text{ T} \cdot \cos 90^\circ$$

Exercise E1 Magnetic Potential

(written test, approx. 8 % of a 120-minute written test, SS2024)

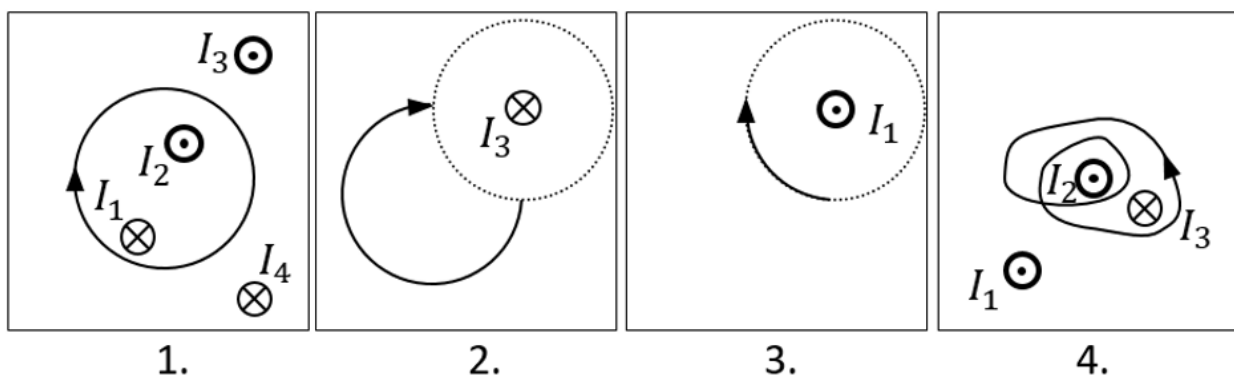
Calculate the magnetic potential difference V_{m} for the following paths as shown by the solid lines.

Dotted lines are only for there for symmetry aspects!

The wires conduct the following currents:

- $|I_1| = 2 \text{ A}$
- $|I_2| = 5 \text{ A}$
- $|I_3| = 11 \text{ A}$
- $|I_4| = 7 \text{ A}$

Pay attention to the signs of the currents (given by the diagrams) and of the results!



Result

Based on the right-hand rule and the part of a full revolution the following results:

1. Task: $+I_1 - I_2 = -3 \text{ A}$
2. Task: $+ \frac{1}{4} I_3 = 11/4 \text{ A}$ (it does not matter which way the path goes from the startpoint to the endpoint, as long as it has the same direction and number of revolutions)

3. Task: $-\frac{1}{4} I_1 = -0.5 \text{ ~\rm A}$

4. Task: $+2 I_2 - 1 I_3 = -1 \text{ ~\rm A}$

Exercise E2 Magnetic Potential

(written test, approx. 8 % of a 120-minute written test, SS2024)

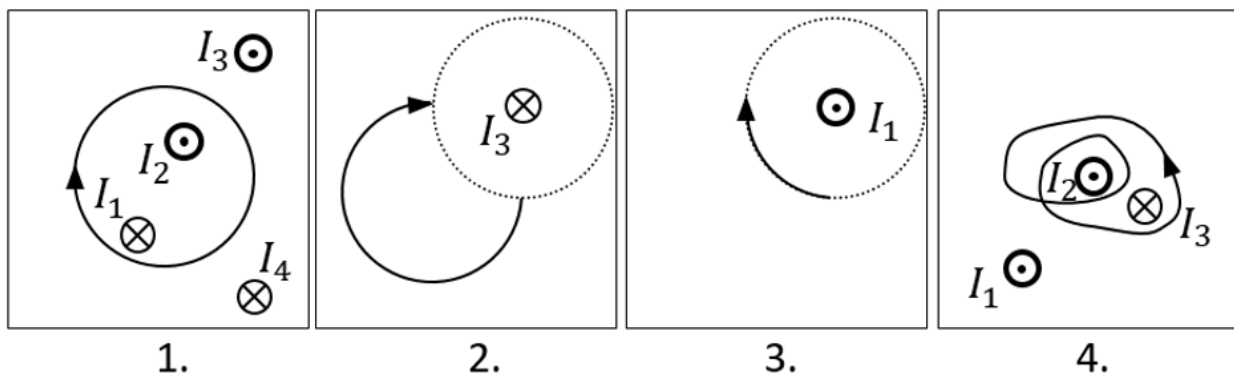
Calculate the magnetic potential difference V_{m} for the following paths as shown by the solid lines.

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3. Task: $-\frac{1}{4} I_1 = -0.5 \text{ ~\rm A}$

4. Task: $+2 \cdot I_2 - 1 \cdot I_3 = -1 \text{ A}$

Exercise E1 Self-Induction

(written test, approx. 8 % of a 120-minute written test, SS2024)

2. Determine the induced voltage in a coil with a radius of 2 cm and 500 turns. The current through the coil changes linearly from 0 A to 3 A in 0.02 ms . The arrangement is located in air ($\mu_r = 1$).
Path

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$$

$$U_{\text{ind}} = 1.32 \text{ mV}$$

.. Calculate the (self-)inductance of the coil.

For the linear change of the current the formula of the induced voltage can also be linearized:
$$u_{\text{ind}} = -L \cdot \frac{di}{dt} \quad \text{or} \quad U_{\text{ind}} = -L \cdot \frac{\Delta i}{\Delta t} \quad \text{or} \quad 1.32 \cdot 10^{-3} \text{ V} = -L \cdot \frac{3 \text{ A}}{0.02 \cdot 10^{-3} \text{ s}}$$

The formula for the induction of a long coil is:
$$L = \mu_0 \mu_r \cdot N^2 \cdot \frac{A}{l} = 4\pi \cdot 10^{-7} \text{ Vs/Am} \cdot (500)^2 \cdot \frac{\pi \cdot (2 \cdot 10^{-2} \text{ m})^2}{2 \cdot 10^{-2} \text{ m}}$$

Exercise E1 Self-Induction

(written test, approx. 8 % of a 120-minute written test, SS2024)

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Path

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$$u_{\text{ind}} = -L \cdot \frac{di}{dt} \quad \text{or} \quad U_{\text{ind}} = -L \cdot \frac{\Delta i}{\Delta t} \quad \text{or} \quad 1.32 \cdot 10^{-3} \text{ V} = -L \cdot \frac{3 \text{ A}}{0.02 \cdot 10^{-3} \text{ s}}$$

The formula for the induction of a long coil is:
$$L = \mu_0 \mu_r \frac{N^2}{l} \cdot A$$

$$= 4\pi \cdot 10^{-7} \cdot 1 \cdot \frac{(500)^2}{2 \cdot 10^{-2}} \cdot \pi \cdot (2 \cdot 10^{-2})^2$$

Exercise E3 Magnetic Circuit

(written test, approx. 9 % of a 120-minute written test, SS2024)

1. Calculate the magnetic resistance of the core with the cross-sectional area of $A = 300 \text{ mm}^2$ and an average circumference of $l = 3 \text{ dm}$.

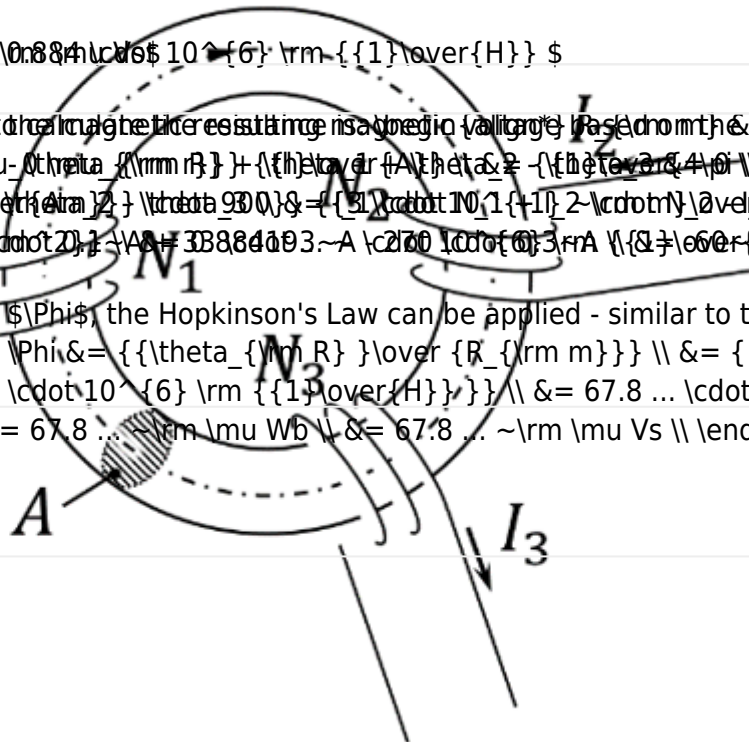
Path

l, Φ

$$R_{\text{m}} = \frac{l}{\mu_0 \mu_r N^2 A} = \frac{3 \cdot 10^{-2} \text{ m}}{4\pi \cdot 10^{-7} \text{ Vs/Am} \cdot 1 \cdot 1200^2 \cdot 300 \cdot 10^{-6} \text{ m}^2} = 0.884 \cdot 10^6 \text{ Vs/Am}$$

The formula for the magnetic resistance is a good one, based on the sources:
$$R_{\text{m}} = \frac{l}{\mu_0 \mu_r N^2 A} = \frac{3 \cdot 10^{-2} \text{ m}}{4\pi \cdot 10^{-7} \text{ Vs/Am} \cdot 1 \cdot 1200^2 \cdot 300 \cdot 10^{-6} \text{ m}^2} = 0.884 \cdot 10^6 \text{ Vs/Am}$$

To get the flux Φ , the Hopkinson's Law can be applied - similar to the Ohm's Law:
$$\Phi = \frac{NI}{R_{\text{m}}} = \frac{1200 \cdot 0.1 \text{ A}}{0.884 \cdot 10^6 \text{ Vs/Am}} = 1.35 \cdot 10^{-5} \text{ Wb}$$



On the core, there are three coils with:

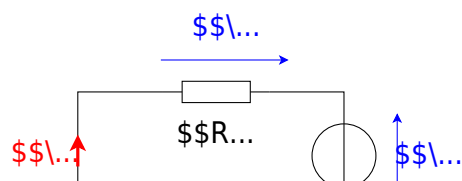
- Coil 1: $N_1 = 1200$, $I_1 = 100 \text{ mA}$
- Coil 2: $N_2 = 33$, $I_2 = 3 \text{ A}$
- Coil 3: $N_3 = 270$, $I_3 = 0.3 \text{ A}$

Refer to the drawing for the direction of the windings, current, and flux!

1. Draw the equivalent magnetic circuit that fully represents the setup. Name all the necessary magnetic resistances, fluxes, and voltages.

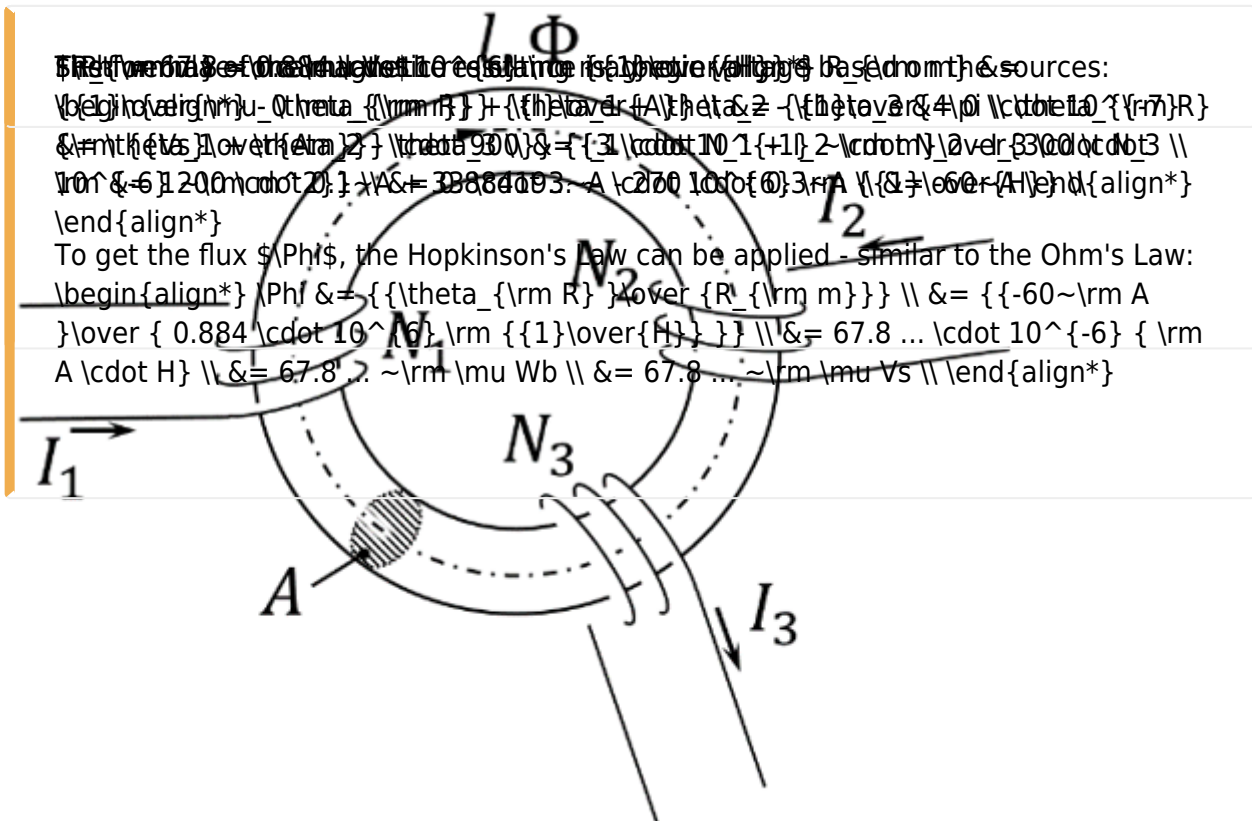
Result

- Since the material, and diameter of the core is constant, one can directly simplify the magnetic resistor into a single R_{m} .
- For the orientation of the magnetic voltages θ_1 , θ_2 , and θ_3 , the orientation of the coils and the direction of the current has to be taken into account by the right-hand rule.
- There is only one flux Φ
- The magnetic voltages are antiparallel to the flux for sources and parallel for the load.

**Exercise E4 Magnetic Circuit****(written test, approx. 9 % of a 120-minute written test, SS2024)**

1. Calculate the resulting resistance R_{m} of the core with a cross-sectional area of $A=300 \text{ mm}^2$ and an average circumference of $l=3 \text{ dm}$.

Path



On the core, there are three coils with:

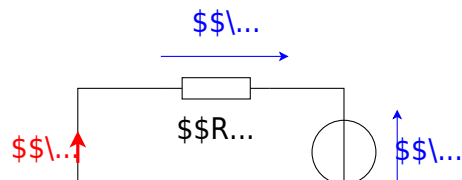
- Coil 1: $\$N_1 = 1200$, $\$I_1 = 100 \sim \text{mA}$
- Coil 2: $\$N_2 = 33$, $\$I_2 = 3 \sim \text{A}$
- Coil 3: $\$N_3 = 270$, $\$I_3 = 0.3 \sim \text{A}$

Refer to the drawing for the direction of the windings, current, and flux!

1. Draw the equivalent magnetic circuit that fully represents the setup. Name all the necessary magnetic resistances, fluxes, and voltages.

Result

- Since the material, and diameter of the core is constant, one can directly simplify the magnetic resistor into a single R_m .
- For the orientation of the magnetic voltages θ_1 , θ_2 , and θ_3 , the orientation of the coils and the direction of the current has to be taken into account by the right-hand rule.
- There is only one flux Φ
- The magnetic voltages are antiparallel to the flux for sources and parallel for the load.



Exercise E1 Magnetic Circuit

(written test, approx. 10 % of a 120-minute written test, SS2024)

2. For a sinusoidal voltage $U_C = 100 \text{ V}$ and a series combination of a resistor $R = 10 \text{ }\Omega$ and a capacitor $C = 10 \text{ nF}$, the resonance frequency f_r is determined. What is the resonance frequency f_r in Hz? What is the voltage U_R across the resistor at resonance? What is the voltage U_C across the capacitor at resonance? What is the voltage U across the series combination at resonance? What is the voltage U across the series combination at resonance? What is the voltage U across the series combination at resonance?

Path

- $U_C = 100 \text{ V}$
- $f_r = 83.0 \text{ m}\Omega$
- $Z_{RLC}(f_r) = 20.0 \text{ m}\Omega$

The formula for the resonance frequency f_r is:
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

 The voltage across the capacitor is the input voltage by the Q -factor increased:

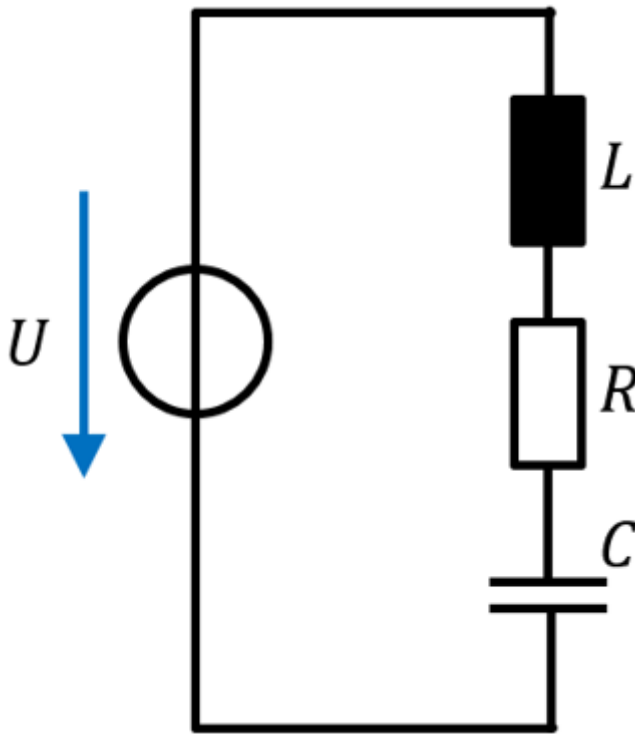
$$U_C = U \cdot Q = 100 \text{ V} \cdot 1.6 \cdot 10^{-9} \cdot 83.0 \cdot 10^{-3} \text{ }\Omega = 1.28 \text{ V}$$

 The voltage across the resistor is the input voltage by the Q -factor increased:

$$U_R = U \cdot Q = 100 \text{ V} \cdot 1.6 \cdot 10^{-9} \cdot 83.0 \cdot 10^{-3} \text{ }\Omega = 1.28 \text{ V}$$

 The voltage across the series combination is the input voltage by the Q -factor increased:

$$U = U_C = U_R = 1.28 \text{ V}$$



A given capacitor shall have the following values:

- $C = 10 \text{ nF}$
- $R = 20 \text{ m}\Omega$
- $L = 1.6 \text{ nH}$

1. What is the impedance Z_{RLC} of this real capacitor for $f_0 = 44 \text{ MHz}$? (Phase and magnitude)

Path

The impedance is based on the resistance R and the reactance $X_{\text{LC}} = \text{Im}(X_L - X_C)$:

$$\underline{Z}_{\text{RLC}} = R + \text{j}(X_L - X_C) = R + \text{j}(\omega L - \frac{1}{\omega C}) = R + \text{j}(2\pi f L - \frac{1}{2\pi f C})$$

The reactive part is

$$X_{\text{LC}} = 2\pi f L - \frac{1}{2\pi f C} = 2\pi \cdot 44 \cdot 10^6 \text{ MHz} \cdot 1.6 \cdot 10^{-9} \text{ nH} - \frac{1}{2\pi \cdot 44 \cdot 10^6 \text{ MHz} \cdot 10 \cdot 10^{-9} \text{ nF}} = +0.08062 \dots \Omega$$

To get the magnitude of the impedance $|\underline{Z}_{\text{RLC}}|$ one can use the Pythagorean Theorem:

$$|\underline{Z}_{\text{RLC}}| = \sqrt{R^2 + X_{\text{LC}}^2} = \sqrt{(0.020 \Omega)^2 + (0.08062 \dots \Omega)^2} = 0.0830 \dots \Omega$$

For the phase φ the $\arctan$ can be applied:

$$\varphi = \arctan\left(\frac{X_{\text{LC}}}{R}\right) = \arctan\left(\frac{0.08062 \dots}{0.020}\right)$$

$$\sim\Omega\over{0.020 \sim\Omega}} \right) \parallel \&= 1.3276 \dots \hat{=} +76^\circ \parallel$$

Exercise E5 Magnetic Circuit

(written test, approx. 10 % of a 120-minute written test, SS2024)

2. For a distribution, the impedance Z shall for the given circuit with an equivalent series resistance R be the equivalent series inductance L in the shown circuit?

Path

$$U_C = 100 \sim\text{V}$$

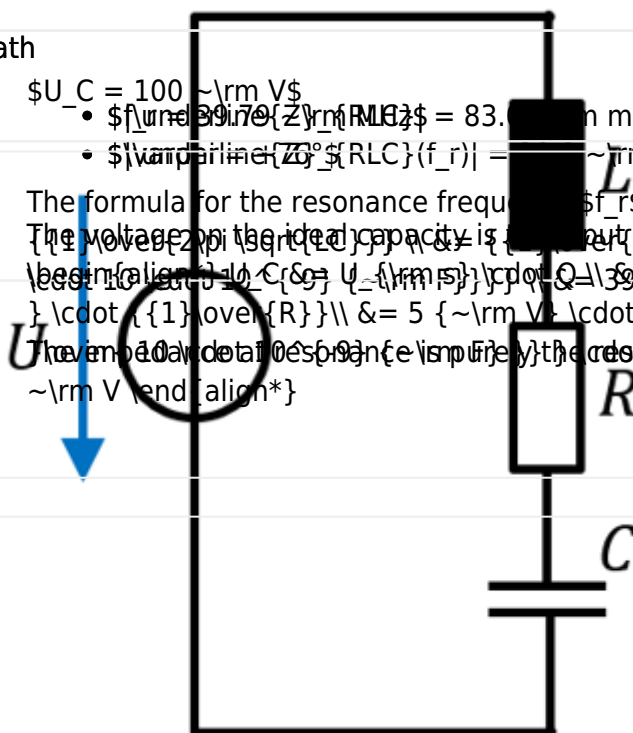
- $f_{\text{res}} = 83.3 \text{ kHz}$
- $Z_{\text{RLC}}(f_r) = \dots$

The formula for the resonance frequency f_r is:
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

The voltage across the ideal capacitor is $U_C = U \cdot \frac{1}{Q}$ and the voltage across the resistor is $U_R = U \cdot Q$.

The voltage across the resistor is $U_R = U \cdot Q = 100 \text{ V} \cdot 1.6 \cdot 10^{-9} \cdot 44 \cdot 10^6 = 704 \text{ V}$.

The voltage across the capacitor is $U_C = U \cdot \frac{1}{Q} = 100 \text{ V} \cdot \frac{1}{1.6 \cdot 10^{-9} \cdot 44 \cdot 10^6} = 15.6 \text{ V}$.



A given capacitor shall have the following values:

- $C = 10 \sim\text{nF}$
- $R = 20 \sim\text{m}\Omega$
- $L = 1.6 \sim\text{nH}$

1. What is the impedance Z_{RLC} of this real capacitor for $f_0 = 44 \sim\text{MHz}$? (Phase and magnitude)

Path

The impedance is based on the resistance R and the reactance $X_{\text{LC}} = \dots$

$$j) \cdot (X_L - X_C) = \begin{aligned} \underline{Z}_{RLC} &= R + j \cdot (X_L - X_C) \\ &= R + j \cdot (\omega L - \frac{1}{\omega C}) \\ &= R + j \cdot (2\pi f \cdot L - \frac{1}{2\pi f \cdot C}) \end{aligned}$$

The reactive part is

$$X_{LC} = 2\pi f \cdot L - \frac{1}{2\pi f \cdot C} = 2\pi \cdot 44 \cdot 10^6 \cdot 1.6 \cdot 10^{-9} - \frac{1}{2\pi \cdot 10^6 \cdot 10 \cdot 10^{-9}} = +0.08062 \dots \Omega$$

To get the magnitude of the impedance $|\underline{Z}_{RLC}|$ one can use the Pythagorean Theorem:

$$|\underline{Z}_{RLC}| = \sqrt{R^2 + X_{LC}^2} = \sqrt{(0.020 \Omega)^2 + (0.08062 \dots \Omega)^2} = 0.0830 \dots \Omega$$

For the phase φ the $\arctan$ can be applied:

$$\varphi = \arctan\left(\frac{X_{LC}}{R}\right) = \arctan\left(\frac{0.08062 \dots \Omega}{0.020 \Omega}\right) = 1.3276 \dots \hat{=} +76^\circ$$

Exercise E1 Magnetic Circuit

(written test, approx. 10 % of a 120-minute written test, SS2024)

2. Calculate and balance the apparent power, and the active power of the 400 V / 50 Hz three-phase power net. Each single string has a resistor $R=5 \Omega$ and an inductance of $L=10 \text{ mH}$.
Path

$L1$

$$|\underline{Z}_{RL}| = 5.90 \Omega$$

$L2$

$$\text{The apparent power } S \text{ is given by } S = 3 \cdot U_s \cdot I_s = 3 \cdot \frac{U_s^2}{|\underline{Z}_{RL}|} = 3 \cdot \frac{(230 \text{ V})^2}{5.90 \Omega} = 26.898 \text{ kVA}$$

$L3$

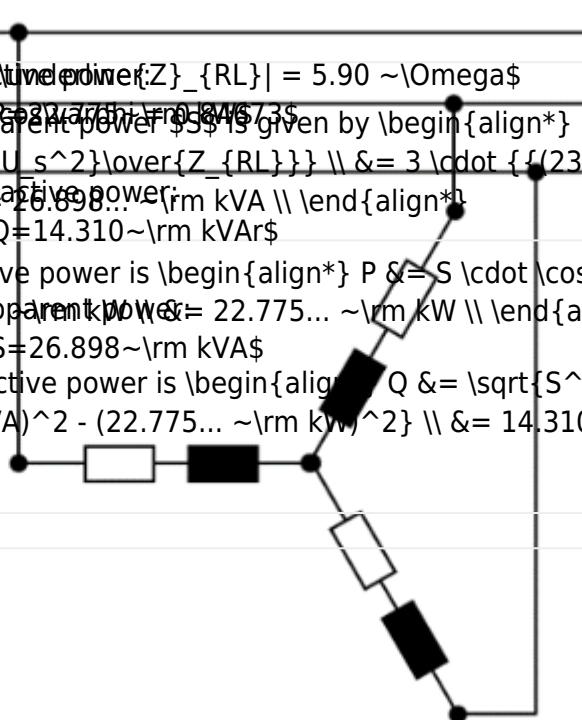
$$\text{The active power is } P = S \cdot \cos \varphi = 26.898 \dots \cdot 0.8463 = 22.775 \dots \text{ kW}$$

$$Q = 14.310 \text{ kVar}$$

$$\text{The reactive power is } Q = \sqrt{S^2 - P^2} = \sqrt{(26.898 \dots \text{ kVA})^2 - (22.775 \dots \text{ kW})^2} = 14.310 \dots \text{ kVar}$$

$$S = 26.898 \text{ kVA}$$

$$\text{The reactive power is } Q = \sqrt{S^2 - P^2} = \sqrt{(26.898 \dots \text{ kVA})^2 - (22.775 \dots \text{ kW})^2} = 14.310 \dots \text{ kVar}$$



1. Calculate the $\cos \varphi$, and the magnitude of the impedance $|Z|$ for a single string.

Path

The phase φ is given by:
$$\varphi = \arctan \left(\frac{X_L}{R} \right) = \arctan \left(\frac{2\pi \cdot f \cdot L}{R} \right) = \arctan \left(\frac{2\pi \cdot 50 \text{ Hz} \cdot 10 \cdot 10^{-3} \text{ H}}{5 \text{ } \Omega} \right) = 0.5609 \dots \hat{=} +32^\circ$$

With this, the $\cos \varphi$ becomes
$$\cos \varphi = \cos(0.5609 \dots) = 0.84673 \dots$$

The impedance is given by:
$$|Z_{RL}| = \sqrt{X_L^2 + R^2} = \sqrt{(2\pi \cdot f \cdot L)^2 + R^2} = \sqrt{(2\pi \cdot 50 \text{ Hz} \cdot 10 \cdot 10^{-3} \text{ H})^2 + (5 \text{ } \Omega)^2} = 5.905 \dots \text{ } \Omega$$

Exercise E6 Magnetic Circuit

(written test, approx. 10 % of a 120-minute written test, SS2024)

2. Calculate and balance the apparent power, and the active power of the 400 V / 50 Hz three-phase power net. Each single string has a resistor $R=5 \text{ } \Omega$ and an inductance of $L=10 \text{ mH}$.

Path

- $|Z_{RL}| = 5.90 \text{ } \Omega$

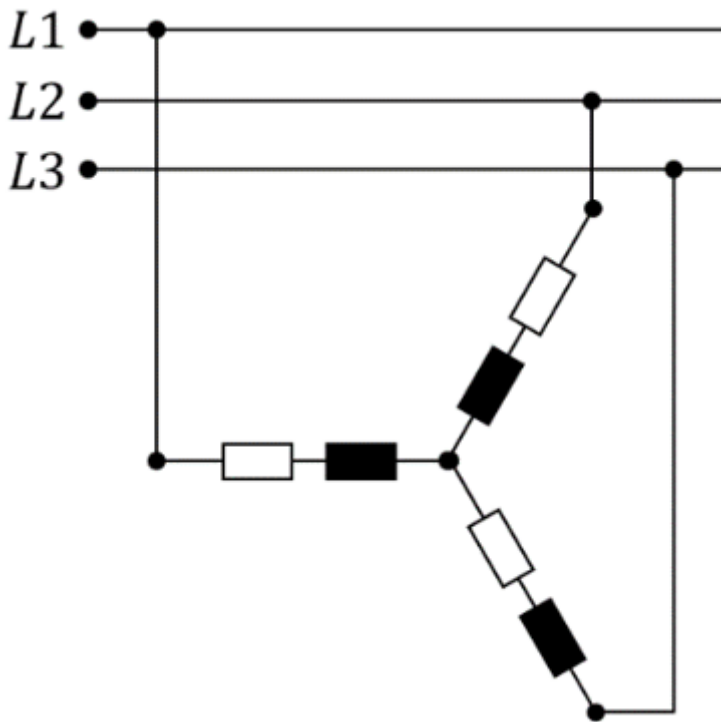
The apparent power S is given by
$$S = 3 \cdot U_s \cdot I_s = 3 \cdot \frac{U_s^2}{|Z_{RL}|} = 3 \cdot \frac{(230 \text{ V})^2}{5.90 \text{ } \Omega} = 26.898 \dots \text{ kVA}$$

- active power: $P=22.775 \text{ kW}$
- reactive power: $Q=14.310 \text{ kVar}$

The active power is
$$P = S \cdot \cos \varphi = 26.898 \dots \cdot 0.84673 \dots = 22.775 \dots \text{ kW}$$

$S=26.898 \text{ kVA}$

The reactive power is
$$Q = \sqrt{S^2 - P^2} = \sqrt{(26.898 \dots \text{ kVA})^2 - (22.775 \dots \text{ kW})^2} = 14.310 \dots \text{ kVar}$$



1. Calculate the $\cos \varphi$, and the magnitude of the impedance $|Z|$ for a single string.

Path

The phase φ is given by:
$$\varphi = \arctan \left(\frac{X_L}{R} \right) = \arctan \left(\frac{2\pi \cdot f \cdot L}{R} \right) = \arctan \left(\frac{2\pi \cdot 50 \text{ Hz} \cdot 10 \cdot 10^{-3} \text{ H}}{5 \Omega} \right) = 0.5609 \dots \hat{=} +32^\circ$$

With this, the $\cos \varphi$ becomes
$$\cos \varphi = \cos(0.5609 \dots) = 0.84673 \dots$$

The impedance is given by:
$$|Z_{RL}| = \sqrt{X_L^2 + R^2} = \sqrt{(2\pi \cdot f \cdot L)^2 + R^2} = \sqrt{(2\pi \cdot 50 \text{ Hz} \cdot 10 \cdot 10^{-3} \text{ H})^2 + (5 \Omega)^2} = 5.905 \dots \Omega$$

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